Entrance Examination Manual,

PART V.—MATHEMATICS.

CONTAINING

Questions on Arithmetic, Algebra, and Geometry set at the Entrance Examinations of the Calcutta University from 1858-1906.

WITH FULL SOLUTINOS

ENTRANCE EXAMINATION MANUAL.

PART V.—MATHEMATICS.

CALCUTTA ENTRANCE EXAMINATION PAPERS

1858.—MORNING.

Examiner,—REV T SMITH.

1. Multiply Rs 18957-13 by Rs 568-11$\frac{3}{4}$; and divide the same sum by the same sum Shew that the one of these operations is absurd and impossible, and perform the other

2 Find the value of the decimal 16854, and deduce the rule arithmetically or algebraically

3 Extract the square roots of 3 and of 3 to 7 decimal places, and explain the rule that in integers the pointing off the periods begins from the right hand and in decimals from the left

4 A plate of metal is beaten to the thickness of $\frac{1}{8}$ of an inch, and the weight of a circular medal cut from it, whose diameter is $1\frac{1}{6}$ inches, is $1\frac{1}{4}$ oz Troy If the same plate be beaten to the thickness of $\frac{1}{9}$ of an inch, what will be the weight of a medal cut out of it of the diameter of $1\frac{1}{7}$ inches (the areas of circles being proportional to the squares of their diameters)?

5 Explain the rule for the signs in algebraical multiplication and multiply

$$7x^{\frac{1}{2}}-3y^{\frac{1}{3}}+2x^{\frac{1}{3}}y^{\frac{2}{3}} \text{ by } 6x^{\frac{1}{3}}-2y^{\frac{2}{3}}+x^{\frac{2}{3}}y^{\frac{1}{3}}.$$

6 Find a fraction, such that if 1 be subtracted from its numerator the value shall be $\frac{3}{4}$, and if 6 be added to the denominator the value shall be $\frac{1}{2}$

7 A and B can do a piece of work in 30 days, A and C in 40 days, and B and C in 50 days All three work together for 10 days If then two be taken away, how long will each of the others take to finish it?

1858.—AFTERNOON.

Examiner,—REV T SMITH

1. If a straight line be bisected, and produced to any point, the sum of the squares of the whole line thus produced and the produced part, are together equal to twice the square of half the line and twice the square of the line made up of half and the part produced

2 If one of the acute angles of a right-angled triangle be double of the other, the hypotenuse is double of the shorter side

3. If any point be taken within an equilateral triangle the sum

of the perpendiculars drawn from it to the sides is equal to the perpendicular from the verte to the base

SOLUTIONS

1858 —MORNING

1 Rs 18257 13as and Rs 568 11¾as are both concrete numbers, and therefore they cannot be multiplied together $\therefore$ the first operation is absurd

Rs 568 11¾as $= 568 \times 16$as $+ 11\frac{3}{4}$as $= 9099\frac{3}{4}$as $= 36399$ ps
and Rs 18957 13as $= (18957 \times 16)$ as $+ 13$as $= 303325$as $= 1213300$ ps.

$$\therefore \text{ the quotient} = \frac{1213300 \text{ ps}}{36399 \text{ ps}} = \frac{100}{3} = 33\frac{1}{3}$$

2 $16854 = \dfrac{16854}{100000} = \dfrac{8427}{50000}$.

Arithmetically

(*a*) $$16854 = \frac{1}{10} + \frac{6}{100} + \frac{8}{1000} + \frac{5}{10000} + \frac{4}{100000}$$
$$= \frac{10000 + 6000 + 800 + 50 + 4}{100000} = \frac{16854}{100000}$$

Hence, we infer that every decimal, and every number composed of integers and decimals, can be put down in the form of a vulgar fraction, with the figures comprising the decimal or those composing the integer and decimal part (the dot being in either cases omitted) as a numerator, and with 1 followed by as many zeros as there are decimal places in the given number for the denominator

Algebraically

Let the decimal 16854 be represented by the symbol x;

then $x = 16854$, $\therefore 100000x = 16854$

$$\therefore x = \frac{16854}{100000}.$$

3 $\sqrt{(3\,00000000000000)} = 1\,7320501$.

(*a*) $\sqrt{(30000000000000)} = 5477225$.

The square root of	1	is	1
	100	is	10
	10000	is	100
	1000000	is	1000
	&c	is	&c

Hence it follows that the square root of any number between 1 and 100 must be between 1 and 10 (*i e*) will have 1 figure in its integral part, of any number between 100 and 10000, must lie between 10 and 100 (*i e*) must have 2 figures in its integral part, and so on.

Wherefore, if a point be placed over the unit's place of the number, and thence over every 2nd figure to the left of that place, the points will shew the number of figures in the integral part of the root

Again, since the square root of 01 is ·1
0001 is 01
000001 is 001
&c is &c

it appears that in extracting the square root of decimals, the decimal places must first of all be made even in number, by affixing a cypher to the right, if this be necessary, and then if points be placed over every 2nd figure to the right beginning as before from the unit's place of whole numbers, the number of such points will show the number of decimal places in the root, and for convenience' sake in decimals the pointing off the periods begins from the left hand

4 $\left.\begin{array}{ll}\frac{1}{8}\text{ in.} & \frac{1}{9}\text{ in} \\ (1\frac{1}{8})^2\text{ sq. in} & (1\frac{1}{7})^2\text{ sq in}\end{array}\right\}$ $1\frac{1}{4}$ oz x

$$\therefore x = \frac{\overset{2}{\cancel{9}} \times \overset{2}{\cancel{6}} \times \overset{2}{\cancel{6}} \times 1 \times 8 \times 8 \times 5}{7 \times 7 \times \underset{3}{\cancel{9}} \times 7 \times 7 \times \cancel{4}} \text{ oz} = \frac{2560}{2401} \text{ oz} = 1\tfrac{159}{2401} \text{ oz.}$$

5
$$7x^{\frac{1}{2}} - 3y^{\frac{1}{3}} + 2x^{\frac{1}{3}}y^{\frac{2}{3}}$$
$$6x^{\frac{1}{3}} - 2y^{\frac{2}{3}} + 7x^{\frac{2}{3}}y^{\frac{1}{3}}$$

$$42x^{\frac{5}{6}} - 14x^{\frac{1}{2}}y^{\frac{2}{3}} + 49x^{\frac{7}{6}}y^{\frac{1}{3}} - 18x^{\frac{1}{3}}y^{\frac{1}{3}} + 6y - 21x^{\frac{2}{3}}y^{\frac{2}{3}} + 12x^{\frac{2}{3}}y^{\frac{2}{3}}$$
$$-4x^{\frac{1}{3}}y^{\frac{4}{3}} + 14yx = 42x^{\frac{5}{6}} - 14x^{\frac{1}{2}}y^{\frac{2}{3}} + 49x^{\frac{7}{6}}y^{\frac{1}{3}} - 18x^{\frac{1}{3}}y^{\frac{1}{3}} + 6y$$
$$-9x^{\frac{2}{3}}y^{\frac{2}{3}} - 4x^{\frac{1}{3}}y^{\frac{4}{3}} + 14xy$$

6 Let x = the numerator and y = denominator of the fraction. Then, by the question,

$$\frac{x-1}{y} = \frac{3}{4} \quad (1) \text{ and } \frac{x}{y+6} = \frac{1}{2} \quad (2)$$

From (1) $4x - 4 = 3y$
" (2) $4x = 2y + 12$ } By substraction $-4 = y - 12$

$\therefore y = 8$ and from (2) $x = \frac{1}{2}(y+6) = 7$

$\therefore$ fraction $= \frac{7}{8}$

7. Let the work done by A in one day $= x$
" " " B " $= y$
" " " C " $= z$

Then, $x + y = \frac{1}{10}$. (1)
$x + z = \frac{1}{40}$ (2)
$y + z = \frac{1}{80}$ (3)

Adding the equations, we get

$$2(x+y+z)=\tfrac{47}{600} \text{ or } x+y+z=\tfrac{47}{1200}. \qquad (4)$$

Subtracting (1), (2) and (3) separately from (4) we have

$$z=\tfrac{7}{1200},\ y=\tfrac{17}{1200},\ x=\tfrac{23}{1200}$$

Now the work done by all in ten days $=\tfrac{47}{1200}\times 10=\tfrac{47}{120}$

Hence the work remaining to be done $=(1-\tfrac{47}{120})=\tfrac{73}{120}$

Then if B and C be taken away,

A can do the remaining work in $\tfrac{73}{120}-\tfrac{23}{1200}$

$$=\tfrac{73}{120}\times\tfrac{1200}{23}=\tfrac{730}{23}=31\tfrac{17}{23} \text{ days}$$

Similarly B can do it in $\tfrac{73}{120}\times\tfrac{1200}{17}=\tfrac{730}{17}=42\tfrac{16}{17}$ days

and C can do it in $\tfrac{73}{120}\times\tfrac{1200}{7}=\tfrac{730}{7}=104\tfrac{2}{7}$ days.

1858.—AFTERNOON.

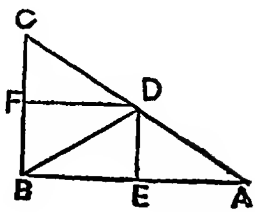
1. Euclid II 10

2. Let ABC be the $\triangle$, rt$\angle$d at B and $\angle C=2\angle A$

At the point B make the $\angle CBD=\angle ACB$

cutting AC in D (I—23)

$\because$ $\angle$ at A $+\angle$ at C $=$ one rt$\angle$,



and the $\angle DBC+\angle DBA=$ one rt$\angle$;

$\therefore$ the $\angle DBA=\angle DAB$ (Ax 3)

and $\therefore$ AD$=$BD (I 6)

Also $\because$ $\angle DBC=\angle DCB$, (*Cons*)

$\therefore$ BD$=$DC, (I 6)

$\therefore$ AD$=$BD$=$DC (Ax-1)

Again $\because$ $\angle$ at C $=2\angle$ at A (Hyp)

$\therefore$ $\angle$ at C $=\tfrac{2}{3}$ of a rt$\angle=\angle DBC$,

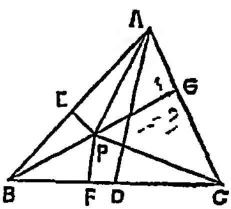
$\therefore$ the rem $\angle BDC=\tfrac{2}{3}$ of rt $\angle$, (I 32)

$\therefore$ the $\triangle DBC$ is equilateral

Wherefore BC$=$CD$=$DA or AC$=$2BC

3. Let ABC be the equilateral $\triangle$ and P the given point within it

From the given point P draw PE, PF and PG $\perp$s to AB, BC, and AC, respectively and from vertex A draw AD $\perp$ to BC



Join PA, PB, PC

To prove that AD$=$PE$+$PF$+$PG

$$\left.\begin{array}{l}\because\ \triangle BPC=\tfrac{1}{2}\ BC, PF\\ \quad \triangle APC=\tfrac{1}{2}\ AC\ PG\\ \quad \triangle APB=\tfrac{1}{2}\ AB\ PE\end{array}\right\}(I\ 41)$$

$\therefore$ $\triangle PBC+\triangle ABP+\triangle APC$ (Ax 2)

or $\triangle ABC=\tfrac{1}{2}$ BC (PE$+$PG$+$PF)

for AB$=$BC$=$AC (Def 24)

But $\triangle ABC=\tfrac{1}{2}$BC AD (I 41)

$\therefore$ $\tfrac{1}{2}$BC AD$=\tfrac{1}{2}$ BC (PF$+$PG$+$PE)

$\therefore$ AD$=$PF$+$PG$+$PE

MARCH 1859.—MORNING.

Examiner—J BURGESS, ESQ

1 What do you mean by a *prime number*, a *factor*, a *ratio*? Resolve 30 and 132 into their prime factors, and find their ratio in its simplest terms

2 How much muslin at 1 rupee 5 ans 8 pies per yard, is equal in value to 143 yards of cambric at 3 rupees 13 ans 8 pies per yard?

3 Whether is the product of $2\frac{1}{4}$ and $3\frac{1}{3}$ or the product of $2\frac{1}{3}$ and $3\frac{1}{4}$ the greater? Extract the square root of the difference

4 If a person get a bequest of $\frac{5}{8}$ of an estate of 2000 acres, and sell $\frac{2}{3}$ of his share, how many acres does he retain?

Simplify the expression $\dfrac{1}{10+\dfrac{1}{2+\frac{1}{30}}}$

5. Find by Practice the rent of 586 acres 1 rood 31 poles, at £4 1*s* $10\frac{1}{2}$*d.* per acre

6. A piece of land is 11·916 poles broad, how long must it be to contain an acre? Divide accurately 0·063 by $0{\cdot}3\dot{6}$

7. How much must be paid for £1250 stock when it sells at 208 per cent?

8. Multiply $1-x+x^2-x^3$ by $1+x+x^2+x^3$;

divide $\left(x^{\frac{2}{3}}-a^{\frac{2}{3}}\right)\left(x^{\frac{2}{3}}+a^{\frac{2}{3}}\right)$ by $x^{\frac{1}{3}}+a^{\frac{1}{3}}$;

and subtract $bcd^2-(a^2-b^2)bd$ from $(a^2+bc)d^2-(a^2-c^2)bd$.

9 Solve the following equations —

(*a*) $10\left(x+\frac{1}{2}\right)-23=6x\left(\frac{1}{x}-\frac{1}{3}\right)$

(*b*) $5+7\sqrt{\left(\frac{2}{3}x-6\right)}=19$

10 I bought 25 yards of cloth for Rs 223 8 annas, for part I paid Rs 8 8 annas a yard, and for the rest Rs 9 8 annas a yard, how many yards of each were there?

11 If m n p q, prove that $\dfrac{(m-n)(m-p)}{m}=(m+q)-(n+p)$.

12. Divide 39 into two such parts that the greater increased by 6 shall be to the less diminished by 3 as 5 to 2

13. In a right-angled triangle, the base is 8, and the sum of the hypotenuse and perpendicular is 12, it is required to find them

14. A person has two horses and a saddle worth 75 rupees; if the saddle be put on the *first* horse, his value becomes *double* that of the *second*; but if the saddle be put on the *second* horse, *his* value will not amount to that of *first* horse by 350 rupees. What is the value of each?

15 There are three numbers, such that the *sum* of the first and second divided by their *product* is $\frac{1}{2}$ the sum of the second and third divided by their product is $\frac{1}{3}$, and the sum of the first and third divided by their product is $\frac{1}{4}$

MARCH 1859 —AFTERNOON.

Examiner,—W S ATKINSON, ESQ, M A

1 Define a *line* and a *straight line* What axiom is required to complete our idea of a straight line? State the axiom which immediately results from the definition of a right angle

2 Define a rhombus, and shew that its diagonals bisect one another, and cut at right angles, what propositions do you assume in your proof?

3 The angles which one straight line makes with another on one side of it, are either two right angles or are together equal to two right angles

4 Construct a triangle of which the sides shall be equal to three given straight lines Is any limitation necessary? If so why?

5 Enunciate three propositions relating to the parallelism of two straight lines, and prove the first of them

6 In any triangle ABC, if the angles at A and B be bisected by straight lines which meet in D, shew that the line joining D and C will bisect the angle ACB

7 If the square described on one side of a triangle be equal to the sum of the squares described on the other two sides of it, the angle contained by these two sides is a right angle

8 (i) If a straight line be divided into two equal parts and also into two unequal parts, the rectangle contained by the unequal parts, together with the square of the line between the points of section, is equal to the square of half the line.

(ii) Prove this also algebraically

9 The squares of the diagonals of a parallelogram are together equal to the sum of the squares of the four sides

10 If in a circle two straight lines cut one another which do not both pass through the centre, they do not bisect each other

SOLUTION

MARCH 1859.—MORNING.

1 'A number which cannot be separated into factors, *i e.* which cannot be divided by any integer except itself and unity, is called a *prime* number

"When a number is made up by multiplying two or more numbers, each of the latter is called a *factor* of the former"

"The relation of one number to another in respect of magnitude is called a *ratio*

$$30 = 6 \times 5 = 3 \times 2 \times 5.$$
$$132 = 4 \times 33 = 2 \times 2 \times 3 \times 11.$$

$$30 : 132 = \frac{30}{132} = \frac{\cancel{3} \times \cancel{2} \times 5}{2 \times \cancel{2} \times \cancel{3} \times 11} = \frac{5}{22} \text{ or } 5 : 22.$$

2. Re 1 5as. 8 pie : Rs 3 13as 8 pie : : 143 yds : x
or 260p : 740p : : 143 yds : x

$$\therefore x = \frac{740 \times 143}{260} \text{ yds} = \frac{37 \times \cancel{2} \times \cancel{10} \times 11 \times \cancel{13}}{\cancel{13} \times \cancel{10} \times \cancel{2}} = 407 \text{ yds}$$

3. $2\frac{1}{4} \times 3\frac{1}{5} = \frac{9}{4} \times \frac{16}{5} = \frac{144}{20}.$
$2\frac{1}{5} \times 3\frac{1}{4} = \frac{11}{5} \times \frac{13}{4} = \frac{143}{20}$
$\frac{144}{20} - \frac{143}{20} = \frac{1}{20}$ ∴ 1st is greater.

The difference of the products $= \frac{1}{20} = \cdot 05$

$\sqrt{(\cdot 05000000)} = \cdot 2236 \ldots$

4 Let the share be 1, then he retains $(1 - \frac{2}{5})$ or $\frac{3}{5}$
∴ he retains $\frac{3}{5}$ of $\frac{5}{7}$ of 2000 acres $= \frac{6000}{7}$ acres $= 857\frac{1}{7}$ acres

$$\frac{1}{10 + \frac{1}{2 + \frac{1}{30}}} = \frac{1}{10 + \frac{1}{\frac{61}{30}}} = \frac{1}{10 + \frac{30}{61}} = \frac{1}{\frac{640}{61}} = \frac{61}{640}$$

	£	s	d	
5 1 ro $= \frac{1}{4}$ of 1 ac	4	1	$10\frac{1}{2}$	= rent of 1 acre
			4	
	16	7	6	= rent of 4 acres.
			10	
	163	15	0	= rent of 40 acres
			14	
	2292	10	0	= rent of 560 acres.
	53	4	$4\frac{1}{2}$	= rent of 13 acres
	53	4	$4\frac{1}{2}$	= rent of 13 acres
	1	0	$5\frac{5}{8}$	= rent of 1 rood
20 po $= \frac{1}{2}$ of 1 ro		10	$2\frac{13}{16}$	= rent of 20 poles
10 po $= \frac{1}{2}$ of 20 po		5	$1\frac{13}{32}$	= rent of 10 poles
1 po. $= \frac{1}{10}$ of 10 po.			$6\frac{9}{64}$	= rent of 1 pole.
	£2400	15	$0\frac{3}{4}d$ $\frac{15}{16}q$	= rent of 586 ac. 1 ro 31 po

6. An acre = 160 sq poles

$$\therefore \text{Length} = \frac{160}{11 \cdot 916} \text{ poles} = 13 \cdot 4273 \text{ poles}$$

$$(a) \quad \frac{\cdot 063}{\cdot 3\dot{6}} = \frac{63}{1000} \times \frac{99}{36} = \frac{693}{4000} = \cdot 17325$$

7 £100 stock : £1250 stock : : £108 : x

$$\therefore x = £\frac{108 \times 1250}{100} = £1350.$$

8 Prod $=\{(1+x^2-(x+x^3)\}\{(1+x^2)+(x+x^3)\}$
$=(1+x^2)^2-(x+x^3)^2=(1+2x^2+x^4)-(x^2+2x^4+x^6)$
$=1+x^2-x^4-x^6$

Dividend $=(x^{\frac{1}{3}}+a^{\frac{1}{3}})(x^{\frac{1}{3}}-a^{\frac{1}{3}})(x^{\frac{2}{3}}+a^{\frac{2}{3}})$

$\therefore$ Quotient $=(x^{\frac{1}{3}}-a^{\frac{1}{3}})(x^{\frac{2}{3}}+a^{\frac{2}{3}})$
$=x-x^{\frac{2}{3}}a^{\frac{1}{3}}-a^{\frac{2}{3}}x^{\frac{1}{3}}-a$

$$\begin{array}{l} a^2d^2+bcd^2-a^2bd+c^2bd \\ \qquad bcd^2-a^2bd+b^3d \\ \hline a^2d^2+c^2bd-b^3d \end{array}$$

9 (a) $10(x+\frac{1}{2})-23=6x\left(\frac{1}{x}-\frac{1}{3}\right)$
or $10x+5-23=6-2x$, $\therefore 12x=24$, $\therefore x=2$.

(b) $5+7\sqrt{(\frac{2}{3}x-6)}=19$
$\therefore 7\sqrt{(\frac{2}{3}x-6)}=14$ or $\sqrt{(\frac{2}{3}x-6)}=2$
or $\frac{2}{3}x-6=4$, $\therefore \frac{2}{3}x=10$, $\therefore x=15$

10 Let x = no of yds. bought at Rs. 8 8as. per yd
Then $25-x$ = ...Rs. 9 8as......
By the question,
$8\frac{1}{2}x+9\frac{1}{2}(25-x)=223\frac{1}{2}$ or $17x+475-19x=447$,
$\therefore 2x=28$, $\therefore x=14$, $25-x=25-14=11$.
Hence 14 and 11 are nos of yds bought

11 $\frac{m}{n}=\frac{p}{q}$.

$\frac{m-n}{m}=\left(\frac{p-q}{p}\right)$ (1)

$=\frac{m-n-(p-q)}{m-p}$

$\therefore \frac{(m-n)(m-p)}{m}=(m+q)-(n+p)$

12 Let x = the greater part
then $39-x$ = the less part
By the question,
$x+6$ $36-x$ 5 2
$\therefore 2x+12=180-5x$, $\therefore 7x=168$, $\therefore x=24$
Hence the parts are 24 and 15

13 Let x = the hypotenuse,
then $12-x$ = the perpendicular.
Then by Euc. I 47,
$8^2+(12-x)^2=x^2$, or $64+144-24x+x^2=x^2$,
$\therefore 24x=208$, $\therefore x=8\frac{2}{3}$
Hence Hyp $=8\frac{2}{3}$ and Perp $=3\frac{1}{3}$.

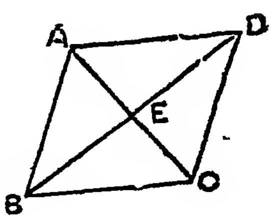
14. Let x = the value of the first horse in Rs
and y = the value of the second horse in Rs.
By the question,
$x+75=2y$ . . (1)
$x-350=y+75$ (2)
By subtraction, $425=y-75$, $\therefore y=$ Rs 500
and $x=2y-75=$ Rs. 925.

15 Let x, y and z be the numbers
By the question,
$\frac{x+y}{xy}=\frac{1}{2}$. (1) From (1) $\frac{1}{y}+\frac{1}{x}=\frac{1}{2}$
$\frac{y+z}{yz}=\frac{1}{3}$ ....(2) " (2) $\frac{1}{z}+\frac{1}{y}=\frac{1}{3}$
$\frac{x+z}{xz}=\frac{1}{4}$.... .(3) " (3) $\frac{1}{z}+\frac{1}{x}=\frac{1}{4}$
Adding the equations $2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=\frac{13}{12}$
or $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{13}{24}$ .... (4)
Subtracting (1), (2) and (3) separately from (4), we have
$\frac{1}{z}=\frac{1}{24}$, $\frac{1}{x}=\frac{5}{24}$ and $\frac{1}{y}=\frac{7}{24}$.
Hence $x=4\frac{4}{5}$, $y=3\frac{3}{7}$, and $z=24$

MARCH 1859.—AFTERNOON.

1. Euc I, Def. 4, Ax. 10, Ax. 11.
2 Euc I, Def. 34.

(a) Let ABCD be a rhombus and let the diagonals AC and BD intersect in E
$\because$ AB=AD and BC=DC and AC com.
$\therefore \angle BAC=\angle DAC$ (I. 8)
And $\because$ BA=AD, and AE com
and $\angle BAE=\angle DAE$
$\therefore$ BE=DE and $\angle AEB=\angle AED$ (I 4)
and they are adjacent angles
$\therefore$ each = a rt$\angle$ (Def 10)
Likewise AE=EC.



∴ the diagonals bisect one another at right angles

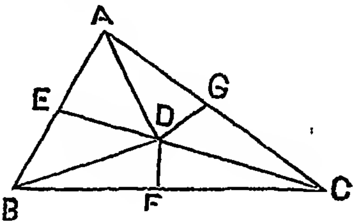
Prop I 4 and I 8 are assumed to prove this proposition.

3 Euclid I. 13

4 Euclid I 22 "But any two whatever of these must be greater than the third,' else the circles used in the construction would not intersect one another and so the construction fails

5 Book I Propositions 27 and 28 and prove proposition 27 of Euclid Book I

6 From D draw DE, DF and DG ⊥rs to AB, BC and CA respectively



∵ $\angle DGA = \angle DEA$ (Ax 11)
and $\angle DAG = \angle DAE$ also DA com
(I 26) ∴ $DG = DE$
Likewise $DE = DF$
∴ $DG = DF$ (Ax 1)
Again ∵ $DG^2 + GC^2 = CD^2$ (I 47)
$= CF^2 + DF^2$ (I 47)
∴ $GC^2 = FC^2$ and $GC = FC$
Again ∵ $GC = FC$ and CD com
and $DG = DF$
∴ $\angle GCD = \angle FCD$ (I 8)

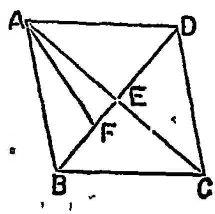
7. Euclid I 48.

8 (1) Euclid II 5

(2) Let AB contain $2a$ linear units, then $BC = a$, also let $CD = m$ then $AD = a + m$ and $DB = a - m$ Now $(a-m)(a+m) = a^2 - m^2$, to each of these equals and m^2

∴ $(a+m)(a-m) + m^2 = a^2$

9 Let ABCD be a ▭ and AC, BD its diagonals intersecting in E In the two triangles AED and BEC



∵ $\begin{cases} \angle ADE = \angle EBC \text{ (I 29)} \\ \text{and } \angle DAE = \angle ACB \text{ (I 29)} \end{cases}$
and $AD = BC$ ∴ $AE = EC$ and $BE = ED$ (I 26).
From A draw AF ⊥ BD
(I. 47) ∵ $AD^2 + AB^2 = 2AE^2 + 2BE^2$
$= 2AE^2 + 2ED^2$
(∵ the sum of the squares on two sides of a triangle is equal to twice the square on half the third side and twice the square on the median which bisects the third side)

Likewise $DC^2 + BC^2 = 2EC^2 + 2ED^2$
∴ $AB^2 + BC^2 + CD^2 + DA^2 = 2AE^2 + 2EC^2 + 4ED^2$
$= 4AE^2 + 4ED^2 = AC^2 + BD^2$ (II. 4 *Cor*)

10 Euclid III 4

$\therefore \frac{2n-4}{n} = \frac{9}{10}$ of 2 rt. $\angle$s $=\frac{9}{5}$

$\therefore 10n-20=9n$, $\therefore n=20$,

or each exterior angle is $\frac{1}{5}$ of a rt angle

But all the exterior angles are 4 rt angles

$\therefore$ no. of angles is $4 \div \frac{1}{5}=20$

$\therefore$ no of sides is 20

7 In prop 15, Book IV AB=AG

1878.—MORNING.

Examiners, { MR W BOOTH, B A / MR M MOWAT, M A }

1. Calculate to three places of decimals the value of $\frac{180 \times 86}{3{\cdot}14159}$

2 Calculate to 5 places of decimals the square root of $1+({\cdot}067)^3$

3. Reduce 483 Rs 11as 6p to the decimals of 1,290 Rs 1as 4p.

4. Give the simple interest on 757 Rs 4as 3p. for 343 days at $3\frac{1}{2}$ per cent per annum

5. Add together $\frac{1}{120}, \frac{1}{5040}, \frac{1}{40320}, \frac{1}{720}$

Express your answer as a decimal

6. Find, by Practice, the value of 99cwt 3qrs 27lbs. at £5 2s. 8d per cwt

7. Divide—

$$x(1+y^2)(1+z^2)+y(1+z^2)(1+x^2)+z(1+x^2)(1+y^2)+4xyz$$
by $1+xy+yz+zx$

8. Extract the square root of

$$(a^2+b^2+c^2)(x^2+y^2+z^2)-(bz-cy)^2-(cx-az)^2-(ay-bx)^2.$$

9. If $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$ find the value of

$$(b-c)x+(c-a)y+(a-b)z$$

10. Solve the equations

(a) $\sqrt{4x^2+20x+17}-\sqrt{16x^2+11x+10}=2(x+2)$

(b) $\frac{4x+2}{9}+\frac{13x}{108}=\frac{8x+19}{9}$.

11 Simplify the expression.

$$(4x^3-3x)^2-\left\{\frac{\frac{3\sqrt{(1-x^2)}}{x}-\frac{(1-x^2)^{\frac{3}{2}}}{x^3}}{1-3\left(\frac{1-x^2}{x^2}\right)}\right\}^2$$

1878.—AFTERNOON.

Examiners,—{Rev, J. P. Ashton, M A. / Mr F J Biden, M.A.

1. (*a*) Explain the meaning of each of the following —definition, axiom, corollary, each to each

(*b*) In representing a point, a line or a superficies, what modification must be made in Euclid's definitions? Explain why such strict definitions are necessary

(*c*) Define a rectangle, and show by numerical examples that for a given perimeter a square has a greater area than any other four-sided figure.

(*d*) What figure is by definition both a sector and segment of a circle?

(*e*) Distinguish between a problem and a theorem.

2 Give the hypotheses and conclusions of IV, V and XXVI, Bk. I Prove in Prop. IV that the bases are equal, and enunciate Prop. XXVI

3. Prove that the difference of the squares on two unequal lines is equal to the rectangle contained by their sum and difference.

4. The straight line which joins the centres of two circles which touch one another internally will, if produced, pass through their point of contact

5 Describe an isosceles triangle, having each of the angles at the base double of the third angle How many degrees are there in the "third angle" and of what regular figure inscribed in a circle is the "the base," a side.

6. (*a*) Two sides of a triangle are 9 and 12 feet respectively, the angle contained by them is equal to the other two, find the length of the third side

(*b*) Show how to describe a circle touching one side of a triangle and the other two produced.

1878 —MORNING.

SOLUTIONS.

1 $\frac{180\times36}{3\ 14159}=\frac{6480}{3\ 14159}$

$6480\ 00000000-3{\cdot}14159=2062\ 649.\quad\ldots$

2. $1+(067)^3=1\ 000300763$

$\sqrt{(1{\cdot}0003007630}=1{\cdot}00015\ .$

3 Rs 483 11as 6 pie$=$92874 pie

and Rs 1290 1a 4 pie = 247696 pie

$\therefore$ fraction reqd $= \dfrac{92874}{247696} = .375$ . ...

4. The int $= \dfrac{\text{Rs } 757 \text{ 4as } 3 \text{ pie} \times 313 \times 7}{100 \times 365 \times 2}$

$= \text{Rs } \dfrac{48465 \times 7 \times 343}{64 \times 2 \times 100 \times 365} = \text{Rs } \dfrac{23272893}{934400}$

$= \text{Rs } 24 \text{ 14as } 6\frac{1470}{14600} \text{ pie}$

5 $\dfrac{1}{120} + \dfrac{1}{5040} + \dfrac{1}{40320} + \dfrac{1}{720} = \dfrac{336 + 8 + 1 + 56}{40320}$

$\dfrac{401}{40320} = .009945436507\dot{9}$

6.

	£	s.	d	
	5	2	8	= cost of 1 cwt.
			9	
	46	4	0	= cost of 9 cwts
			11	
	508	4	0	= cost of 99 cwts
2 qrs = $\frac{1}{2}$ of 1 cwt	2	11	4	= cost of 2 qrs
1 qr = $\frac{1}{2}$ of 2 qrs	1	5	8	= cost of 1 qr
14 lbs = $\frac{1}{2}$ of 1 qr		12	10	= cost of 14 lbs
4 lbs = $\frac{1}{7}$ of 1 qr		3	8	= cost of 4 lbs
4 lbs = $\frac{1}{7}$ of 1 qr.		3	8	= cost of 4 lbs.
4 lbs = $\frac{1}{7}$ of 1 qr		3	8	= cost of 4 lb
1 lb = $\frac{1}{4}$ of 4 lbs			11	= cost of 1 lb
	£513	5s	9	= cost of 99 cwts 3 qrs. 27 lbs.

7 The dividend $= (x + xy^2)(1 + z^2) + (y + yz^2)(1 + x^2) + (z + zx^2)$
$(1 + y^2) + 4xyz$

$= x + xy^2 + xz^2 + xy^2z^2 + y + yz^2 + x^2y + x^2yz^2 + z + zx^2$
$+ zy^2 + y^2zx^2 + xyz + xyz + xyz + xyz$

$= x(1 + xy + yz + zx) + y(1 + xy + yz + zx) + z(1 + xy + yz + zx)$
$+ xyz(1 + yx + yz + xz)$

$= (x + y + z + xyz)(1 + xy + yz + xz)$

$\therefore$ the quotient $= x + y + z + xyz$

8 The expression

$= a^2x^2 + b^2x^2 + c^2x^2 + a^2y^2 + b^2y^2 + c^2y^2 + a^2z^2 + b^2z^2 + c^2z^2$
$- (b^2z^2 + c^2y^2 + c^2x^2 + a^2z^2 + a^2y^2 + b^2x^2)$
$+ 2(bcyz + acxz + abxy)$

$= a^2x^2 + b^2y^2 + c^2z^2 + 2(bcyz + acxz + abxy)$

$= (ax + by + cz)^2$

$\therefore$ square root $= ax + by + cz$

9 Let $\dfrac{x}{b+c-a} = \dfrac{y}{c+a-b} = \dfrac{z}{a+b-c} = k.$

Then $x = (b+c-a)k$, $y = (c+a-b)k$, $z = (a+b-c)k$

$\therefore$ Ans $= (b-c)(b+c-a)k + (a-b)(a+b-c)k$
$\quad + (c-a)(c+a-b)k$

$= k\{b^2 - c^2 - a(b-c) + c^2 - a^2 - b(c-a) + a^2 - b^2 - c(a-b)\}$

$= k \times 0 = 0$

10 $\sqrt{\{4x^2 + 20x + 17 - \sqrt{(16x^2 + 11x + 10)}\}} = 2(x+2)$

sq $4x^2 + 20x + 17 - \sqrt{(16x^2 + 11x + 10)} = 4(x^2 + 4x + 4)$

$\therefore \sqrt{(16x^2 + 11x + 10)} = 4x + 1$

sq. $16x^2 + 11x + 10 = 1 + 16x^2 + 8x$

$\therefore 11x - 8x = 1 - 10 \qquad \therefore 3x = -9 \qquad \therefore x = -3$

(b) $\dfrac{4x+3}{9} + \dfrac{13x}{108} = \dfrac{8x+19}{18}$

clearing of fractions,

$48x + 36 + 13x = 48x + 114$

$\therefore 61x - 48x = 114 - 36 \qquad \therefore 13x = 78 \qquad \therefore x = 6.$

11 The expression,

$= \dfrac{1}{(4x^3 - 3x)^2} - \left\{\dfrac{\sqrt{(1-x^2)}\{3x^2 - (1-x^2)\}}{x^3} \times \dfrac{x^2}{x^2 - 3 + 3x^2}\right\}^2$

$= \dfrac{1}{(4x^3 - 3x)^2} - \left\{\dfrac{(1-x^2)(4x^2-1)^2}{(4x^3-3x)^2}\right\}$

$= \dfrac{1 - (1-x^2)(4x^2-1)^2}{(4x^3-3x)^2} = \dfrac{1 - (1-x^2)(16x^4 - 8x^2 + 1)}{(4x^3 - 3x)^2}$

$= \dfrac{1 - 16x^4 + 8x^2 - 1 + x^2(16x^4 - 8x^2 + 1)}{(4x^3 - 3x)^2}$

$= \dfrac{x^2(16x^4 - 24x^2 + 9)}{(4x^3 - 3x)^2} = \dfrac{x^2(4x^2-3)^2}{x^2(4x^2-3)^2} = 1$

1878.—AFTERNOON.

1 (a) A definition is a statement which distinguishes the thing defined from every other of the same kind.

An axiom is a selfevident truth which cannot be rendered more evident by demonstration.

A corollary is a theorem which is deduced from the demonstration of a proposition.

If of two series of equal magnitudes, the first of the former is equal to the first of the latter, the second of the former to the second of the latter and so on, then they are said to be equal, *each to each.*

(*b*) In representing a point it is supposed to have *size* as well as *position*, a line, to have breadth or thickness, as well as *length*, *a surface*, to have thickness, as length and breadth

(*c*) Euclid II Def 1 Let 8 and 12 represent the two adjacent sides of a rectangle then its perimeter=40, and hence the side of a square of equal perimeter=10 Now since 100 > 96, ∴ area of the square > the area of the rectangle

(*d*) A semi circle

(*e*) A problem is a proposition when some geometrical construction is required to be effected, and a theorem is a proposition when some geometrical property is to be demonstrated.

3 See Question 5 of 1870

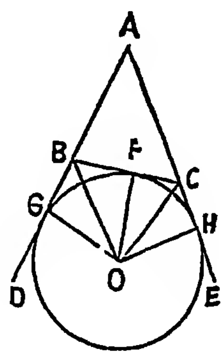
4. Euclid III. 11.

5 Euclid IV 10

Let x=the no of degrees in the third angle, and $2x$=the no of degrees in each of the angles at the base The $x+2x+2x=2$ rt ∠s, ∴ $5x=2$ rt. ∠s, ∴ x=one fifth of 2 rt ∠s The base is the side of a regular decagon inscribed in the ⊙

6 (*a*) The angle contained by the sides is a rt ∠ (I 32)
∴ Hyp $=\sqrt{(9^2+12^2)}=\sqrt{(81+144)}$
$=\sqrt{(225)}=15$ ft

(*b*) Let ABC be the △
Prod AB, AC to D, E.
Bisect the ∠CBD and ∠BCE by BO, and CO meeting in O
Draw OF⊥BC and OG, OH to AB, AC prod
Then OG, OF, OH are equal (I 26)
∴ the ⊙ desc with O as centre and with OF as rad is the reqd ⊙



1879.—MORNING.

Examiners,—{Mr S F Downing B A.
Mr F. J. Biden, M.A

1 What is the local value of each of the figures composing the number 456 654?

2 Rs. 49 was divided amongst 150 children, each girl had as. 8, and each boy as 4, how many boys were there?

3. Simplify—

(a) $8-8\times\dfrac{2\frac{1}{3}-1\frac{3}{7}}{2-\dfrac{1}{6-\frac{1}{8}}}$

(b) $\frac{1}{5}+\frac{1}{6}\div\frac{1}{7}-\frac{1}{7}\times\frac{1}{8}-\frac{1}{8}$

(c) $15\dot{9}\dot{0}\times 172-2\dot{7}$

(d) What decimal of £4 3s. 4d is $\frac{1}{1300}$ of £5 8s 4d ?

4 A tank 75 yards long, 50 yards broad, and 11 ft. deep, is full of water; how many times can each of 16 water carts, length 5 ft, breadth 4 ft. and depth 27 inches, be filled from the tank before the water in it falls 6 inches?

5 If 17 men can build a wall 100 yards long, 12 ft. high, and 2½ ft thick in 25 days, how many will build a wall twice the size in ½ the time?

6 Find the change of income when a person transfers £2616 5s from 5 per cents. at 95⅛ to 4 cents at 83 Brokerage as usual

7. In a game of skill A can gain give B, and B can give C, 10 points out of a game of 50. How many should A give C?

8. Multiply $a^{2n}-a^{n}x^{n}+x^{2n}$ by $a^{n}+x^{n}$ and find the greatest common measure of

$$x^2+\tfrac{7}{8}x+\tfrac{1}{8} \text{ and } x^2+\tfrac{3}{4}x+\tfrac{1}{12}$$

9 Divide $x^{2^n}-y^{2^n}$ by $x^{2^{n-1}}+y^{2^{n-1}}$, and

Simplify $\dfrac{x-y}{x-z}+\dfrac{x-z}{x-y}-\dfrac{(y-z)^2}{(x-y)(x-z)}$.

10 Solve the equations—

(a) $x-k+\sqrt{k^2+x^2}=m$.

(b) $\begin{cases} a^{x}a^{y}+1=a^{7} \\ a^{2y}a^{3x}+5=a^{21} \end{cases}$

(c) $\begin{cases} \dfrac{4}{x}+\dfrac{10}{y}=2 \\ \dfrac{3}{x}+\dfrac{2}{y}=\dfrac{19}{20}. \end{cases}$

11 If $\dfrac{a}{b}=\dfrac{c}{d}$ prove that $\dfrac{a^2+b^2}{a^2-b^2}=\dfrac{ac+bd}{ac-bd}$

Two armies number 11,000 and 7,000 men respectively, before they fight each is re- inforced by 1,000 men in favour of which army is the increase?

12 From two towns 261 miles apart two men start, one from each, at the same time one goes 24 and the other 27 miles a day; in how many days will they meet?

1879.—AFTERNOON

Examiners,— { MR J H. GILLILAND, M A
REV. FATHER VAN J IMPE, S.J }

1 Define an angle, an isosceles triangle, a rhombus and a gnomon If two triangles have two sides of the one equal to two sides of the other, each to each, but the base of the one greater than the base of the other the angle contained by the sides of that which has the greater base shall be greater than the angle contained by the sides equal to them, of the other

2 Divide a straight line into two parts such that the rectangle contained by the whole line and one of the parts shall be equal to the square on the other part

3 If in two circles which touch each other externally, two parallel diameters are drawn, shew that one extremity of each diameter and the point of contact lie in the same straight line

4 Inscribe a circle in a given triangle

A circle is described to touch BC, a side of the triangle ABC, in D and the other two sides produced in E and F respectively. Prove that AF is equal to one half of the sum of the sides of the triangle ABC.

5 Two fixed points A and B lie on the same side of a fixed straight line CD of unlimited length. P is any point in CD Prove that the sum of the lengths AP and BP is least when the angles which AP and BP make with CD are equal

SOLUTIONS.

1879 —MORNING.

1 $456\cdot654=456+\cdot654=400, 50, 6, \frac{6}{10}, \frac{5}{100}, \frac{4}{1000}$.

2 Had all the children been girls, they would have received altogether 150×8as $=$Rs 75

Hence the deficiency Rs $(75-49)=$Rs 26 ought to be divided among the boys

$$\therefore \text{ the No of boys } \frac{26\times16}{4}=104$$

3 (*a*) $$8-8\times\frac{2\frac{1}{5}-1\frac{2}{7}}{2-\frac{1}{6-\frac{1}{6}}}=8-8\times\frac{\frac{11}{5}-\frac{9}{7}}{2-\frac{1}{\frac{36-1}{6}}}=8-8\times\frac{\frac{77-45}{35}}{2-\frac{1}{\frac{35}{6}}}$$

$$=8-8\times\frac{\frac{32}{35}}{2-\frac{6}{35}}=8-8\times\frac{\frac{32}{35}}{\frac{64}{35}}=8-8\times\tfrac{1}{2}=8-4=4.$$

DECEMBER 1859.—MORNING.

1 A man can count at the rate of 100 a minute,—how long will it take him to count five hundred lacs?

2. A shopkeeper purchased $250\frac{3}{4}$ yards of cloth for 900 Rs. and paid expenses amounting to 103 Rs what must he charge per yard in order to make a profit of 50 per cent?

3 Reduce 005 of a pound to the fraction of a penny, and extract the square root of 00006241

4 Add together $\frac{2\frac{1}{5}}{3\frac{1}{3}}$, $\frac{1\frac{1}{4}}{3}$, 9 and $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{5\frac{5}{8}}{7}$.

5 State the rules for pointing in the multiplication and division of decimals, and multiply 256 by 0025 and divide 0036 by 4 and 4 by 00001

6 Show that

$$\{(ax+by)^2+(ay-bx)^2\}\times\{(ax+by)^2-(ay+bx)^2\}$$
$$=\{a^4-b^4\}\{x^4-y^4\}$$

7, Divide $x^6+2x^3y^2+y^6$ by $(x+y)^2$

8 Resolve $x^{12}-a^{12}$ into its simplest factors, and simplify

$$\frac{1+\dfrac{a-b}{a+b}}{1-\dfrac{a-b}{a+b}}-\frac{1-\dfrac{a^2-b^2}{a^2+b^2}}{1-\dfrac{a^4-b^2}{a^4+b^4}}$$

9 Find the Greatest Common Measure of x^3+3x^2-9x+5 and $x^3-19x+30$

10. Solve the equations

(i) $\frac{1}{2}\left(x-\frac{a}{3}\right)-\frac{1}{3}\left(x-\frac{a}{4}\right)+\frac{1}{4}\left(x-\frac{a}{5}\right)=0$

(ii) $\frac{a}{x-a}-\frac{b}{x-b}=\frac{a-b}{x-c}$

DECEMBER 1859.—AFTERNOON

1 If one side of a triangle be produced, the exterior angle is greater than either of the interior opposite angles

2 From the same point there cannot be drawn more than two equal straight lines to meet a given straight line

3 The opposite sides and angles of parallelograms are equal to one another, and the diameters bisect them into two equal parts

4 Prove that the four triangles, into which a parallelogram is divided by its diagonals, are equal to one another

(b) $=\frac{1}{5}+\frac{7}{6}-\frac{1}{56}-\frac{1}{8}=\frac{168+980-15-105}{840}$

$=\frac{1148-120}{840}=\frac{1028}{840}=\frac{257}{210}=1\frac{47}{210}$

(c) $1\dot{5}9\dot{0}\times\cdot47\dot{2}\div27=\frac{1590-15}{9900}\times\frac{472-47}{900}\div\frac{27-2}{9}$

$=\frac{1575}{9900}\times\frac{425}{900}\div\frac{25}{9}=\frac{1575}{9900}\times\frac{425}{900}\times\frac{9}{25}=\frac{119}{4400}=\cdot027045$

(d) $\frac{1}{1300}$ of £5 8s 4d. $=\frac{1}{1300}$ of $1300d=1d$

and £4 3s. 4d $=1000d$

$\therefore$ fraction reqd $=\frac{1}{1000}=\cdot001$

4. The cubical content of water drawn off

$=75\times50\times\frac{1}{2}\times3\times3$ cub ft $=75\times25\times9$ cub ft

The cubical content of each of the 16 carts $=5\times4\times\frac{27}{12}$ cub ft

$=45$ cub. ft

$\therefore$ the no of times filled $=\frac{75\times25\times9}{45\times16}=\frac{375}{16}=23\frac{7}{16}$

5. 17 men $\times 25d$: x men $\times\frac{25}{2}d$. :: 1 size : 2 size

$\therefore\ \frac{17\times25\times2\times2}{25}$ men $=68$ men

6 £100 : £2616¼ :: £5 : income in 1st case

$\therefore$ income $=£\frac{10465\times5}{100\times4}=£130$ 16s 3d.

£100 : £2616¼ :: (£95) : selling price,

$\therefore$ selling price $=£\frac{10465\times95}{4\times100}=£\frac{2093\times19}{16}$

£83⅛ : £$\frac{2093\times19}{16}$ :: £4 : income in 2nd case

$\therefore$ income $=\frac{2093\times19\times4\times8}{16\times665}=£\frac{598}{5}=£119$ 12s.

$\therefore$ Decrease in income $=£130$ 16s 3d $-£119$ 12s

$=£11$ 4s 3d.

7 When A can pass over 50 points, B can pass 40 and when B can pass over 50 points C can pass 40

$\therefore$ When B passes over 40 points, C can 32

$\therefore$ When A can pass 50 points, B can 40 and C can 32.

$\therefore$ A can give to C $(50-32)$ or 18 points

8. $(a^{2n}-a^n x^n+x^{2n})(a^n+x^n)$

$$=a^{3n}-a^{2n}x^n+a^n x^{2n}+a^{2n}x^n-a^n x^{2n}+x^{3n}$$

$$=a^{3n}+x^{3n}$$

(a) $x^2+\frac{2}{3}x+\frac{1}{12}\big)\ x^2+\frac{7}{6}x+\frac{1}{3}\ \big(1$

$$x^2+\frac{2}{3}x+\frac{1}{12}$$

$$\frac{1}{2}\big|\ \frac{1}{2}x+\frac{1}{4}$$

$$x+\frac{1}{2}$$

$x+\frac{1}{2}\big)\ x^2+\frac{2}{3}x+\frac{1}{12}\ \big(x+\frac{1}{6}$

$$x^2+\frac{1}{2}x$$

$$\frac{1}{6}x+\frac{1}{12}$$

$$\frac{1}{6}x+\frac{1}{12}$$

$\therefore x+\frac{1}{2}$ is the G C M.

9 Since $2^n=2^{n-1+1}=2^{n-1}\times 2$,

$$\therefore x^{2^n}-y^{2^n}=(x^{2^{n-1}})^2-(y^{2^{n-1}})^2$$

$$=(x^{2^{n-1}}+y^{2^{n-1}})(x^{2^{n-1}}-y^{2^{n-1}})$$

Quotient $=x^{2^{n-1}}-y^{2^{n-1}}$.

(a) The expression

$$=\frac{x-y}{x-z}+\frac{x-z}{x-y}-\frac{(y-z)^2}{(x-y)(x-z)}$$

$$=\frac{(x-y)^2+(x-z)^2-(y-z)^2}{(x-y)(x-z)}=\frac{2(x^2-xy-xz+yz)}{(x-y)(x-z)}$$

$$=\frac{2(x-y)(x-z)}{(x-y)(x-z)}=2$$

10 (a) $x-k+\sqrt{(k^2+x^2)}=m$

$\therefore x-k-m=-\sqrt{(k^2+x^2)}$

sq $x^2+k^2+m^2-2kx-2xm+2km=k^2+x^2$

$\therefore m^2+2km=2x(k+m)$

$$\therefore x=\frac{m(m+2k)}{2(k+m)}$$

(a) $a^x a^{y+1}=a^7$ . . (1),

From (1), $a^{x+y+1}=a^7$, $\therefore x+y+1=7$

or $x+y=6$....... .(3)

From (2), $x^{2y+3x-5}=a^{20}$, $\therefore 2y+3x+5=20$

or $2y+3x=15$ ......(4)

Multiply (3) by 2, $2y+2x=12$.

By subtraction $x=3$, and $y=6-x=3$

(c) $\frac{4}{x}+\frac{10}{y}=2$ ......(1), $\frac{3}{x}+\frac{2}{y}=\frac{19}{20}$ ....(2)

Multiply (1) by 3, and (2) by 4

$$\left.\begin{aligned}\frac{12}{x}+\frac{30}{y}&=6\\ \frac{12}{x}+\frac{8}{y}&=\frac{19}{5}\end{aligned}\right\}$$

By subtraction $\frac{22}{y}=\frac{11}{5}$, $\therefore y=10$

From (1) $\frac{4}{x}=2-\frac{10}{y}=1$, $\therefore x=4$.

11 Since $\frac{a}{b}=\frac{c}{d}$, $\therefore \frac{a^2}{b^2}=\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$.

$\therefore \frac{a^2+b^2}{b^2}=\frac{ac+bd}{bd}$ and $\frac{a^2-b^2}{b^2}=\frac{ac-bd}{bd}$

By division, $\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$

(a) In the 1st case, in every one man, the increase is $\frac{1}{11}$.

In the 2nd case, in every one man the increase is $\frac{1}{7}$.

Now since $\frac{1}{7}$ is greater than $\frac{1}{11}$,

$\therefore$ increase is in favour of the latter.

12, Let x be the no of days when they meet.

By the question, $24x+27x=561$,

$\therefore 51x=561$ $\therefore x=11$ days.

1879 —AFTERNOON.

1. Euclid I Defs. 8, 26, 34, Euclid II Def. 2

(a) Euclid I 24

2. Euclid II 11

3. Let the two ⊙s touch each other at E,

and let AOB, CQD be the ∥ diameters

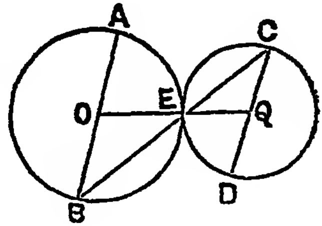
O and Q being the centres.

Join BE, CE

Then BE shall be in the same str. line with EC

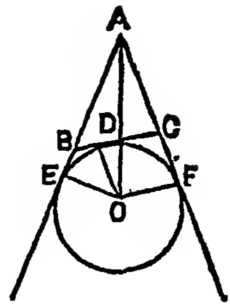
The str line joining OQ shall pass through E. (III, 11)

∵ ∠BOE=∠EQC. (I, 29)



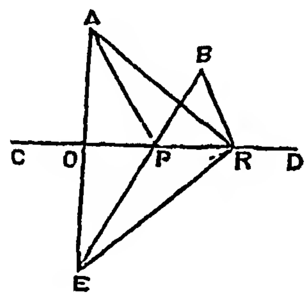
∴ the rem $\angle OBE+\angle OEB=\angle QEC+\angle QCE$. (I 32)
But $\angle OBE=\angle BEO$, for $OB=OE$,
likewise $\angle QEC=\angle QCE$
∴ $\angle BEO=\angle CEQ$.
Add to these equals $\angle BEQ$
∴ $\angle BEO+\angle BEQ=\angle BEQ+\angle QEC$
But $\angle BEO+\angle BEQ=2$ rt. $\angle$s,
∴ $\angle BEQ+\angle QEC=2$ rt $\angle$s,
∴ CE is in the same str line with EC (I 14)

4 Euclid IV 4



Let O be the centre of the ⊙
Join OE, OD, OF and AO
∵ $AO^2=AE^2+EO^2=AF^2+FO^2$ (I 47)
But $EO^2=FO^2$, ∵ $EO=FO$
∴ $AE^2=AF^2$, $AE=AF$
Likewise $BE=BD$ and $DC=FC$.
∴ $AB+BD=AC+CD$
But $AE=AB+BD$
∴ AE or AF=half the sum of the sides

5 Draw AO⊥ to the fixed str. line and prod AO to meet BP prod in E



∵ $\angle OPE=\angle BPD$, (I 15)
∴ $\angle APO=\angle OPE$,
also $\angle AOP=\angle POE$ and OP com (Ax 11)
∴ $AP=PE$
Let R be any other position of P in the fixed str line
Then $AR=ER$
∴ $AR+BR=ER+BR$
But $ER+BR>EB$ (I 20)
∴ $AR+BR>AP+BP$

1880 —MORNING

Examiners,— { MR E J BIDEN, M A
REV FATHER VAN J IMPF, S J.

N B —*Algebraical symbols are not to be employed in solving the first six questions*

1 Express each of the figures composing the number 128 456 as a multiple or sub-multiple of 10

What fraction must be added to—

$2\frac{1}{3}+\dfrac{3\frac{1}{3}-\frac{1}{6}}{3\frac{1}{3}+\frac{1}{6}}-2\frac{5}{7}$ of $\frac{4}{19}$ that the sum may be equal to 3

2 (*a*) What fraction of $\frac{2}{3}$ of Rs. 187 annas 5 is Rs 28 annas 8 ?
(*b*) Of what sum of money will 325 be 13*l* ?
(*c*) Extract the square root of 7 0225

3 Divide 127*l* 8*s* among 2 men, 3 women, and 7 boys giving each of the boys one-third of what a woman receives, and each of the men twice as much as a woman

4 A leaky cistern is filled in 5 hours with 30 pails of 3 gallons but in 3 hours with 20 pails of 4 gallons each, the pails being poured in at intervals Find how much the cistern will hold and in what time the water would waste away ?

5 A race-course is half-a-mile long A and B run a race and A wins by 10 yards C and D run over the same course and C wins by 30 yards: B and D run over it and B wins by 20 yards if A and C run over it, which should win, and by how much ?

6 A tradesman puts two prices on his goods one for ready money, the other for 6 months' credit, interest being calculated at $12\frac{1}{2}$ per cent per annum If the credit price of an article be Rs 26 annas 6, what is its cash price ?

7. Simplify—

$$\left\{\frac{x}{a}+\frac{2x^2}{a(b-x)}\right\}\left\{\frac{x}{a}-\frac{2ax}{x(b+x)}\right\}$$

8. Find the highest common factor and the least common multiple of—

$$3x^2-10ax+7a^2 \text{ and } x^3-5ax^2+7a^2x-3a^3$$

9 Solve the equations—

(*a*) $15+\sqrt{x+7}=19$

(*b*) $4x-\dfrac{x-1}{2}=x+\dfrac{2x-2}{5}+24$

(*c*) $\left\{\begin{aligned}\frac{7+x}{5}-\frac{2x-y}{4}&=3y-5\\ \frac{5y-7}{2}+\frac{4x-3}{4}&=18-5x\end{aligned}\right\}$

10 If $a \quad b \quad c \quad d$, shew that—

$$ma+nc \quad mb+md \qquad (a^2+c^2)^{\frac{1}{2}} \quad (b^2+d^2)^{\frac{1}{2}}$$

11 Extract the square root of—

$$x^{\frac{8}{5}}-2a^{-\frac{3}{5}}x^{\frac{11}{5}}+2a^{\frac{4}{5}}x^{\frac{4}{5}}+a^{-\frac{6}{5}}x^{\frac{14}{5}}-2a^{\frac{1}{5}}x^{\frac{7}{5}}+a^{\frac{8}{5}}$$

12 A boat goes up stream 30 miles and down stream 44 miles in ten hours it also goes up stream 40 miles and down stream 55 miles in 13 hours Find the rate of the stream and of the boat.

1880.—AFTERNOON.

1 Enunciate and prove I 6 Hence show that equiangular triangles are equilateral.

2 The side BC of the triangle ABC is produced to D, show that the angle ACD is greater than the angle ABC without showing that it is greater than the angle BAC

3 Give the particular enunciation and the construction *only* of the following proposition —

(*a*) I 48

(*b*) II 9. If a straight line be divided into two equal, and also into two unequal parts are double, &c

(*c*) II 11. To divide a given straight line, &c

4. Prove that if two circles touch one another internally the straight line which joins their centres being produced shall pass through the point of contact

5 If two straight lines cut one another within a circle, the rectangle contained by the segments of the one of them shall be equal to the rectangle contained by the segments of the other. Prove that *only* when one of the lines passes through the centre, and cuts the other which does not pass through the centre, but not at right angles

AOC and BQD are two triangles having the angle AOC equal to the angle BQD and the angle ACO equal to the angle DBQ: show that the rectangle contained by AO and QB is equal to that contained by CO and QD.

6 Describe a circle about a given triangle, and show that the square on the side of an equilateral triangle described about a circle is four times the square on the side of an equilateral triangle inscribed in the same circle.

SOLUTIONS

1880.—MORNING.

1 $123 \cdot 456 = 123 + \cdot 456 = 100 + 20 + 3 + \frac{4}{10} + \frac{5}{100} + \frac{6}{1000} = 10 \times 10 + 10 \times 2 + 3 + \frac{4}{10} + \frac{5}{100} + \frac{6}{1000}$

$2\frac{1}{3} + \dfrac{3\frac{1}{3} - \frac{1}{6}}{3\frac{1}{3} + \frac{1}{6}} - 2\frac{5}{7} \text{ of } \frac{4}{15} = 2\frac{1}{3} + \dfrac{3\frac{1}{6}}{3\frac{1}{2}} - \frac{19}{7} \text{ of } \frac{4}{15}$

$= 2\frac{1}{3} + \frac{19}{6} \times \frac{2}{7} - \frac{4}{7} = 2\frac{1}{3} + \frac{19}{21} - \frac{4}{7} = 2\frac{1}{3} + \dfrac{19 - 12}{21}$

$= 2\frac{1}{3} + \frac{7}{21} = 2\frac{1}{3} + \frac{1}{3} = 2\frac{2}{3}$

$\therefore$ the fraction reqd $= 3 - 2\frac{2}{3} = \frac{1}{3}$.

2 (*a*) Rs 28 8as = 456as

and $\frac{2}{3}$ of Rs 187 5as = Rs 124 14as = 1998as.

$\therefore$ fraction reqd $= \frac{456}{1998} = \frac{76}{333}$

(*b*) The sum reqd $= £\,\dfrac{13}{325} - £40$

(*c*) $\sqrt{(\dot{7}\,0\dot{2}25)} = 2\,65$

3. 2 men $= 2 \times 3 \times 2$ boys $= 12$ boys
3 women $= 3 \times 3$ boys $= 9$ boys
7 boys $= 7$ boys.

$\therefore$ 2 men + 3 women + 7 boys = 28 boys

$\therefore$ each boy should receive $\dfrac{£127\ 8s}{28} = £4\ \ 11s$

$\therefore$ each woman should receive $3 \times £4\ \ 11s = £13.\ 13s$

$\therefore$ each man should receive $6 \times £4\ \ 11s. = £27.\ 6s$

4. 3×30 or 90 gals of water are poured in 5 hrs } at intervals.
4×20 or 80 gals . .. . in 3 hrs }

$\therefore$ 10 gals. of water leak out in 2 hrs
$\therefore$ 5 gals . ... in 1 hr
$\therefore$ 25 gals ... ... in 5 hrs

Hence the cistern holds (90—25) or 65 gals of water.

5 65 1 hr. time the water would waste away

$\therefore$ Required time $= \frac{65}{5}$ or 13 hours.

5. Since $\frac{1}{2}$ a mile = 880 yds
$\therefore$ 870 of A = 880 of B
860 of B = 880 of D
880 of D = 850 of C
x of C = 880 of A

$$\therefore x = \frac{880 \times 880 \times 850 \times 880}{880 \times 860 \times 870} = 879\tfrac{2861}{3741}$$

Hence C wins by $1 - \frac{2861}{3741}$ or $\frac{880}{3741}$ yds

6. The interest on Rs. 100 for 6 mo $= (12\frac{1}{2} \times \frac{1}{2})$ Rs
$= \frac{25}{4}$ Rs $= 6\frac{1}{4}$ Rs.

$\therefore$ $(100 + 6\frac{1}{4})$ Rs Rs 26 9as Rs 100 reqd cash price.

$$\therefore \text{reqd. cash price} = \frac{4 \times 425 \times 100}{16 \times 425}\text{Rs} = 25 \text{ Rs}$$

7. $$\text{Fraction} = \frac{x(b-x)+2x^2}{a(b-x)} \times \frac{a(b+x)-2ax}{x(b+x)}$$
$$= \frac{x(b+x)}{a(b-x)} \times \frac{a(b-x)}{x(b+x)} = 1.$$

8 $\quad 3x^3-10ax+7a^2\big)\, x^2-5ax^2+7a^2x-3a^3 \big(\, x-5a$

$$\begin{array}{l} \quad\quad\quad\quad\quad\quad\quad\quad\quad 3 \\ \hline 3x^3-15ax^2+21a^2x-9a^3 \\ 3x^3-10ax^2+7a^2x \\ \hline -5ax^2+14a^2x-9a^3 \\ \quad\quad\quad\quad\quad\quad\quad 3 \\ \hline -15av^2+42a^2x-27a^3 \\ -15ax^2+50a^2x-35a^2 \\ \hline -8a^2\big)\ \underline{-8a^2x+8a^2} \\ \quad\quad\quad x-a \end{array}$$

$$x-a\big)\, 3x^2-10ax+7a^2 \big(\, 3x-7a$$

$$\begin{array}{l} 3x^2-3ax \\ \hline -7ax+7a^2 \\ -7ax+7a^2 \\ \hline \end{array} \qquad .\text{G C M} = x-a$$

and L C M $=(x-a)^2(x-3a)(3x-7a)$

9 $\quad$ (1) $15+\sqrt{(x+7)}=19$

$\quad .\ \sqrt{(x+7)}=4$ sq $x+7=16 \qquad \therefore x=9$

(2) $4x-\dfrac{x-1}{2}=x+\dfrac{2x-2}{5}+24$

$\therefore 40x-5x+5=10x+4x-4+240$

$\therefore 35x+5=14x+236, \quad$ or $21x=231 \qquad \therefore x=11.$

(3) $\left.\begin{array}{l} \dfrac{7+x}{5}-\dfrac{2x-y}{4}=3y-5. \ . \quad (1) \\ \dfrac{5y-7}{2}+\dfrac{4x-3}{6}=18-5x \quad 2) \end{array}\right\}$

From (1) $28+4x-10x+5y=60y-100$

$\therefore 6x+55y=128 \quad . \ . \ . \quad (3)$

From (2), $15y-21+4x-3=108-30x$

$\therefore 34x+15y=132 \ \ldots \ \ldots\ldots \quad (4)$

$$\begin{array}{ll} (3)\times 17, & 102x+935y=2176 \\ (4)\times 3, & 102x+\ \ 45y=\ \ 396 \\ \hline \end{array}$$

By subtı $\quad 890y=1780 \qquad \cdot\ y=2.$

From (3) $6x=128-55y=18, \qquad \therefore x=3$

10 Since $\frac{a}{b}=\frac{c}{d}$, $\therefore \frac{a}{c}=\frac{b}{d}$ or $\frac{ma}{nc}=\frac{mb}{nd}$.

$$\therefore \frac{ma+nc}{nc}=\frac{mb+nd}{nd}, \qquad \therefore \frac{ma+nc}{mb+nd}=\frac{nc}{nd}=\frac{c}{d}.$$

Again, $\therefore \frac{a^2}{c^2}=\frac{b^2}{d^2}$, $\therefore \frac{a^2+c^2}{c^2}=\frac{b^2+d^2}{d^2}$.

$$\therefore \frac{a^2+c^2}{b^2+d^2}=\frac{c^2}{d^2} \text{ or } \frac{(a^2+c^2)^{\frac12}}{(b^2+d^2)^{\frac12}}=\frac{c}{d}.$$

Hence $\frac{ma+nc}{mb+nd}=\frac{(a^2+c^2)^{\frac12}}{(b^2+d^2)^{\frac12}}$

or $ma+nc \quad mb+nd \qquad (a^2+c^2)^{\frac12} \,,\, (b^2+d^2)^{\frac12}$

11. Arranging the terms, we get

$$\begin{array}{l|l}
 & a^{-\frac65}x^{\frac{14}{5}}-2a^{-\frac35}x^{\frac{11}{5}}+x^{\frac85}-2a^{\frac15}x^{\frac75}+2a^{\frac45}x^{\frac45}+a^{\frac85}\,\big(a^{-\frac35}x^{\frac75}-x^{\frac45}-a^{\frac45} \\
 & a^{-\frac65}x^{\frac{14}{5}} \\ \hline
2a^{-\frac35}x^{\frac75}-x^{\frac45} & -2a^{-\frac35}x^{\frac{11}{5}}+x^{\frac85} \\
 & 2a^{-\frac35}x^{\frac{11}{5}}+x^{\frac85} \\ \hline
2a^{-\frac35}x^{\frac75}-2x^{\frac45}-a^{\frac45} & -2a^{\frac15}x^{\frac75}+2a^{\frac45}x^{\frac45}+a^{\frac85} \\
 & -2a^{\frac15}x^{\frac75}+2a^{\frac45}x^{\frac45}+a^{\frac85} \\ \hline
\end{array}$$

12 Let x = rate of the boat per hour in miles,

and y .. .. stream . . . . . .

then $x+y$ and $x-y$ are the rates with and against the stream respectively.

By the question,

$$\left.\begin{array}{ll}\frac{30}{x-y}+\frac{44}{x+y}=10 \quad . & (1) \\ \frac{40}{x-y}+\frac{55}{x+y}=13 \quad .. & (2)\end{array}\right\}$$

$$(1)\times 4,\ \frac{120}{x-y}+\frac{176}{x+y}=40$$

$$(2)\times 3,\ \frac{120}{x-y}+\frac{165}{x+y}=39$$

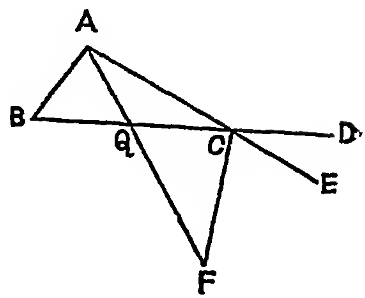
By substraction, $\frac{11}{x+y}=1$, or $x+y=11$

From (1)$\frac{30}{x-y}=10-\frac{44}{x+y}=10-4=6$, or $x-y=5$

Hen $x=8$, and $y=3$

1880.—AFTERNOON.

1. Euclid Prop Book I Cor. I 6

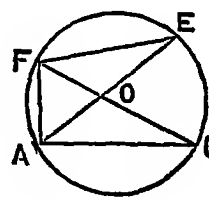


2 Prod AC to E
Bisect BC at Q
Join AQ and prod. it to F making QF=AQ
Join CF,
Then ∠ABQ=∠QCF, (I 4)
but ∠BCF>∠BCE
∴ ∠ABC>∠BCE
and ∠BCE=∠ACD (I 15)
∴ ∠ACD>∠ABC.

3 (*a*), (*b*), (*c*) See Hall and Steven's *Euclid.*

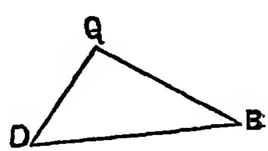
4. Euclid III. 11.

5 Euclid III 35. Third case,



Prod AO. CO to E F respectively making EO=BQ, FO=QD
Join FE
∴ △OFE=△DQB; (I 4)
∴ ∠FEO=∠QBD
But ∠QBD=∠ACO
∴ ∠FEO=∠ACO
∴ a circle described about △ ACF shall pass through E (III 21 con)
∴ AO OE=CO OF III. 35
Wherefore AO QB=CO QD.

6 Euclid IV 5.



The side of the equil △ desc about the ⊙ is double of the side of the equil △ inscribed in the same ⊙. Therefore the square on the side of the desc △ is four times the square on the side of the inscribed figure

5 In obtuse-angled triangles, if a perpendicular be drawn from any of the acute angles, to the opposite side produced, the square of the side subtending the obtuse angle is greater than the sum of the squares of the sides containing the obtuse angle, by twice the rectangle contained by the side, upon which when produced, the perpendicular falls, and the straight line intercepted, without the triangle between the perpendicular and the obtuse angle

6 The angle at the centre of a circle is double of the angle at the circumference upon the same base, that is, upon the same part of the circumference

7 If two chords of a circle intersect at right angles, the portions of the circumference, taken alternately, are together equal to half of the circumference

8 If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles made by this line with the line touching the circle shall be equal to the angles which are in the alternate segments of the circle

9 If two circles cut one another, find a point from which the straight lines drawn to touch the two circles shall be equal to one another

SOLUTIONS

DECEMBER 1859.—MORNING.

1 100)50000000 = 500000 min.

= 347 days 5 hrs 20 min.

2 His total cost = (900 + 103) Rs = 1003 Rs

∴ 100 Rs   1003 Rs   150 Rs   the selling price

∴ selling price $= \frac{150 \times 1003}{100}$ Rs

∴ Charge per yard = Rs. $\frac{150 \times 1003}{100 \times 250\frac{3}{4}}$ = Rs $\frac{\overset{6}{\cancel{150}} \times \cancel{1003} \times 4}{\underset{25}{\cancel{100}} \times \cancel{1003}}$ = Rs 6.

3, 005 of £1 $= \frac{5}{1000} \times 20 \times 12d = \frac{6}{5}d.$

∴ the fraction reqd $= \frac{\frac{6}{5}}{1} = \frac{6}{5}$

(a) $\sqrt{(0000624\dot{1})} = \cdot 0079$

4 $\frac{2\frac{1}{2}}{3\frac{1}{3}} + \frac{1\frac{1}{4}}{3} + 9 + \frac{3}{4}$ of $\frac{8}{9}$ of $\frac{5\frac{5}{6}}{7}$

$= \frac{5}{2} \times \frac{3}{10} + \frac{5}{4} \times \frac{1}{3} + 9 + \frac{3}{4} \times \frac{8}{9} \times \frac{35}{6} \times \frac{1}{7},$

$\frac{3}{4} + \frac{5}{12} + 9 + \frac{5}{9} = 9 + \frac{27 + 15 + 20}{36} = 9 + \frac{31}{18} = 9 + 1\frac{13}{18} = 10\frac{13}{18}.$

1881.—MORNING.

N.B.—Algebraical symbols are not to be employed in solving the first five questions.

1. What do you mean by *Multiplication?* Define *quotient, factor, power, expression, and dimension.*

2. Add together $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{6}{7}$, and simplify

$$\frac{\frac{3}{7}-\frac{2}{9}}{\frac{3}{7}+\frac{1}{5}} \text{ of } 2\frac{11}{18} \div \frac{4}{13-3\frac{8}{9}}+3\frac{11}{16}-\frac{3}{3-1\frac{10}{13}}$$

3. What decimal of 45 Rs. is 33 Rs. 2as. 6 p.? Find the value of $\frac{1.074}{.0015}$ of $8\frac{1}{2}$ annas.

4. Express $37.8\dot{4}6\dot{3}$ as an improper vulgar fraction in its lowest terms and find, correct to 4 places of decimals, the result of dividing the square root of this number by the square root of 11.

5. A man who has a certain capital calculates that if he invest in $3\frac{1}{2}$ per cent. stock at 91 his income will be £25 more than if he invest it in 3 per cent. stock at 88. What is his capital?

6. What do you mean by a negative quantity?

Prove that $a-(b-c)=a-b+c$.

7. Simplify $\frac{1}{abx}+\frac{1}{a(a-b)(x-a)}+\frac{1}{b(b-a)(x-b)}$, and resolve into elementary factors the expressions —

$x^2-5ax-66a^2$ and $(1-c^2)(1+a)^2-(1-a^2)(1+c)^2$

8. A man receives $\frac{x}{y}$ths of 10 Rs. and afterwards $\frac{y}{x}$ths of 10 Rs. He then gives away 20 Rs. Show that he cannot lose by the transaction.

9. What is an *equation?* Prove that a simple equation has only one root.

10. Solve the equations—

(1) $\sqrt{x^2+11x+20}-\sqrt{x^2+5x-1}=3$

(2) $\frac{4.05}{9x}-\frac{3}{8-2x}=\frac{1.8}{x}-\frac{3.6}{2.4-6x}$

(3) $ax+by=c,\ a^2x+b^2y=c^2$

11. A challenged B to ride a bicycle race of 1,040 yds. He first gave B 120 yds. start, but lost by 5 seconds; he then gave B 5 seconds start and won by 120 feet. How long does each take to ride the distance?

1881.—AFTERNOON.

1 Define *an angle*, *an obtuse angled triangle*, *a circle*, and a *parallelogram* Write out the 2nd and 8th axioms

2 Write out the 12th axiom and prove its converse

3 To make a triangle of which the sides shall be equal to three given straight lines, but any two whatever of these must be greater than the third

Point out how the construction fails if the above condition be not complied with

4 Triangles upon the same base and between the same parallels are equal to one another

Shew how to make a triangle equal to a given quadrilateral which shall have its base on one side of the quadrilateral produced if necessary, and its vertex at one of the opposite angles

5 If a straight line be divided into two equal, and also into two unequal parts, the squares on the two unequal parts are together double of the square on half the line and of the square on the line between the points of section

Also prove this algebraically

6 The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles

BC is a given arc of a circle whose centre is O, A is any point in BC, AD, AE are drawn perpendiculars to OB, OC Prove that the line DE is of constant length

7 To inscribe an equilateral and equiangular pentagon in a given circle

SOLUTIONS

1881.—MORNING.

1. *Multiplication* is a short method of finding the sum of any given number repeated as often as there are units in another given number

(*a*) The number which expresses the number or times which the dividend contains the divisor is called the *Quotient*

(*b*) When a number is composed of the product of two or more numbers, each is called a *factor* of the first number

(*c*) The product of several of the same number is called a *power* of that number

(*d*) An algebraical quantity made up of several letters is called an *expression*

(*e*) The sum of the indices of the letters in a term is called the number of *dimension* of the term.

2 $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$

$$=\frac{210+280+315+336+350+360}{420}$$

$$=\frac{1851}{420}=4\frac{171}{420}=4\frac{57}{140}.$$

(a) $\dfrac{\frac{3}{7}-\frac{2}{9}}{\frac{3}{7}+\frac{2}{9}}$ of $2\frac{11}{26} \div \dfrac{4}{13-3\frac{8}{9}}+3\frac{11}{16}-\dfrac{3}{3-1\frac{10}{13}}$

$$=\frac{\frac{27-14}{63}}{\frac{27+14}{63}}\text{of}\,\frac{63}{26}-\frac{4}{10-\frac{8}{9}}+3\tfrac{11}{16}-\frac{3}{2-\frac{10}{13}}$$

$$=\frac{13}{63}\times\frac{63}{41}\times\frac{63}{26}-\frac{4}{\frac{90-8}{9}}+3\tfrac{11}{16}-\frac{3}{\frac{26-10}{13}}$$

$$=\tfrac{13}{41}\times\tfrac{63}{26}\div\left(\tfrac{4}{1}\times\tfrac{9}{82}\right)+3\tfrac{11}{16}-\tfrac{3}{1}\times\tfrac{13}{16}$$

$$=\frac{63}{82}\times\frac{82}{4\times9}+3\tfrac{11}{16}-\frac{39}{16}=\frac{7}{4}+3\tfrac{11}{16}-2\tfrac{7}{16}=1\tfrac{3}{4}+1\tfrac{1}{4}=3$$

3 45 Rs $=45\times16\times12$ p, and Rs 35 2 as 6 p. $=6750$ p.

$\therefore$ decimal reqd $=\dfrac{6750}{45\times16\times12}=\dfrac{25}{32}=.78125$

(a) $\dfrac{1.074}{.0015}$ of $8\frac{1}{2}$as $=\dfrac{10740}{15}\times\dfrac{17}{2}$as $=6086$as $=$ Rs 380 6as.

4. $37.8\dot{4}6\dot{3}=\dfrac{378463-3784}{9900}=\dfrac{374679}{9900}=\dfrac{41631}{1100}$

$$\sqrt{\left(\frac{41631}{1100}\right)}\div\sqrt{11}=\frac{\sqrt{(41631)}}{10\sqrt{11}}\times\frac{1}{\sqrt{11}}$$

$$=\frac{\sqrt{(41631)}}{10\times11}=\frac{204.0367\ldots}{10\times11}=\frac{18.5488}{10}=1.8548\ldots$$

5 £91 : £100 :: $3\frac{1}{4}$£ : Int of £100 capital

$\therefore$ Int in 2nd case $=£\frac{13}{4}\times\frac{100}{91}=£\frac{25}{7}$

£88 : £100 :: 3£ : Int. of £100 capital

$\therefore$ Int in 1st case $=£\,\dfrac{3\times100}{88}=£\frac{75}{22}$

$\therefore$ Diff. in income $=\left(\dfrac{25}{7}-\dfrac{75}{22}\right)£=£\dfrac{25}{7\times22}$

$\therefore \dfrac{25}{7\times22}£$: £25 :: £100 : capital reqd.

$\therefore$ Capital reqd. $=\dfrac{100\times25\times7\times22}{25}=£15400$

6 Those quantities that are preceded by the sign *minus* are called negative quantities.

(1) To subtract $b-c$ from a, we consider that if b alone were taken from a, the remainder would be expressed by $a-b$. but inasmuch as we have by this process taken away from a a quantity too *large* by c, it follows that the remainder will be too *small* by the same quantity. and therefore the proper result will be $a-b$ increased by c that is, $a-(b-c)$ is equivalent to $a-b+c$.

7. Fraction $= \frac{1}{abx} + \frac{1}{a(a-b)(x-a)} - \frac{1}{b(a-b)(x-b)}$

$$= \frac{(a-b)(x-a)(x-b)+bx(x-b)-ax(x-a)}{abx(a-b)(x-a)(x-b)}$$

Num $=(a-b)\{x^2-(a+b)x+ab\}-x^2(a-b)+x(a^2-b^2)$

$=(a-b)x^2-(a^2-b^2)x+ab(a-b)-x^2(a-b)+x(a^2-b^2)$

$=ab(a-b).$

Ans $= \frac{1}{x(x-a)(x-b)}$

(a) $x^2-5ax-66a^2=x^2-11ax+6ax-66a^2$

$=x(x-11a)+6a(x-11a)$

$=(x+6a)(x-11a)$

(b) $(1-c^2)(1+a)^2-(1-a^2)(1+c)^2$

$=(1+c)(1+a)\{(1-c)(1+a)-(1-a)(1+c)\}$

$=(1+a)(1+c)\{(1-c+a-ac)-(1-a+c-ac)\}$

$=(1+a)(1+c)2(a-c)=2(1+a)(1+c)(a-c)$

8. Since $(x-y)^2>0$, or $x^2-2xy+y^2>0$.

$\therefore x^2+y^2>2xy$, or $\frac{x^2+y^2}{xy}>2$.

$\therefore \frac{x}{y}+\frac{y}{x}>2$ $\qquad \therefore 10\left(\frac{x}{y}+\frac{y}{x}\right)>20$

Hence he cannot lose by the bargain.

9 An *equation* is an expression of equality between two sets of quantities only for a particular value or values of the unknown quantities

If possible, let the simple equation $ax+b=c$, have two values of the unknown quantity x, viz. α and δ

then $a\alpha+b=c$ and $a\delta+b=c$ } By subtraction, $a(\alpha-\delta)=0$.

but a is not$=0$, $\therefore \alpha-\delta=0$, $\therefore \alpha=\delta$

Hence the two values are equal; and a simple equation cannot have more than one value of x

10. (1) $\sqrt{(x^2+11x+20)}-\sqrt{(x^2+5x-1)}=3$

$\sqrt{(x^2+11x+20)}=3+\sqrt{(x^2+5x-1)}$

sq $x^2+11x+20=9+6\sqrt{(x^2+5x-1)}+x^2+5x-1$

$\therefore 6x+12=6\sqrt{(x^2+5x-1)}$, or $x+2=\sqrt{(x^2+5x-1)}$

sq $x^2+4x+4=x^2+5x-1$. $\therefore x=5$

(2) $\dfrac{4\cdot05}{9x}-\dfrac{3}{\cdot8-2x}=\dfrac{1\cdot8}{x}-\dfrac{3\cdot6}{2\cdot4-6x}$

$\therefore \dfrac{\cdot45}{x}-\dfrac{3}{8-2x}=\dfrac{1\cdot8}{x}-\dfrac{12}{8-2x}$

or $-\dfrac{1\cdot35}{x}=\dfrac{-9}{8-2x}$ $\therefore -1\cdot35\times(8-2x)=-9x$

or $-1\cdot08+2\cdot7x=-9x$. $\therefore 3\cdot6x=1\cdot08$, $\therefore x=\cdot3$

(3) $ax+by=c$... ..(1) $a^2x+b^2y=c^2$ .... (2)

(1) $\times a$, $a^2x+aby=ac$

From (2) $a^2x+b^2y=c^2$

By subtr $b(a-b)y=c(a-c)$

$$\therefore y=\frac{c(a-c)}{b(a-b)}.$$

From (1) $ax=c-by=c-\dfrac{c(a-c)}{a-b}=\dfrac{c(c-b)}{a-b}$

$$\therefore x=\frac{c(c-b)}{a(a-b)}$$

11. Let $x=$ A's speed in yds per second

and $y=$ B's....... ....

By the questions,

$$\left.\begin{aligned}\frac{1040}{x}&=\frac{920}{y}+5\ .....(1)\\ \frac{1040}{x}&=\frac{1000}{y}-5......(2)\end{aligned}\right\}$$

By subtraction,

$\dfrac{80}{y}=10$, $\therefore y=8$.

Hence B's time $=\frac{1040}{8}$ or 130 seconds.

From (1) A's time $=\dfrac{1040}{x}=115+5=120$ seconds.

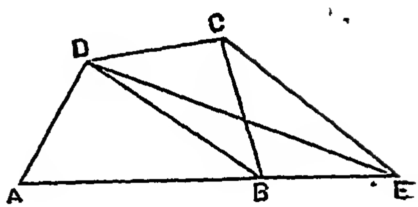
1881.—AFTERNOON.

1. Euclid, B. I. Defs. 8, 28, 15, A.
2. Euclid, Prop. 17, Book 1
3. Euclid, B. I Prop. 22. If the condition be omitted, the circles employed in the construction may or may not cut each other. Hence the construction will fail.

4 Euclid, B I Prop. 37.

Let ABCD be the quad

It is reqd to make a Δ, whose base shall be in AB or AB prod and its vertex at the opposite $\angle$ D



Join BD and from C draw CE $\parallel$ BD meeting AB or AB prod in E

Join DE

Then ADE is the reqd Δ

$\therefore$ Δ BCD=BDE (I 37)

to each add the Δ ADB

$\therefore$ quad ABCD=ΔADE

5 Euclid B II Prop 9

Let AB contain $2a$ linear units, its half AC or BC will contain a units,

and let CD, the line between the points of section contain m units,

Then AD contains $a+m$ units, and BD contains $a-m$ units,

$\because$ $(a+m)^2=a^2+2am+m^2$, and $(a-m)^2=a^2-2am+m^2$

$\therefore$ adding these equals, $(a+m)^2+(a-m)^2=2a^2+2m^2$

6 Euclid, III Prop 22

Join DE

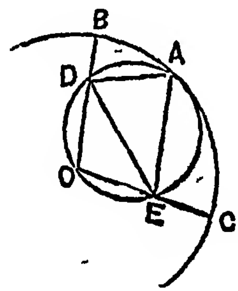
(Hyp) $\because$ $\angle$ADO+$\angle$AEO=2 rt $\angle$s

$\therefore$ a $\odot$ will go round the fig ADOE (III 22 Con)

Again, $\because$ the arc BC is given

$\therefore$ $\angle$BOC at the centre is also given, and is of constant magnitude

Hence the arc DE is constant, and also the line DE



7 Euclid, B IV Prop 11.

1882 —MORNING.

1 The quotient arising from the division of 6739546 by a certain number is 1559 and the remainder is 3107 : find the divisor

2 Subtract $\frac{2}{3}$ of $\frac{5}{9}$ of $\frac{4}{15}$ of £ 31 5*s* from $\frac{4}{7}$ of $\frac{2}{11}$ of $\frac{49}{81}$ of £100 16*s* 8*d*, and express the remainder as the decimal of £10 8*s* 4*d*.

3 Seven bells begin to strike simultaneously and strike at intervals of 2, 3, 5, 15, 21, 65, 77 seconds respectively After what time will they again strike simultaneously, and how often will each have struck ?

4 (i) Simplify $\dfrac{2\frac{2}{3}+5\frac{7}{9}}{1\frac{1}{2}-\frac{4}{9}}-(\frac{2}{25}$ of $\frac{7}{12}$ of $\frac{23}{36})$

(ii) Find the value of $\dfrac{\sqrt{(15)}+\sqrt{(13)}}{\sqrt{(15)}-\sqrt{(13)}}$ to five places of decimals.

5 A besieged garrison consists of 300 men, 120 women, and 40 children, and has provisions enough for 200 men for 30 days If a woman eats $\frac{2}{3}$ as much as a man and a child $\frac{1}{2}$ as much, and if after 6 days 100 men with all the women and children escape, for how long will the remaining provisions last the garrison?

6 A person begins to speculate with a certain sum of money in his first transaction he losses $\frac{1}{7}$th of this sum, in his second he gains 10 per cent on his investment in his third he loses $\frac{9}{11}$th of the sum invested in his fourth he gains $66\frac{2}{3}$ per cent. If he then has Rs 10,000, with what sum did he start?

7 Divide $x^n - a^n$ by $x-a$, and find the continued product of $x-a$, x^2+ax+a^2, x^3+a^3.

8 Resolve into factors $x^2+13x+42$, x^2+x-42, $343x^3+512y^3$.

9 Find the highest common factor of $x^3-7x^2-80x+576$, and $3x^2-14x-80$, and the lowest common multiple of these two expressions and $3x^2+17x-90$

10 If $a \quad b \quad c \quad d \quad e \quad f$, shew that each of these ratios is equal to

$$\sqrt[3]{a^3+c^3+e^3} \qquad \sqrt[3]{b^3+d^3+f^3}$$

11 Solve the equations—

(i) $(6x+9)^2+(8x-7)^2=(10x+3)^2-71$

(ii) $$65x+\frac{585x-975}{6}=\frac{156}{2}-\frac{39x-78}{9}$$

(iii) $$\left.\begin{aligned} \frac{x+2}{2}-\frac{x+y}{14}&=\frac{x-y-1}{8}-\frac{y+12}{4} \\ \frac{x+7}{3}+\frac{y-5}{10}&=1-x-\frac{5(y+1)}{7} \end{aligned}\right\}$$

12 The distance from a place P to another place Q is $3\frac{1}{2}$ miles two persons A and B start together from P to go to Q, the former by carriage which travels at the rate of 6 miles an hour, the latter walking at the rate of 3 miles an hour If A remains at Q for 15 minutes and then returns by the carriage to P, find where he will meet B

1882.—AFTERNOON.

1. Bisect a given rectilineal angle

(*a*) OC is a straight line which bisects the angle AOB, and OD is any other straight line without the angle AOB, show that the angles DOA, DOB are together double of the angle DOC

2 If a straight line falling on two other straight lines makes the alternate angles equal to each other, these two straight lines shall be parallel.

(*b*) ABC is a triangle, straight lines AD, CE bisect the angles

at A and C, and from B, BE is drawn equal to BC, and BD equal to BA, shew that EBD is a straight line.

3 If a straight line be divided into two equal and also into two unequal parts, the squares on the two unequal parts are together double of the square on half the line and of the square on the line between the points of section

(c) Prove the same algebraically

(d If A, B be fixed points and O any other point, the sum of the squares on AO and BO is least when O is the middle point of AB

4 The angle at the centre of a circle is double of the angle at the circumference upon the same base, that is, upon the same part of the circumference

(e) If two straight lines AB, CD in a circle intersect in E, the angles subtended by AC and BD at the centre are together double the angle AEC

5 If from any point without a circle two straight lines be drawn, one of which cuts the circle and the other touches it, the rectangle contained by the whole line which cuts the circle and the part of it without the circle shall be equal to the square on the line which touches it

(*f*) AO, BO are radii of a circle at right angles to each other, ACD is a straight line meeting BO in C and the circle in D then *the rectangle contained by AC, AD is double of the square on OB.*

6. Inscribe a triangle is a given circle

(*g*) O is the centre of the circle inscribed in the triangle ABC, OC is joined and OD drawn perpendicular to OC, meeting the circle in D towards BC, E is the centre of an equal circle touching BC and AC produced; shew that ED touches the first circle at D.

SOLUTIONS

1882.—MORNING.

1.

```
          6739546
             3107
          -------
   1559 ) 6736439 ( 4321
          6236
          ----
           5004
           4677
           ----
            3273
            3118
            ----
             1559
             1559
             ----
```

2 $\frac{4}{7}$ of $\frac{2}{11}$ of $\frac{49}{64}$ of £100 16*s*. 8*d*.

$= \frac{7}{88}$ of £100 16*s* 8*d*.

$=$ 7 of £1 2*s* 1*d*

$=$ £7 14*s*. 7*d*

and $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{4}{25}$ £31 5*s*.

$= \dfrac{\text{£}31\ 5s.}{15} =$ £2 1*s* 8*d*.

∴ the remainder $=$ £7 14*s* 7*d*.
 $-$£2 1*s*. 8*d* $=$ £5 12*s* 11*d*

and, £5 12*s*. 11*d*. $=$ 1355*d*

and, £10 8*s* 4*d* $=$ 2500*d*.

∴ the fraction reqd. $= \frac{1355}{2500} = \cdot542$.

3. Since the L C M. of 2, 3, 5, 15, 21, 65, 77 is 30030

$\therefore$ the reqd. time $=30030$ seconds.

and the bells strike 15016, 10011, 6007, 2003, 1431, 463, 391 times respectively.

4. (i) $\dfrac{2\frac{2}{3}+5\frac{7}{9}}{1\frac{1}{2}-\frac{4}{9}} \div (\frac{2}{25} \text{ of } \frac{7}{42} \text{ of } \frac{21}{56}) = \dfrac{2\frac{6}{9}+5\frac{7}{9}}{1\frac{9}{18}-\frac{8}{18}} \div (\frac{2}{25} \text{ of } \frac{1}{6} \text{ of } \frac{3}{8})$

$=\dfrac{7+\frac{13}{9}}{1\frac{1}{18}} \div \frac{1}{200} = \frac{76}{9} \times \frac{18}{19} \times 200 = 1600.$

(iii) $\dfrac{\sqrt{15}+\sqrt{13}}{\sqrt{15}-\sqrt{13}} = \dfrac{(\sqrt{15}+\sqrt{13})(\sqrt{15}+\sqrt{13})}{(\sqrt{15}-\sqrt{13})(\sqrt{15}+\sqrt{13})}$

$=\dfrac{28+2\sqrt{(15\times 13)}}{15-13} = \dfrac{28+2\sqrt{195}}{2} = 14+\sqrt{195}$

$=14+13\ 96424 \ldots \ldots \quad =27\ 96424 \ldots\ldots$

5 1 man $=\frac{3}{2}$ woman $=2$ child.

$\therefore$ 300 men + 120 women + 40 children $=(300+80+20)$ men $=400$ men.

and 100 men + 120 momen + 40 children $=(100+80+20)$ men $=$ 200 men.

After 6 days 200 men escape, $\therefore$ there remain $(400-200)$ or 200 souls, and they had provisions enough for 200 men for 30 days, *i.e.* they had provisions enough for 15 days. Now there remain $(15-6)$ or 9 day's provisions

$\therefore$ 200 men : 400 men : : 9 days : reqd. no of days.

$\therefore$ reqd. no of days $\dfrac{9\times 400}{200}=18$

6. Let Rs. 100 be sum he started with.

Then $\frac{2}{7}$ of Rs 100 $=$ Rs $\frac{200}{7}$ $=$ sum after 1st transaction

$\frac{110}{100}$ of Rs. $\frac{200}{7}=$ Rs $\frac{220}{7}=$ sum after 2nd.........

$\frac{6}{11}$ of Rs. $\frac{220}{7}=$ Rs. $\frac{120}{7}=$ sum after 3rd ....

$\dfrac{166\frac{2}{3}}{100}$ of Rs $\frac{120}{7}=$ Rs. $\frac{200}{7}=$ sum after 4th............

$\therefore$ Rs $\frac{200}{7}$: Rs 10,000 : : Rs. 100 : roqd. sum of money

$\therefore$ reqd. sum of money $=$ Rs $\dfrac{100\times 10000\times 7}{200}=$ Rs. 35000.

7. $x-a\,\big)\,x^n-a^n\,\big(\,x^{n-1}+ax^{n-2}+a^2x^{n-3}+\&c \ldots\ldots\ldots + a^{n-3}x^2+a^{n-2}x+a^{n-1}$

See Student's Algebra, Part I. Art. 71

(i) Product $=(x^3-a^3)(x^3+a^3)=x^6-a^6$

8 $x^2+13x+42=x^2+7x+6x+42$

$$=x(x+7)+6(x+7)=(x+6)(x+7)$$

$x^2+x-42=x^2+7x-6x-42$

$$=x(x+7)-6(x+7)=(x-6)(x+7)$$

$343x^3+512y^3=(7x)^3+(8y)^3=(7x+8y)\{(7x)^2-7x\cdot 8y+(8y)^2\}$

$$=(7x+8y)(49x^2-56xy+64y^2).$$

9
$$\begin{array}{r|l}
3x^2-14x-80 & x^3-7x^2-80x+576\;(x-7 \\
 & 3 \\ \hline
 & 3x^3-21x^2-240x+1728 \\
 & 3x^3-14x^2-80x \\ \hline
 & -7x^2-160x+1728 \\
 & 3 \\ \hline
 & -21x^2-480x+5184 \\
 & -21x^2+98x+560 \\ \hline
-578 & -578x+4624 \\ \hline
 & x-8
\end{array}$$

$$\begin{array}{r|l}
x-8 & 3x^2-14x-80\;(3x+10 \\
 & 3x^2-24x \\ \hline
 & 10x-80 \\
 & 10x-80 \\ \hline
\end{array}$$

$\therefore$ G C M $=x-8$

Again, $x^3-7x^2-80x-576=(x-8)(x^2+x-72)$

$$=(x-8)(x-8)(x+9)$$

$3x^2-14x-80=(3x+10)(x-8)$

$3x^2+17x-90=(3x-10)(x+9)$,

$\therefore$ L C M $=(x-8)(x-8)(x+9)(3x+10)(3x-10)$

$$=(x-8)^2(x+9)(9x^2-100)$$

10 Let $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{e}{f}=x$

then $a=bx$, $c=dx$, and $e=fx$

$\therefore a^3=b^3x^3$, $c^3=d^3x^3$, and $e^3=f^3x^3$

$\therefore a^3+c^3+e^3=(b^3+d^3+f^3)x^3$

$\therefore \dfrac{a^3+c^3+e^3}{b^3+d^3+f^3}=x^3$ or $\sqrt[3]{\left(\dfrac{a^3+c^3+e^3}{b^3+d^3+f^3}\right)}=x=$ each ratio.

11 (i) $(6x+9)^2+(8x-7)^2=(10x+3)^2-71$

$36x^2+108x+81+64x^2-112x+49=100x^2+60x+9-71$

5. "Multiply the numbers together as if they were whole numbers, and point off in the product from the right hand side as many decimal places as there are decimal places in both the multiplicand and the multiplier together . if there are not figures enough, supply the deficiency by prefixing cyphers on the left hand side"

First When the number of decimal places in the dividend exceeds the number of decimal places in the divisor

Divide as in whole numbers, and mark off in the quotient a number of decimal places equal to the excess of the number of decimal places in the dividend over the number of decimal places in the divisor, if there are not figures sufficient, prefix cyphers.

Secondly —When the number of decimal places in the dividend is less than the number of decimal places in the divisor

Affix cyphers to the dividend until the number of decimal places in the dividend equals the number of decimal places in the divisor; the quotient up to this point of the division will be a whole number, if there be a remainder, and the division be carried on further, the figures in the quotient after this point will be decimals

$$\begin{array}{r} 256 \\ 0025 \\ \hline 1280 \\ 512 \\ \hline 0006400 \\ = 00064 \end{array}$$

$$4)\ 0036\ (\ 009 \qquad \underline{36}$$

$$00001)\ 4\,00000\ (\ 400000. \qquad \underline{4}$$

6 The left side $=\{a^2x^2+b^2y^2+2axby+a^2y^2+b^2x^2-2abxy\}$

$\times\{a^2x^2+b^2y^2+2axby-a^2y^2-b^2x^2-2abxy\}$

$=\{x^2(a^2+b^2)+y^2(a^2+b^2)\}\times\{x^2(a^2-b^2)-y^2(a^2-b^2)\}$

$=\{(a^2+b^2)(x^2+y^2)\}\{(a^2-b^2)(x^2-y^2)\}$

$=(a^4-b^4)(x^4-y^4)$

7. Dividend $=(x^3+y^3)^2=(x+y)^2(x^2-xy+y^2)^2$

$\therefore$ Quotient $=(x^2-xy+y^2)^2$

8 (*a*) $x^{12}-a^{12}=(x^6+a^6)(x^6-a^6)$

$=(x^2+a^2)(x^4-a^2x^2+a^4)(x^3+a^3)(x^3-a^3)$

$=(x^2+a^2)(x^4-a^2x^2+a^4)(x+a)(x^2-ax+a^2)(x-a)\cdot$

(x^2+ax+a^2).

(*b*) $\text{Ans.}=\dfrac{\dfrac{2a}{a+b}}{\dfrac{2b}{a+b}}-\dfrac{\dfrac{2a^2}{a^2+b^2}}{\dfrac{2b^2}{a^2+b^2}}=\dfrac{2a}{2b}\times\dfrac{2b^2}{2a^2}=\dfrac{b}{a}$

$100x^2-4x+130=100x^2+60x-62$

$\therefore 60x+4x=130+62,\qquad \therefore 64x=192,\qquad \therefore x=3$

(ii) $\quad {\cdot}65x+\dfrac{{\cdot}585x-{\cdot}975}{6}=\dfrac{1{\cdot}56}{2}-\dfrac{{\cdot}39x-{\cdot}78}{9}$

Multiply both sides by 18

$1{\cdot}17x+1{\cdot}755x-2{\cdot}925=14{\cdot}04-{\cdot}078x+{\cdot}156$

or $2{\cdot}925x-2{\cdot}925={\cdot}156-{\cdot}078x+14{\cdot}04$

or $2{\cdot}925x+{\cdot}078x+2{\cdot}925+{\cdot}156+14{\cdot}04$

or $3{\cdot}705x=18{\cdot}525,\quad \therefore x=\dfrac{18{\cdot}525}{3{\cdot}705}=5$

(iii) $\left.\begin{array}{ll}\dfrac{x-2}{2}-\dfrac{x+y}{11}=\dfrac{x-y-1}{8}-\dfrac{y+12}{4} & (1)\\ \dfrac{x+7}{3}+\dfrac{y-5}{10}=1-x-\dfrac{5(y+1)}{7}\quad .. & (2)\end{array}\right\}$

From (1), $28x-56-4x-4y=7x-7y-7-14y-168$,

$\therefore 17x+17y=-119$, or $x+y=-7$ . . (3)

From (2), $70x+490+21y-105=210-210x-150y-150$

$\therefore 280x+171y=-325$ . . . .(4)

and $280x+280y=-1960$, from (3)

By subtr $\quad -109y=1635 \qquad y=-15$ and $x=8$, from (3)

12 Let x = distance (in miles) from P where A meets B, then $3\frac{1}{2}-x$ = distance from Q

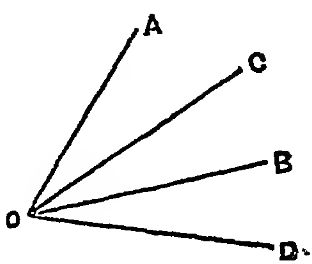
By the question,

$\dfrac{3\frac{1}{2}}{6}+\dfrac{1\frac{1}{2}}{6}+\dfrac{3\frac{1}{2}-x}{6}=\dfrac{x}{3}$

$\therefore 7+3+7-2x=4x$ or $6x=17$

$\therefore x=\frac{17}{6}=2\frac{5}{6}$ miles from P

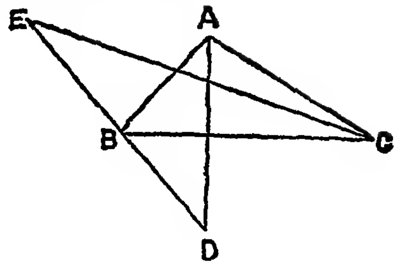
1882.—AFTERNOON



1 Euclid, B I Prop 9

(a) ∵ ∠AOB=2∠COB, (Hyp.)
add to each 2∠DOB.
∴ ∠AOB+2∠DOB
=2∠COB+2∠DOB (Ax. 2)
or ∠AOD+∠DOB=2∠COD

2 Euclid, B I. Prop. 27.



(*b*) $\therefore$ EB=BC (Hyp)
$\therefore$ BEC=BCE (I 5)
but $\angle$BCE=ACE (Hyp)
$\therefore$ $\angle$BEC=$\angle$ACE (Ax I)
Hence BE $\parallel$ AC (I 27)
$\therefore$ EBA=$\angle$BAC (I 29).
Similarly BD $\parallel$ AC
$\therefore$ $\angle$DBC=$\angle$BCA (I 29)
Hence $\angle$EBA+$\angle$DBC=
$\angle$BAC+BCA
Add to each $\angle$ABC
$\therefore$ $\angle$EBA+$\angle$DBC+$\angle$ABC
=$\angle$BAC+$\angle$BCA+$\angle$ABC=2rt $\angle$s (I 32).
$\therefore$ EBD is a str line (I. 14).

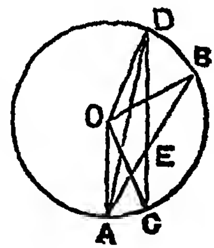
3 Euclid, B II Prop 9

(*c*) See Question 5 of 1881.

(*d*) Divide AB at any other point D.

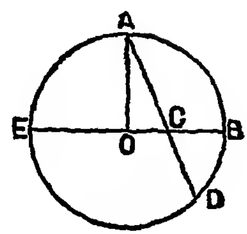
(2 9) Then $\because$ $AD^2+DB^2=2AO^2+2OD^2$
$\therefore$ $AD^2+DB^2>2AO^2$ by $2OD^2$
$>AO^2+AO^2$ (*i e*) AO^2+BO^2 ($\because$ AO=BO)
Hence AO^2+BO^2 is the least.

4. Euclid B III Prop 20



(*e*) Find O the centre
and join AO, BO, CO, DO, AD
$\therefore$ $\angle$AOC=2 $\angle$ADC and $\angle$ BOD=2$\angle$BAD (III 20)
$\therefore$ $\angle$AOC+$\angle$BOD=2$\angle$ADC+2 $\angle$BAD (Ax 2)
but $\angle$ADC+$\angle$BAD=$\angle$AEC (I. 32)
$\therefore$ $\angle$AOC+$\angle$BOD=2 $\angle$AEC

5. Euclid B. III Prop 36



(*f*) Prod BO to meet the circum in E.
AC AD=AC^2+AC CD (II 3)
=AO^2+OC^2+EC CB (III 35)
=AO^2+OB^2 (II. 5)=$2OB^2$.

6 Euclid B. IV Prop 2.

(*g*) There is some ambiguity in the wording of the question.

1883.—MORNING

Examiners, $\left\{\begin{array}{l}\text{Rev. J. P. Ashton.}\\ \text{Mr. J. H. Gilliland.}\end{array}\right.$

1 Divide $2\frac{1}{4}+8\frac{6}{11}-\frac{1}{2}$ of $(7\frac{1}{2}-3\frac{1}{3})$ by $11+\cfrac{1}{1-\cfrac{1}{1+\cfrac{1}{8\frac{1}{11}}}}$

2 Divide the square root of 122·257249 by ·3695$\dot{6}$, and multiply the quotient by the square root of ·000625

3 What decimal of a square yard is 9 square inches? Add together 1·032 of 5 Rs ·6$\dot{4}$ of Rs 1·25 and ·08 of half a rupee. What is the value of £10·541$\dot{6}$?

4. Find by "Practice" the value of 6 tons 3 cwt 21lbs 14oz at £3 10*s* per ton

5 If it costs Rs 200 to build a wall 6 ft high by 1 ft 3 in broad by 166 ft. 8 in long, what will be the costs of building a wall $3\frac{1}{2}$ by $1\frac{1}{2}$ ft by 115 ft.

6 When will the interest amount to the principal at $3\frac{1}{2}$ per cent per annum? What will the interest on Rs 150 at one anna per rupee per month amount to in 5 years? and how much is that rate per cent per annum?

7. Divide $(a-b)^2c^2+(a-b)c^2-(c^3-b^2)b^2+(c-a)b^3$
by $(a-b)c^2-(c-a)b^2$.

8. Find the Greatest Common Measure of
$x(6x^3-8y^2)-y(3x^2-4y^2)$ and $2xy\,(2y-x)+4x^3-2y^3$

9 Simplify $\frac{2}{a+x}-\frac{1}{a-x}-\frac{3x}{a^2-x^2}+\frac{ax}{a^3+x^3}$.

10. Find the value of $\frac{x^2-y^2+x}{y^2-x^2+y}$ when $x=\frac{a-b}{a+b}$ and $y=\frac{a+b}{a-b}$

11 Solve the equation —

(a) $\left.\begin{array}{r} x^2+y^2=a^2\\ xy=b^2\end{array}\right\}$

(b) $\frac{x^2-2\frac{1}{2}}{4}-\frac{x-3\frac{1}{2}}{5}=\frac{2x^2-3}{8}-\frac{x-5\frac{1}{2}}{3}$

(c) $\frac{x}{\cdot 5}-\frac{1}{\cdot 05}+\frac{x}{\cdot 005}-\frac{1}{\cdot 0005}=0$

(d) $\left.\begin{array}{l} 3y+x-2=0\\ 3z-4y=x+15\\ 2x+7z=7\end{array}\right\}$

12 Reverse the digits of a number and it will become five-sixths of what it was before also the difference between the two digits is one Find the number Also find that number of three digits which is the same when reversed, the the sum of whose digits is 16 and the difference 2

1883.—AFTERNOON.

Examiners— Mr A Macdonell, Mr R Carter, Babu Gauri Sanker De

1 If two triangles have two sides of the one equal to two sides of the other each to each, and have likewise the angles contained by these sides equal to one another, they shall likewise have their bases or third sides, equal, and the two triangles shall be equal and their other angles shall be equal each to each, namely those to which the equal sides are opposite.

2 To describe a parallelogram that shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle

3 ABC is a triangle The line bisecting the angle B meets the line bisecting the angle C in the point G and the line bisecting the external angle at A in the point D Prove that the angle ADG is equal to the angle ACG

4 In every triangle, the square on the side subtending an acute angle is less than the squares on the sides containing that angle by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall upon it from the opposite angle and the acute angle

5 If two straight lines cut one another within a circle, the rectangle contained by the segments of one of them shall be equal to the rectangle contained by the segments of the other

6 Through one extremity of the common chord of two intersecting circles two straight lines are drawn terminated by these circles Prove that the lines joining the extremity of the common chord and the two terminal points of the two straight lines on each circle together with the lines joining these terminal points form two equiangular triangles

7. In a given circle to inscribe a triangle equiangular to a given triangle.

SOLUTIONS.

1883.—MORNING.

1 $2\frac{1}{3}+8\frac{6}{11}-\frac{1}{5}$ of $(7\frac{1}{2}-3\frac{1}{3})=2\frac{1}{3}+8\frac{6}{11}-\frac{1}{5}$ of $4\frac{1}{6}=2\frac{1}{3}+8\frac{6}{11}-\frac{1}{5}$ of $\frac{25}{6}$

$$=2\frac{1}{3}+8\frac{6}{11}-\frac{5}{6}=10+\frac{22+36-55}{66}=10\frac{3}{66}=10\frac{1}{22}.$$

$$\text{and } 11+\cfrac{1}{1-\cfrac{1}{1+\cfrac{1}{8\frac{1}{11}}}}=11+\cfrac{1}{1-\cfrac{1}{1+\frac{11}{89}}}$$

$$=11+\cfrac{1}{1-\cfrac{1}{\frac{100}{89}}}=11+\frac{1}{1-\frac{89}{100}}$$

$$=11+\frac{1}{\frac{11}{100}}=11+\frac{100}{11}=\frac{221}{11}$$

$\therefore$ the quotient $=10\frac{1}{22}\div\frac{221}{11}=\frac{221}{22}\times\frac{11}{221}=\frac{1}{2}$

2 $\sqrt{122\ 257249}=11\ 057$, and $36856=\frac{11057}{30000}$

$\therefore$ the quotient $=\frac{11057}{1000}\times\frac{30000}{11057}=30$

and $30\times\sqrt{(\ 000625)}=30\times\ 025=\ 75$

3. $\left.\begin{array}{l}9 \text{ sq in.}=\frac{9}{144}\times\frac{1}{9}\text{sq yd}\\ \text{and } 1 \text{ sq yd.}=1\text{sq yd}\end{array}\right\}$

$\therefore$ decimal reqd $=\dfrac{\frac{9}{144}\times\frac{1}{9}}{1}=\dfrac{1}{144}=\ 00694$

1 032 of Rs 5 = Rs 5 16

64 of 1 25 Rs = Rs 8

and 08 of half a rupee Rs 04

$\therefore$ sum = Rs 5 16 + Rs 8 + Re 04 = Rs 6

$£\ 10\ 541\dot{6}=10+\dfrac{5416-541}{9000}£=£10+\frac{13}{24}£=£10\ 10s.\ 10d.$

4 2 cwt $=\frac{1}{10}$ of 1 ton

	£	s	d	
	3	10	0	
			6	
	21	0	0	= value of 6 tons.
		7	0	= value of 2 cwt
1 cwt. = $\frac{1}{2}$ of 2 cwt.		3	6	= value of 1 cwt.
14 lbs = $\frac{1}{8}$ of 1 cwt			$5\frac{1}{4}$	= value of 14 lbs.
7 lbs = $\frac{1}{2}$ of 14 lbs.			$2\frac{5}{8}$	= value of 7 lbs.
14 oz = $\frac{1}{8}$ of 7 lbs.			$\frac{21}{64}$	= value of 14oz.
	£21	11	$2\frac{13}{64}$	= value of 6 tons. 3 cwt. 21 lbs 14 oz.

5 $(6\times1\frac{1}{4}\times166\frac{2}{3})$ cub ft : $(3\frac{1}{2}\times1\frac{1}{2}\times115)$ cub. ft :: Rs 200 : x Rs.

$$\therefore x=\text{Rs.}\frac{200\times7\times3\times115\times4\times3}{2\times2\times6\times5\times500}=\text{Rs }\frac{483}{5}$$

$=$ Rs. 96 9as $7\frac{1}{5}$p

6 Let £ 100 be the principal

$\therefore$ £$3\frac{1}{2}$ £100 1 year reqd no of years

$\therefore$ reqd no of years$=\frac{1\times 100\times 2}{7}=\frac{200}{7}=28\frac{4}{7}$

The interest is one anna per rupee per month, $\therefore$ 12 annas is the int. per rupee every twelve month or one year, or $\frac{3}{4}$ Rs is the int. per rupee.

$\therefore$ the rate per cent. per annum is $\frac{3}{4}$ of 100 or 75

$\therefore$ the int. reqd $=$Rs $\frac{150\times 75\times 5}{100}$Rs. 562 8as

7. Divide$=(a-b)c^2(a-b+c)-(c-a)b^2(c+a-b)$

$=(a-b+c)\{(a-b)c^2-(c-a)b^2\}$,

$\therefore$ quotient$=a-b+c$

8. Ist quantity$=(2x-y)(3x^2-4y^2)$

2nd quantity$=4x^3-2x^2y+4xy^2-2y^3$

$=2x^2(2x-y+2y^2(2x-y)=(2x-y)(2x^2+2y^2)$

$\therefore$ G C M $=2x-y$

9 Fraction$=\frac{2(a-x)-(a+x)+3x}{a^2-x^2}+\frac{ax}{a^3+x^3}$

$$=\frac{a}{a^2-x^2}+\frac{ax}{a^3+x^3}=\frac{a\,(a^2-ax+x^2)+ax(a-x)}{(a-x)(a^3+x^3)}$$

$$=\frac{a^3}{(a-x)(a^3+x^3)}$$

10. $\frac{(x^2-y^2)+x}{-(x^2-y^2)+y}=\frac{(x+y)(x-y)+x}{-(x+y\,(x-y)+y}$

$$=\frac{\left(\frac{a-b}{a+b}+\frac{a+b}{a-b}\right)\left(\frac{a-b}{a+b}-\frac{a+b}{a-b}\right)+\frac{a-b}{a+b}}{-\left(\frac{a-b}{a+b}+\frac{a+b}{a-b}\right)\left(\frac{a-b}{a+b}-\frac{a+b}{a-b}\right)+\frac{a+b}{a-b}}$$

$$=\frac{\frac{a-b}{a+b}-\frac{8ab(a^2+b^2)}{(a^2-b^2)^2}}{\frac{a+b}{a-b}+\frac{8ab\,(a^2+b^2)}{(a^2-b^2)^2}}+\frac{\frac{a^4-90a^3b-6ab^3-b^4}{(a^2-b^2)^2}}{\frac{a^4+10a^3b+6ab^3-b^4}{(a^2-b^2)^2}}$$

$$=\frac{a^4-10a^3b-6ab^3-b^4}{a^4+10a^3b+6ab^3-b^4}$$

11. (a) $x^2+y^2=a^2$ and $xy=b^2$. $\therefore$ $2xy=2b^2$ $\therefore$

$\therefore$ $x^2+y^2+2xy=a^2+2b^2$, or $x+y=\sqrt{(a^2+2b^2)}$......(1)

and $x^2+y^2-2xy=a^2-2b^2$, or, $x-y=\sqrt{(a^2-2b^2)}$ .....(2)

Adding (1) and (2) $2x=\sqrt{(a^2+2b^2)}+\sqrt{(a^2-2b^2)}$

$\therefore x=\frac{1}{2}\{\sqrt{(a^2+2b^2)}+\sqrt{(a^2-2b^2)}\}.$

Subtracting (2 from (1) $2y=\sqrt{(a^2+2b^2)}-\sqrt{(a^2-2b^2)}$

$\therefore y=\frac{1}{2}\{\sqrt{(a^2+2b^2)}-\sqrt{(a^2-2b^2)}\}.$

(b) $\frac{x^2-2\frac{1}{2}}{4}-\frac{x-3\frac{1}{2}}{5}=\frac{2x^2-3}{8}-\frac{x-5\frac{1}{2}}{3}$

$\frac{x^2}{4}-\frac{5}{8}-\frac{x}{5}+\frac{7}{10}=\frac{x^2}{4}-\frac{3}{8}-\frac{x}{3}+\frac{11}{6}$

or, $\frac{x}{3}-\frac{x}{5}=\frac{5}{8}-\frac{7}{10}-\frac{3}{8}+\frac{11}{5}$ or $\frac{2x}{15}=\frac{83}{60}$

$\therefore 8x=83 \;\therefore x=\frac{83}{8}=10\frac{3}{8}.$

(c) $\frac{x}{\cdot5}-\frac{1}{\cdot05}+\frac{x}{\cdot005}-\frac{1}{\cdot0005}=0$

$\frac{x}{\frac{1}{2}}-\frac{1}{\frac{1}{20}}+\frac{x}{\frac{1}{200}}=\frac{1}{\frac{1}{2000}}$

or $2x+200x=2000+20$ or $202x=2020 \quad \therefore x=10$

(d) $x+3y-2=0.$ ... (1), $3z-4y=x+15$..... (2)

$2x+7z=7$.........(3)

Multiply (1) by 4 and (2) by 3, then

$$\left.\begin{array}{l}4x+12y=8\\ 9z-12y-3x=45\end{array}\right\}\quad \text{By addition}\quad x+9z=53 \;\ldots\;(4)$$

Multiply (4) by 2 and subtract (3)

$\therefore 11z=99 \qquad \therefore z=9$

From (3) $2x=7-7z=-56, \quad \therefore x=-28$

From (1) $3y=2-x=30, \quad \therefore y=10$

12. Let $x=$ the digit in the ten's place

and $y=$ the digit in the unit's place.

Then $10x+y$ is the number

By the question,

$10y+x=\frac{5}{6}(10x+y)$ .. .. (1), $x-y=1$......(2),

From (1) $60y+6x=50x+5y$, or $55y=44x \quad \therefore x=\frac{5y}{4}.$

Substitute x in (2), then

$\frac{5y}{4}-y=1$, or $\frac{y}{4}=1$, $\therefore y=4$, and $x=5.$

$\therefore$ the number is 54

(1) Let x, y and z be the three digits in the hundred's, ten's and unit's place respectively

Then the no is $100x+10y+z$,

Since by the question,

$100x+10y+z=100z+10y+x$, $\therefore x=z$

Also $2x+y=16$ ..(1), and $x-y=2$......(2)

Adding (1) and (2), $3x=18$ $\therefore x=6$

and $y=x-2=4$

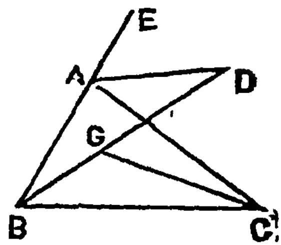
Hence the number $=646$

1883.—AFTERNOON.

1 Euclid I. Prop. 4

2 Euclid I. Prop. 42

3 Produce BA to E



$\because \angle EAC=\angle ABC+\angle ACB$ (I 32)

$\therefore \angle EAD=\angle ABD+\angle ACG$ (Ax. 7).

But $\angle EAD=\angle ABD+\angle ADB$ (I. 32)

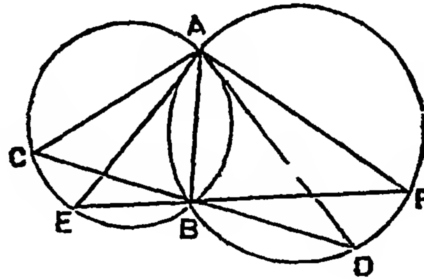
$\therefore \angle ABD+\angle ACG=\angle ABD+\angle ADB$ (Ax 1)

Hence $\angle ACG=\angle ADB$. (Ax. 3)

4 Euclid II. Prop 13

5. Euclid III. Prop. 35

6 Let the two ⊙s cut each other in A and B, and join AB



Through B draw any two lines CBD, EBF, meeting the circumference of each in the points C, E, and D, F.

Join AC, AE, AD and AF.

Then $\triangle ACD$ is equiangular to $\triangle AEF$

$$\left.\begin{array}{l}\because \angle ACB=\angle AEB\\ \text{and } \triangle ADB=\angle AFB\end{array}\right\}\text{(III. 21)}$$

rem $\angle CAD=\angle EAF$ (I. 32)

Hence $\triangle ACD$ is equiangular to AEF.

7. Euclid IV. Prop. 2.

1885.—MORNING.

Examiners,—{ REV FATHER E H YOUNAN, S J. / BABU BIPIN BIHARI GUPTA.

1. Of what number is $2\frac{1}{6}$ the $\frac{5}{6}$th part?

By what fraction must $\frac{1\frac{1}{4}}{1\frac{1}{12}}$ of $\frac{2}{3}+\frac{2\frac{1}{2}-1\frac{5}{8}}{\frac{1}{4}+1\frac{5}{6}}-\frac{8\frac{1}{5}}{7\frac{1}{3}}$ be divided in order to give a quotient $=\frac{2}{3}$?

2. Simplify $\frac{12 \text{ of } (\cdot0104-\cdot002)+\cdot36\times\cdot002}{12\times\cdot12}$; and express your result as a fraction of $\cdot6$

Reduce $\frac{5}{8}$ of 16*s* $4\frac{1}{2}d.$ to the decimal of £1 9*s*. $10\frac{1}{4}d$

3 What circulating decimal multiplied by $\frac{2\frac{3}{5}}{4\frac{1}{5}}$ will give 2 for a product?

(*a*) If $\cdot42857\dot{1}$ of a barrel of beer be worth $\cdot72$ of £2 10*s*. what is the value of $\cdot625$ of the remainder?

4. Find the price of 10lbs. 11 ozs 16 dwts. 16 grs. of gold at £3 17*s* $10\frac{1}{2}d.$ per oz

(*a*) Extract the square root of $9\frac{1}{4}$ and $\frac{1}{12\cdot5}$ to 4 places

5 If 27 men can perform a piece of work in 15 days, how many men must be added to the number that the work may be finished in three-fifths of the time?

I buy a horse for £40 and sell it for £45 at a credit of 8 months. What do I gain per cent, reckoning money worth 6 per cent per annum?

6 Which is the better investment, bank stock paying 10 per cent at 319 or 3 per cent. consols at 96?

What will be the cost of £1,500 3 per cent consols at $89\frac{3}{8}$, brokerage being $\frac{1}{8}$ per cent? What rate of interest will such investment obtain?

7 Find the co-efficient of x^4 in the product of

$$x^4-ax^3+bx^2-cx+d \text{ and } x^2+px+q$$

Divide $\frac{83}{80}$ into 2 parts, differing by $\frac{14}{15}$,

8 Resolve into their simplest possible factors

$$x^8-a^8,\quad x^2-\left(a-\frac{1}{x}\right)x+1,\ x^4+a^2x^2+a^4,$$

and find the Least Common Multiple of

$$x^2-1, x^2+1, (x-1)^2, (x+1)^2, x^3-1, x^3+1.$$

9 Find the value of

(i) $\frac{x^{3n}}{x^n-1} - \frac{x^{2n}}{x^n+1} - \frac{1}{x^n-1} + \frac{1}{x^n+1}$

(ii) $\left(\frac{x}{x-1}\right)^2 + \left(\frac{x}{x+1}\right)^2$, when $x = \sqrt{\left(\frac{n-1}{n+1}\right)}$.

10 Solve the following equations —

(i) $\frac{x-ax}{\sqrt{x}} = \frac{\sqrt{x}}{x}$. (ii) $\frac{ax-1}{\sqrt{(ax)}+1} = 4 + \frac{\sqrt{(ax)}-1}{2}$.

(iii) $(1+x)^{\frac{1}{3}} + (1-x)^{\frac{1}{3}} = (2)^{\frac{1}{3}}$

(iv) $\frac{m}{x} - \frac{n}{y} = a$, $px = qy$

11 Find to 4 terms the square root of $1 - x - x^2$

12 Two sums of money are together equal to £54 12*s* and there are as many pounds in the one as shillings in the other. What are the sums?

A boy buys a certain number of oranges at 3 for 2*d*, and one-third of that number at 2 for 1*d*, at what price must he sell them to get 20 per cent profit? If his profit be 5*s* 4*d*, find the number bought.

1885.—AFTERNOON

Examiners,— { Babu Saradaranjan Ray
" Gouri Sankar De
" Prassanakumar Sarbadhikari.

1 (*a*) Define a *plane*, a *right angle*, *parallel straight lines*, a *rhombus*, a *gnomon*, and a *segment of a circle*,

(*b*) If at a point in a straight line, two other straight lines on the opposite sides of it, make the adjacent angles together equal to two right angles, then these two straight lines shall be in one and the same straight line

2 (*a*) In one right-angled triangle, the square which is described on the side subtending the right angle, is equal to the squares described upon the sides which contain the right angle

(*b*) Divide a given straight line into two parts, such that the difference between the squares described upon the two parts may be equal to the square on a given straight line

3 In obtuse-angled triangles if a perpendicular be drawn from either of the acute-angles upon the opposite side produced, the square on the side subtending the obtuse angle is greater than the square on the sides containing the obtuse angle, by twice the rectangle contained by the side upon which, when pro-duced, the perpendicular falls, and the straight line intercepted without the triangle, between the perpendicular and the obtuse angle.

9 $x^3-19x+30\,)\,x^3+3x^2-9x+5\,(\,1$

$\underline{x^3 \qquad\quad -19x+30}$

$3x^2+10x-25$

$3x^2+10x-25\,)\,3x^3-57x+90\,(\,x-10$

$\underline{3x^3+10x^2-25x}$

$-10x^2-32x+90$

$\underline{3}$

$-30x^2-96x+270$

$\underline{-30x^2-100x+250}$

$4\,)\,4x+20$

$x+5$

$x+5\,)\,3x^2+10x-25\,(\,3x-5$

$\underline{3x^2+15x}$

$-5x-25$

$\underline{-5x-25}$

$\therefore x+5$ is the G C M

10 (i) or $\frac{1}{2}x-\frac{a}{6}-\frac{1}{4}x+\frac{a}{12}+\frac{1}{5}x-\frac{a}{20}=0$

$\therefore \frac{9}{20}x=\frac{2}{15}a, \qquad x=\frac{8}{27}a$

(ii) or $\frac{a(x-b)-b(x-a)}{(x-a)(x-b)}=\frac{a-b}{x-c}, \quad \therefore \frac{x(a-b)}{(x-a)(x-b)}=\frac{a-b}{x-c}.$

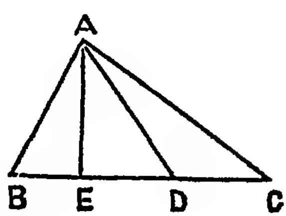
Dividing by $a-b$, and simplifying, we get

$x(x-c)=(x-a)(x-b)$, or $x^2-cx=x^2-(a+b)x+ab$

$\therefore x(a+b-c)=ab, \qquad \therefore x=\frac{ab}{a+b-c}$

DECEMBER, 1859.—AFTERNOON.

1. Euclid I 16.



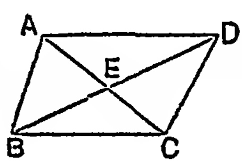
2 If possible, from A draw three lines AB, AC and AD equal to one another.

$\because$ AB=AC $\therefore$ ∠ABC=∠ACB

and $\because$ AB=AD $\therefore$ ABC=∠ADB

$\therefore$ ∠ACB=∠ADB, which is impossible (I 16)

3. Euclid I 34



4 Let ABCD be a ▭ and AC and BD its diagonals intersecting in E

$\because$ diagonal of a ▭ bisect one another

$\therefore$ AE=EC and BE=ED

$\therefore$ △ AED=△DEC and △ AED=△ EBC and △ AEB=△AED

$\therefore$ The four triangles are equal

5 Euclid II 12.

6. Euclid III. 20

4. (*a*) In a circle, the angle in a semicircle is a right angle; but the angle in a segment greater than a semicircle is less than a right angle; and the angle in a segment less than a semicircle is greater than a right angle

(*b*) AB is a diameter of a circle, and AC a tangent at A equal in length to AB, CB is joined cutting the circle in D; prove that CB is bisected in D, and AD equal to half of CB.

5. (*a*) Describe a circle touching three given straight lines lying in one plane, no two of which are parallel. Show that *four such* circles can be described.

(*b*) ABC is an acute-angled triangle Perpendiculars AD, BE, CF, are drawn from the angular points A, B, C, upon the oposite sides respectively, intersecting in O · prove that O is the centre of the circle inscribed in the triangle DEF, and A, B, C are the centres of circles *described* to the same triangle.

6 Inscribe a regular hexagon in a given circle

SOLUTIONS

1885.—MORNING.

1. Number $= 2\frac{1}{6} \div \frac{5}{6} = \frac{13}{6} \times \frac{6}{5} = \frac{13}{5} = 2\frac{3}{5}$

(*a*) Here, $\dfrac{1\frac{1}{4}}{1\frac{1}{12}}$ of $\frac{2}{3} + \dfrac{2\frac{1}{2}-1\frac{5}{6}}{\frac{1}{4}+1\frac{5}{6}} - \dfrac{8\frac{1}{5}}{7\frac{4}{5}}$

$$= \frac{5}{4} \times \frac{12}{13} \times \frac{2}{3} + \frac{\frac{5}{2}-\frac{11}{6}}{\frac{1}{4}+\frac{11}{6}} - \frac{41}{5} \times \frac{5}{39}$$

$$= \frac{10}{13} + \frac{\frac{15-11}{6}}{\frac{3+22}{12}} - \frac{41}{39} = \frac{10}{13} + \frac{4}{6} \times \frac{12}{25} - \frac{41}{39}$$

$$= \frac{10}{13} + \frac{8}{25} - \frac{41}{39} = \frac{8}{25} - \frac{11}{39} = \frac{312-275}{25 \times 39} = \frac{37}{25 \times 39}$$

$\therefore$ Fraction required $= \dfrac{37}{25 \times 39} \div \frac{2}{3} = \dfrac{37}{25 \times 39} \times \frac{3}{2} = \frac{37}{650}$

2. Ans. $= \dfrac{\cdot12 \times \cdot0084 + \cdot36 \times \cdot002}{\cdot12 \times 12} = \dfrac{12(\cdot0084 + 3 \times \cdot002)}{12 \times \cdot12}$

$$= \frac{\cdot0084 + \cdot006}{\cdot12} = \frac{\cdot0144}{\cdot12} = \cdot12.$$

and fraction required $= \dfrac{\cdot12}{\cdot6} = \dfrac{12}{60} = \frac{1}{5}$.

(*a*) Here $\frac{5}{9}$ of 16*s*. $4\frac{1}{2}$*d*. $= \frac{5}{9}$ of $196\frac{1}{2}$*d*. $= \frac{5}{9} \times \frac{393}{2}$*d*. $= \dfrac{5 \times 131}{6}$*d*.

and £1 9s. $10\frac{1}{4}d = 358\frac{1}{4}d = \frac{1433}{4}d$

$\therefore$ Decimal reqd. $= \frac{5\times 131}{6} \times \frac{4}{1433} = \frac{1310}{4299} = 30472\ldots\ldots$

3. Here, $\frac{2\frac{3}{5}}{4\frac{1}{2}} = \frac{13}{5} \times \frac{2}{9} = \frac{26}{45}$

$\therefore$ Decimal reqd. $= 2 - \frac{26}{45} = 2 \times \frac{45}{26} = \frac{45}{13} = 3\ 46153\dot{8}$

(a) Here, $\dot{4}2857\dot{1} = \frac{428571}{999999} = \frac{3}{7}$,

therefore ·625 of the remainder $= \frac{625}{1000}$ of $(1 - \frac{3}{7}) = \frac{5}{8}$ of $\frac{4}{7} = \frac{5}{14}$.

Since $\frac{3}{7}$ of a barrel costs (72 of £$2\frac{1}{2}$ or £$\frac{9}{5}$)

$\therefore$ $\frac{1}{7}$ of barrel costs $\frac{1}{3}$ of £$\frac{9}{5}$ or £$\frac{3}{5}$

$\therefore$ $\frac{1}{14}$ of a barrel costs of $\frac{1}{2}$ of £$\frac{3}{5}$ of £$\frac{3}{10}$

$\therefore$ $\frac{5}{14}$ of a barrel costs £ $\frac{5\times 3}{10}$ £$\frac{3}{2}$ = £1 10s.

4.

	£.	s.	d.	
	3	17	$10\frac{1}{2}$	= price of 1 oz
			11	
	42	16	$7\frac{1}{2}$	= price of 11 oz.
	46	14	6	= price of 12 oz or 1 lbs.
			10	
	467	5	0	= price of 10 lbs
10 dwts $= \frac{1}{2}$ of 1 oz	42	16	$7\frac{1}{2}$	= price of 11 ozs.
5 dwts $= \frac{1}{2}$ of 10 dwts	1	18	$11\frac{1}{4}$	= price of 10 dwts.
1 dwt $= \frac{1}{5}$ of 5 dwts.		19	$5\frac{5}{8}$	= price of 5 dwts
12 grs $= \frac{1}{2}$ of 1 dwts		3	$10\frac{29}{40}$	= price of 1 dwt.
4 grs $= \frac{1}{3}$ of 12 dwts		1	$11\frac{29}{80}$	= price of 12 grs
			$7\frac{189}{240}$	= price of 4 grs
	£513	6	$6\frac{1}{4}d.$	= price of 10 lbs 11 ozs. 16 dwts. 16 grs.

(a) $\sqrt{(9\frac{3}{4})} = \sqrt{(\frac{39}{4})} = \frac{\sqrt{(39)}}{2} = \frac{6\ 2449}{2} = 3\ 1225$

$\sqrt{\left(\frac{1}{12\ 5}\right)} = \left(\frac{2}{25}\right) = \frac{\sqrt{2}}{5} = \frac{1\ 4142}{5} = 2828.\ \ldots$

5 $\frac{3}{5}$ of 15 days $= 3 \times 3$ days $= 9$ days

Now, 9 15 . 27 men no of men,

$\therefore$ no of men $= \frac{27\times 15}{9} = 45$.

Hence additional no. of men $= 45 - 27 = 18$

(a) Here, amount of £100 for 8 months at 6 per cent.

$= £\ (100 + \frac{8}{12} \times 6) = £104$

£104 . £45 £100 present worth of £45,

$\therefore$ present worth $= £\dfrac{100 \times 45}{104} = £\frac{1125}{26}$

$\therefore$ gain $= £(\frac{1125}{26} - 40) = \frac{85}{26}$

£40 £100 · · £$\frac{85}{26}$ gain per cent.

$\therefore$ gain per cent. $= \dfrac{85 \times 100}{26 \times 40} = \frac{425}{52} = 8\frac{9}{52}$

6. In the first case, interest per £ $= £\frac{10}{319}$.

In the second case..... ...... $= \frac{3}{96}$.

Now, $\frac{10}{319}, \frac{3}{96} = \dfrac{960}{319 \times 96}, \quad \dfrac{957}{319 \times 96}$

But $960 > 957$,

$\therefore$ The first is the better investment

(a) £100 · £1500 $(89\frac{3}{8} + \frac{1}{8})$£ · cost

$\therefore$ cost $= £\dfrac{179 \times 1500}{2 \times 100} = £\frac{2685}{2}£ = 1342$ 10s

£89½ £100 £3 rate of interest

$\therefore$ Rate of interest $= £\dfrac{100 \times 3 \times 2}{179} = £\frac{600}{179} = £3\frac{63}{179}$ per cent.

7. $x^4 - ax^3 + bx^2 - cx + d$

$x^2 + px + q$

The terms involving $x^4 = bx^4 - apx^4 + qx^4 = x^4(b - ap + q)$

$\therefore$ Co-efficient reqd. $= b - ap + q$.

(a) The greater part $= \frac{1}{2}\ (\frac{83}{89} + \frac{14}{19}) = \frac{2823}{3382}$.

The lesser part $= \frac{1}{2}(\frac{83}{89} - \frac{14}{19}) = \frac{331}{3382}$

8. $x^3 - a^3 = x^3 - ax^2 + ax^2 - a^2x + a^2x - a^3$

$= x^2(x-a) + ax(x-a) + a^2(x-a)$

$= (x-a)(x^2 + ax + a^2)$.

$x^2 - \left(a + \dfrac{1}{a}\right)x + 1 = x^2 - ax - \dfrac{x}{a} + 1,$

$= x(x-a) - \dfrac{1}{a}(x-a)$

$= (x-a)\left(x - \dfrac{1}{a}\right)$.

$$x^4+a^2x^2+a^4=(x^2+a^2)^2-a^2x^2$$
$$=(x^2+a^2+ax)(x^2+a^2-ax).$$

(a)
$$\begin{array}{r|l} x-1 & x^2-1,\ x^2+1,\ (x-1)^2,\ (x+1)^2,\ x^3-1,\ x^3+1 \\ \hline x+1 & x+1,\ x^2+1,\ x-1,\ (x+1)^2,\ x^2+x+1,\ x^3+1 \\ \hline & 1,\ x^2+1,\ x-1,\ x+1,\ x^2+x+1,\ x^2-x+1 \end{array}$$

$\therefore$ L C M $=(x-1)(x+1)(x^2+1)(x-1)(x+1)$
$$(x^2+x+1)(x^2-x+1)$$
$$=(x^2-1)(x^2+1)(x^3-1)(x^3+1)$$
$$=(x^4-1)(x^6-1)=x^{10}-x^6-x^4+1$$

9 (i) Ans $=\dfrac{x^{3n}-1}{x^n-1}-\dfrac{x^{2n}-1}{x^n+1}=x^{2n}+x^n+1-(x^n-1)=x^{2n}+2.$

(ii) Fraction $=\dfrac{x^2(x+1)^2+x^2(x-1)^2}{(x^2-1)^2}=\dfrac{2x^2(x^2+1)}{(x^2-1)^2}$

Now, $x^2+1=\dfrac{n-1}{n+1}+1=\dfrac{2n}{n+1}$

and $x^2-1=\dfrac{n-1}{n+1}-1=-\dfrac{2}{n+1}$

$\therefore$ Ans. $=2\left(\dfrac{n-1}{n+1}\right)\times\dfrac{2n}{n+1}\div\dfrac{4}{(n+1)^2}$

$$=\frac{4n(n-1)}{(n+1)^2}\times\frac{(n+1)^2}{4}=n(n-1).$$

10. (i) Hence $\dfrac{x(1-a)}{\sqrt{x}}=\dfrac{\sqrt{x}}{x}$, or $\dfrac{(1-a)\sqrt{x}}{1}=\dfrac{1}{\sqrt{x}}$

$\therefore (1-a)x=1$, $\therefore x=\dfrac{1}{1-a}$.

(ii) Since $ax-1=\{\sqrt{(ax)}+1\}\{\sqrt{(ax)}-1\}$,

$\therefore \sqrt{(ax)}-1=4+\dfrac{\sqrt{(ax)}-1}{2}$, or $\dfrac{\sqrt{(ax)}-1}{2}=4.$

$\therefore \sqrt{(ax)}-1=8$, $\therefore \sqrt{(ax)}=9.$

$\therefore ax=81$, $\therefore x=\dfrac{81}{a}$

(iii) Cubing, $2+3(1-x^2)^{\frac{1}{3}}\{(1+x)^{\frac{1}{3}}+(1-x)^{\frac{1}{3}}\}=2.$

$\therefore 3(1-x^2)^{\frac{1}{3}}2^{\frac{1}{3}}=0$, by substitution.

$\therefore (1-x^2)^{\frac{1}{3}}=0$, cubing, $1-x^2=0.$

$\therefore x^2=1$, $\therefore x=1.$

(iv) From (1) $my - nx = ayx$.

" (2) $y = \frac{px}{q}$.

Hence by substitution, $\frac{mpx}{q} - nx = ax\frac{px}{q}$,

or $mpx - nqx = apx^2$, or $mp - nq = apx$

$\therefore x = \frac{mp - nq}{ap}$.

And $y = \frac{p}{q}x = \frac{p}{q} \times \frac{mp - nq}{ap} = \frac{mp - nq}{aq}$

11

$$
\begin{array}{l}
1 - x - x^2 \big) \; 1 - \frac{x}{2} - \frac{5x^2}{8} - \frac{5x^3}{16} \\
\qquad 1 \\
\hline
2 - \frac{x}{2} \big) \; -x - x^2 \\
\qquad -x + \frac{x^2}{4} \\
\hline
2 - x - \frac{5x^2}{8} \big) \; -\frac{5x^2}{4} \\
\qquad -\frac{5x^2}{4} + \frac{5x^3}{8} + \frac{25x^4}{64} \\
\hline
2 - x - \frac{5x^2}{4} - \frac{5x^3}{16} \big) \; -\frac{5x^3}{8} - \frac{25x^4}{64} \\
\qquad -\frac{5x^3}{8} + \frac{5x^4}{16} + \frac{25x^5}{64} + \frac{25x^6}{256} \\
\hline
\end{array}
$$

12 Let x = no of pounds in one sum,

then $54\frac{3}{5} - x$ = no of pounds in the other

$\therefore$ By the question,

$x = 20\,(54\frac{3}{5} - x)$,

$\therefore x = 1092 - 20x$, $\therefore 21x = 1092$,

$\therefore x = £52$.

Hence the sums are £52 and £2 12*s.*

(*a*) Let x = no of oranges bought at first.

and y = selling price of an orange in *d*,

then $x + \frac{1}{3}x$ or $\frac{4x}{3}$ = whole no. of oranges bought.

Cost of oranges $=\left(\frac{2x}{3}+\frac{x}{6}\right)d$. or $\frac{5x}{6}d$

∴ By the question,

$$(1+\tfrac{1}{5})\left(\frac{5x}{6}\right)=\frac{4x}{3}\times y, \text{ or } y=\tfrac{3}{4}d.$$

Hence he must sell 4 oranges for $3d$.

Again, selling price of $\frac{4x}{3}$ oranges $=\left(\frac{4x}{3}\times\frac{3}{4}\right)d=xd$.

∴ By the question,

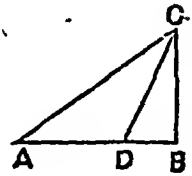
$$x-\frac{5x}{6}=64, \quad \therefore \frac{x}{6}=64, \quad \therefore x=384.$$

Hence no. of oranges bought $=\frac{4x}{3}=\frac{4}{3}\times384=512$

1885.—AFTERNOON.

1 (*a*) A plane is that which has length, breadth but no thickness. See Euclid, Defs, 10, 35, 32, II Defs 2, 1, Def. 19.
(*b*) Euclid, Book I. Prop 14.

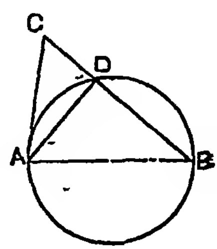
2. (*a*) Euclid Book 1 Prop 47.
(*b*) Let AB be the given straight line
From B draw BC⊥AB, and make BC= the side of the given square
Join AC, and at C in AC make the ∠ACD =∠CAD,
and let CD cut AB at D
Then D is the reqd. point of section such that AD^2-DB^2=the square on the given straight line BC



∴ ∠ACD=∠CAD (Cons) ∴ AD=CD (I. 6)
Again, ∵ ∠DBC=a rt ∠ (Cons)
∴ $CD^2=DB^2+BC^2$ (1 47) Hence $CD^2-DB^2=BC^2$.
But AD=CD
(Ax 1) ∴ $AD^2-DB^2=BC^2$
=the given square.

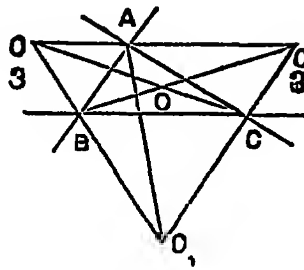
3 Euclid, Book II Prop 12

4 (*a*) Euclid, Book III Prop 31.
(*b*) ∴ ∠ADB=a rt ∠ (I 32)
∴ ∠DAB+∠DBA=a rt ∠ (III 31)
But ∠BAC=a rt ∠. (III 16, Cor)
∴ ∠DAB+∠DBA=∠BAC.
Take away the com ∠DAB
∴ ∠DBA=∠DAC (Ax 3)
But ∠DBA=∠ACD,
for AB=AC (Hyp)
∴ ∠ACD=∠DAC (Ax. 1)



$\therefore$ CD=AD (I 6)
(3. 16, Cor.) Again, $\because$ $\angle$BAC=a rt $\angle$=$\angle$ABC+ACB (I 32)
(proved) and $\angle$DAC=$\angle$ACD
$\therefore$ $\angle$DAB=$\angle$ABC (Ax. 3)
$\therefore$ AD=DB (I 6)
(Ax. 1) Hence CD=AD=DB
Wherefore CB is bisected at D, and AD=$\frac{1}{2}$CB

5. (*a*) Let the three given, str lines, no two of which are parallel, form by their intersections the triangle ABC.



Bisect the $\angle$s at B and C by BO and CO meeting a O.

(Euc IV 4) Then O is the centre of the circle touching the three given str lines

Again, if BO and CO be produced to meet the line bisecting the exterior angle at A to O_2 and O_3,

then O_2 and O_3 are the centres of two of the circles

Also, O_2B and O_3C be joined and produced to meet at O_1, then O_1 is the centre of another circle

Hence O, O_1, O_2 and O_3 are the centres of the four circles
See Hall and Steven's Euclid III Book Pedal triangle

(*b*) $\therefore$ $\angle$BEC=a rt $\angle$BFC (Hyp)
$\therefore$ a $\odot$ will go round the fig. BFEC (III. 21)
$\therefore$ $\angle$FEB=FCB (III 21)

Again, $\because$ $\angle$CEO=a rt$\angle$=$\angle$CDO (Hyp)
$\therefore$ a $\odot$ will go round the fig DOEC (III. 22)
$\therefore$ $\angle$DEO=$\angle$DOC (III 21)
Hence $\angle$FEB=$\angle$DEO or OE bisects the $\angle$FED (Ax 1)

In the same manner, it can be shewn that DO and FO severally bisect the $\angle$s EDE and DFE

Wherefore O is the centre of the circle inscribed in the ΔDEF (Euc IV 4)

Again, produce FE to any point P
(Ax 11) $\therefore$ $\angle$BEC=a rt$\angle$=$\angle$BEA
(proved) and $\angle$BED=$\angle$BEF
(Ax 3) $\therefore$ $\angle$DEC=FEA=$\angle$PEC (I 15)
Hence EC bisects the ext $\angle$ at E.
Wherefore C is the centre of one of the escribed circles of the ΔDEF.

Similarly, A and B are the centres of two other of the escribed circles of the ΔDEF.

6 Euclid, Book IV 15

1886 —MORNING

Head Examiner,—REV J. P ASHTON, M A

1 Divide $\dfrac{1\frac{2}{3}-1\frac{3}{4}}{1\frac{4}{5}-1\frac{5}{6}} \div \dfrac{1\frac{6}{7}-1\frac{7}{8}}{1\frac{8}{9}-1\frac{9}{10}}$ by $\dfrac{\frac{1}{7}\div\frac{1}{8}}{\frac{1}{11}\div\frac{1}{13}} \div \dfrac{\frac{1}{17}-\frac{1}{18}}{\frac{1}{19}\div\frac{1}{20}}$

2 Simplify $\dfrac{3\cdot125}{2\cdot16}$ of $\dfrac{24}{125}-\dfrac{2\cdot\dot{2}}{1\cdot5}$ of $\dfrac{187\cdot5}{3\cdot42}$.

3 Reduce £1 11*s*. 10¾*d* to the fraction of £7 18*s* 6½*d*.
What fraction of £10 must be added to £16 10*s*. 3*d*. to make it £20?

4 What decimal of 9 maunds 20 seers is ⅜ of 7 maunds 5 seers? Reduce 5½ square yards to the decimal of an acre

5 Find the value, by Practice of 2 tons 15 cwt 35lbs. at £13 6*s* 8*d* per ton

6 What sum of money at 4 per cent. simple interest will secure the same income as Rs. 25475 at 4½ per cent.?

7 If a rupee is equivalent to 1*s* 6¾*d*, what is the price of a sovereign in rupees? If, after buying 250 sovereigns at this price, I sell them again when the rupee is equivalent to 1*s* 6*d*. how much shall I gain or lose by the transaction?

8 Simplify $\dfrac{\dfrac{1}{m-n}-\dfrac{1}{m-s}}{\dfrac{1}{(m-n)^2}-\dfrac{1}{(m-s)^2}}$ and

$$\left\{\frac{x}{1-\frac{1}{x}}-x-\frac{1}{1-x}\right\}\div\left\{\frac{x}{1-\frac{1}{x}}+x-\frac{1}{1+x}\right\}$$

9 Find the Least Common Multiple of

$9x^4-28x^2+3$, $27x^4-12x^2+1$, $27x^4+6x^2-1$, and x^4-6x^2+9

10 Extract the square root of

$$\frac{(a^2+b^2)^2}{a^4+b^4-2a^2b^2}+4\frac{a}{a+b}\times\frac{b}{a-b}$$

11. Solve the following equations:—

(*a*) $\dfrac{a-x^2}{bx}-\dfrac{b-x}{c}=\dfrac{c-x}{b}-\dfrac{b-x^2}{cx}$;

(*b*) $\cdot011x+\dfrac{\cdot001x-\cdot125}{6}-\dfrac{5-x}{\cdot03}=145$;

(*c*) $x-y-z=-15$,
$y+x+2z=40$,
$4z-5x-6y=-150$,

(*d*) $16\left(\dfrac{a-x}{a+x}\right)^3=\dfrac{a+x}{a-x}$

12 A bankrupt paid only 17*s.* 6*d* in the pound to his creditors, and then gave four-fifth of what he still owed to the lawyers This left him £20 for his current expenses What was the amount of his debt?

13 At what time are the hands of a watch together between 5 and 6 o'clock?

1886.—AFTERNOON.

Head Examiner,—REV J P. ASHTON, M.A.

1. Define *angle*, *circle*, *acute-angled triangle*, *chord*, *sector* and *plane surface.*

2. If a quadrilateral has two opposite sides equal and parallel, it is a parallelogram

3 The square described on the difference of two straight lines together with twice the rectangle contained by the lines is equal to the sum of the squares described on them.

4. (*a*) If two circles touch internally the centre of the interior circle lies in that radius of the exterior circle which passes through the point of contact.

(*b*) Also, show that any chord of the exterior circle drawn from the point of contract is bisected by the interior circle, if that circle passes through the centre of the exterior circle.

5 When the aid of an isosceles triangle such that each of the angles at its base is seven times the angle at the vertex to inscribe a regular quindecagon in a given circle Give the geometrical proof

3 If the middle points of three sides of a triangle be joined to the opposite angles by three straight lines, prove that the sum of those three lines is less than the sum of the three sides

7. What is the angle in a quadrant of a circle, and why?

8 Two equal circles intersect in A and B Let CD and EF be chords of the circles each equal to the chord AB and so placed on opposite sides of AB that all the three chords meet in H Then AH bisects the angle CHE.

SOLUTIONS

1886.—MORNING.

1 $$\frac{1\frac{2}{3} \div 1\frac{3}{4}}{1\frac{4}{5} \div 1\frac{5}{6}} = \frac{\frac{5}{3} \times \frac{4}{7}}{\frac{9}{5} \times \frac{6}{11}} = \frac{20}{21} \times \frac{55}{54} = \frac{10 \times 55}{21 \times 27}$$

$$\frac{1\frac{6}{7} \div 1\frac{7}{8}}{1\frac{8}{9} - 1\frac{9}{10}} = \frac{\frac{13}{7} \times \frac{8}{15}}{\frac{17}{9} \times \frac{10}{19}} = \frac{104}{105} \times \frac{171}{170} = \frac{52 \times 171}{105 \times 85}$$

$$\text{Dividend}=\frac{10\times 55}{21\times 27}\times\frac{105\times 85}{52\times 171}=\frac{25\times 55\times 85}{27\times 26\times 171}$$

$$\frac{\frac{1}{7}\div\frac{1}{9}}{\frac{1}{11}\div\frac{1}{13}}=\frac{\frac{1}{7}\times\frac{9}{1}}{\frac{1}{11}\times\frac{13}{1}}=\frac{9}{7}\times\frac{11}{13}=\frac{9\times 11}{7\times 13}$$

$$\frac{\frac{1}{17}\div\frac{1}{18}}{\frac{1}{19}\div\frac{1}{20}}=\frac{\frac{1}{17}\times\frac{18}{1}}{\frac{1}{19}\times\frac{20}{1}}=\frac{18}{17}\times\frac{19}{20}=\frac{9\times 19}{17\times 10}$$

$$\therefore\ \text{Divisor}=\frac{9\times 11}{7\times 13}\times\frac{17\times 10}{9\times 19}=\frac{11\times 17\times 10}{7\times 13\times 19}$$

$$\text{Hence, Ans.}=\frac{25\times 55\times 85}{27\times 26\times 171}\times\frac{7\times 13\times 19}{11\times 17\times 10}=\frac{875}{972}$$

2 $\text{First Fraction}=\frac{3125}{2160}\times\left(\frac{24-2}{90}-\frac{125}{1000}\right)$

$$=\frac{3125}{2160}\times\frac{22}{90}\times\frac{1000}{125}=\frac{125\times 11}{54\times 9}$$

$$\text{Second fraction}=\frac{20}{9}\times\frac{10}{15}\times\frac{18750}{342}=\frac{125000}{9\times 171}$$

$$\therefore\ \text{Ans}=\frac{125\times 11}{54\times 9}\times\frac{9\times 171}{125000}=\frac{209}{6\times 1000}=03583$$

3 £1 11s $10\frac{1}{2}d=382\frac{1}{2}d$ and £7 18s $6\frac{1}{2}d=1902\frac{1}{2}$

$$\text{fraction reqd}=\frac{382\frac{1}{2}}{1902\frac{1}{2}}=\frac{765}{2}\times\frac{2}{3805}=\frac{153}{761}$$

(a) Hence £20 − £16 10s 3d − £3 9s 9d $=3\frac{39}{80}$£

$\therefore$ Fraction reqd $=3\frac{39}{80}-10=\frac{279}{80}\times\frac{1}{10}=\frac{279}{800}$.

4. $\frac{2}{3}$ of 7 mds 5 srs $=\frac{2}{3}\times 285$ seers $=190$ seers

9 mds 20 srs = 380 seers

$\therefore$ Decimal reqd $=\frac{190}{380}=5$

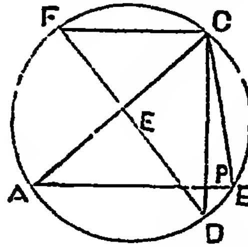
(a) $5\frac{1}{2}$ sq yds $\times\frac{11}{2}$ sq yds

1 acre = 4840 sq yds

$\therefore$ Decimal reqd. $=\frac{11}{2}\times\frac{1}{4840}=\frac{1}{880}=00113\dot{6}$

	£	s	d	
5 10 cwts $=\frac{1}{2}$ of 1 ton	13	6	8	= value of 1 ton,
			2	
	26	13	4	= value of 2 tons.
5 cwts $=\frac{1}{2}$ of 10 cwts	6	13	4	= value of 10 cwts.
35 lbs $=\frac{1}{16}$ of 5 cwts	3	6	8	= value of 5 cwts.
		4	2	= value of 35 lbs.
	£36	17	6	= value of 2 tons 14 cwts 35lbs.

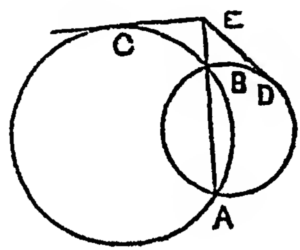
7. Let the chords AB and CD intersect in P at rt. $\angle$s to each other.



Find E the centre.
Join DE and prod it to meet the circumference in E
Join FC AC and BC
Then $\angle$FCD is a rt. $\angle$. (III 31)
$\therefore$ FC $\parallel$ AB (I 28)
and hence $\angle$FCA $=$ $\angle$CAB (I 26)
$\therefore$ the arc AF $=$ the arc BC (III 29)
Add to each of these equals the arcs FC $+$ DB,
then the arcs AC $+$ DB $=$ arc FCD, which is $\frac{1}{2}$ of the whole circumference

7. Euclid III 32

8 Let the two $\odot$s cut one another at A and B.



The str lines drawn from any point in AB produced, to touch two $\odot$s are equal to one another.
Let E be any point in AB produced
draw EC and ED tangents to the two $\odot$s ABC and ABD,
$\because$ $CE^2 = AE \cdot EB$ and
$ED^2 = AE \cdot EB$ (III 36)
(Ax 1) $\therefore$ $CE^2 = ED^2$ $\therefore$ CE $=$ ED

1860 —MORNING.

Examiners,— { Dr Mouat, / Mr. Thwaytes

1. If the price of bricks depends upon their magnitude, and if 100 bricks, of which the length, breadth and thickness are 16, 8 and 10 inches respectively, cost 2 Rs. 9as, what will be the price of 9,21,600 bricks, which are one-fourth less in every dimension?

2 Explain the method of pointing in extracting the square root of whole numbers and decimals Find the square root of 57214096 and also the square root of ·5 to four places of decimals

3 Simplify $(1+\frac{5}{6}+\frac{7}{8}+\frac{11}{12})-(\frac{7}{4}-\frac{2}{5})$,

$$\text{and } \frac{7\frac{1}{2}}{6\frac{1}{2}}+\frac{11\frac{1}{3}-2\frac{2}{3}}{11\frac{1}{3}+2\frac{2}{3}}+10\frac{2}{13}-6\frac{4}{2[illegible]}.$$

4 A tea dealer buys a chest of tea containing 2 maunds and 16 seers at 4 Rs. 2as. per seer, and two chests more each containing 3 maunds and 24 seers at Rs 4 10as per seer, at what rate per seer must he sell the whole in order to gain 576 Rupees?

6. Annual income $=$ Rs $\dfrac{25475 \times 4\frac{1}{2}}{100} =$ Rs $\dfrac{1019 \times 9}{8}$

$\therefore$ sum reqd $=$ Rs. $\dfrac{100 \times 1019 \times 9}{8 \times 4} =$ Rs $28659\frac{3}{8} =$ Rs 28659 6as.

7. $1s\ 6\frac{3}{4}d = 18\frac{3}{4}d$. and a sov. $= 240d$

$\therefore$ Price of a sovereign $= \dfrac{240}{18\frac{3}{4}} =$ Rs. $\dfrac{240 \times 4}{75}$ Rs

$=$ Rs $12\frac{4}{5} =$ Rs 12 12as. $9\frac{3}{5}$p.

Value of 250 sov. at Rs $12\frac{4}{5} =$ Rs. $250 \times 12\frac{4}{5} =$ Rs 3200.

Again, the value of a sov at 1s 6d per rupee $= \frac{240}{18}$ Rs $=$ Rs $\frac{40}{3}$.

$\therefore$ value of 250 sovereigns $=$ Rs $250 \times \frac{40}{3} =$ Rs. $3333\frac{1}{3}$.

$\therefore$ gain $=$ Rs. $(3333\frac{1}{3} - 3200) =$ Rs $133\frac{1}{3}$

$=$ Rs. 133 5as. 4p.

8. Since $\dfrac{1}{(m-n)^2} - \dfrac{1}{(m-s)^2} = \left(\dfrac{1}{m-n} + \dfrac{1}{m-s}\right)\left(\dfrac{1}{m-n} - \dfrac{1}{m-s}\right)$

$$\therefore \text{Ans} = \frac{1}{\dfrac{1}{m-n} + \dfrac{1}{m-s}} = \frac{1}{\dfrac{2m-n-s}{(m-n)(m-s)}} = \frac{(m-n)(m-s)}{2m-n-s}$$

(a) First fraction $= \dfrac{x^2}{x-1} - x - \dfrac{1}{1-x} = \dfrac{x^2}{x-1} - x + \dfrac{1}{x-1}$

$$= \frac{x^2 - x(x-1) + 1}{x-1} = \frac{x+1}{x-1}.$$

Second fraction $= \dfrac{x^2}{x+1} + x - \dfrac{1}{1+x}$

$$= \frac{x^2 + x(x+1) - 1}{x+1} = \frac{2x^2 + x - 1}{x+1}$$

$$= \frac{(2x-1)(x+1)}{x+1} = 2x - 1$$

$$\therefore \text{Ans.} = \frac{x+1}{x-1} \times \frac{1}{(2x-1)} = \frac{x+1}{2x^2 - 3x + 1}.$$

9. $9x^4 - 28x^2 + 3 = 9x^4 - 27x^2 - x^2 + 3 = 9x^2(x^2-3) - (x^2-3)$

$= (x^2-3)(9x^2-1)$

$27x^4 - 12x^2 + 1 = 27x^4 - 3x^2 - 9x^2 + 1 = 3x^2(9x^2-1) - (9x^2-1)$

$= (9x^2-1)(3x^2-1)$

$27x^4 + 6x^2 - 1 = 27x^4 + 9x^2 - 3x^2 - 1$

$= 9x^2(3x^2+1) - (3x^2+1) = (3x^2+1)(9x^2-1)$.

$$x^4-6x^2+9=x^4-3x^2-3x^2+9=x^2(x^2-3)-3(x^2-3)$$
$$=(x^2-3)(x^2-3)=(x^2-3)^2$$
$$\therefore \text{L C M.}=(9x^2-1)(3x^2-1)(3x^2+1)(x^2-3)^2$$
$$(9x^2-1)(9x^4-1)(x^2-3)^2$$

10. The expression $=\dfrac{(a^2+b^2)^2}{(a^2-b^2)^2}+\dfrac{4ab}{a^2-b^2}$

$$=\frac{(a^2+b^2)^2+4ab(a^2-b^2)}{(a^2-b^2)^2}$$
$$=\frac{(a^2-b^2)^2+4a^2b^2+4ab(a^2-b^2)}{(a^2-b^4)^2}$$
$$=\frac{\{(a^2-b^2)+2ab\}^2}{(a^2-b^2)^2}$$
$$\therefore \text{sq root}=\frac{a^2-b^2+2ab}{a^2-b^2}.$$

11. (*a*) Multiplying both sides by bcx, we have

$$ac-cx^2-b^2x+bx^2=c^2x-cx^2-b^2+bx^2$$
$$\therefore b^2x+c^2x=b^2+ac, \text{ or } x\,(b^2+c^2)=b^2+ac$$
$$\therefore x=\frac{b^2+ac}{b^2+c^2}$$

(*b*) Multiply both sides by 6

$$\cdot0066x+\cdot001x-\cdot125=100-20x-\cdot087$$
$$\cdot0076x-\cdot125=99\cdot913-20x$$
$$\therefore 20\cdot0076x=99\cdot913+\cdot125=100\cdot038$$
$$\therefore x=\frac{100\cdot038}{20\cdot0076}=5.$$

(*c*) Adding (1) and (2)

$$2x+z=25 \quad \ldots\ldots\ldots\ldots(4)$$

Multiply (2) by 6, $\quad 6y+6x+12z=240$

From (3) $\quad 4z-5x-6y=-150$

By addition, $\quad x+16z=90\ldots\ldots(5)$

Multiply (5) by 2 $\quad 2x+32z=180$

From (4) $\quad 2x+z\;=\;25$

By subtraction, $\quad 31z=155$

$$\therefore z=5$$

From (4) $2x=25-z=20,\quad \therefore x=10.$

From (1) $y=x-z+15=10-5+15=20$

(*d*) Multiply both sides by $\frac{1}{16}\left(\frac{a-x}{a+x}\right)$ ∴ $\left(\frac{a-x}{a+x}\right)^4=\frac{1}{16}$.

Taking the fourth root, $\frac{a-x}{a+x}=\frac{1}{2}$ ∴ $\frac{2a}{2x}=\frac{3}{1}$, ∴ $x=\frac{1}{3}a$.

12. Let x = the amount of the debt in £,

then the sum paid away to creditors $=£\frac{17\frac{1}{2}x}{20}=£\frac{7x}{8}$

and therefore $£\left(x-\frac{7x}{8}\right)$ or $£\frac{x}{8}$ is still left unpaid.

The sum paid to lawyers $=\frac{4}{5}\times£\frac{x}{8}=£\frac{x}{10}$.

Therefore the sum still left $=£\left(\frac{x}{8}-\frac{x}{10}\right)=£\frac{x}{40}$

∴ By the question, $\frac{x}{40}=20$, ∴ $x=£800$

13. Let x = no of minute divisions passed over by the hour-hand after 5,

then $12x$ = no of minutes passed over by minute hand after 5

At 5 o'clock the hands are distant from each other 5×5 or 25 minutes

∴ By the question,

$$12x=x+25, \qquad ∴ 11x=25$$

∴ $x=\frac{25}{11}$ and $12x=\frac{25\times12}{11}$ min $=\frac{300}{11}$ min. $=27\frac{3}{11}$ min past 5.

1886.—AFTERNOON.

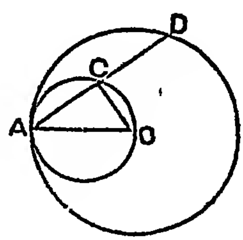
1 Euclid, Book I Defs. 8, 15, 29, III. 10, I 7.

Any straight line cutting a circle and terminated by the circumference is called a chord

2 Euclid, Book I Prop 33

3 Euclid, Book II prop. 7

4. (*a*) Euclid, Book III Prop 11.



(*b*) Let the two circles touch each other internally at the point A, whereof the inner passes through the centre O of the outer From A draw any chord ACD to the outer circle, cutting the inner at C Then AD is bisected at C Join CO

∵ ∠ACO = a rt ∠ (III 31)

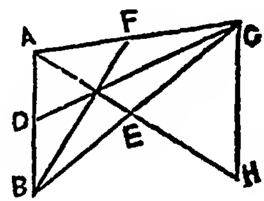
∴ AD is bisected at C (III. 3)

5 Let ABC be the given circle In the circle inscribe an isosceles $\triangle$ABC having each of the angles at B and C seven times the angle A (IV 2) Divide each of the equal angles at B and C into seven equal angles, each $=\angle$ at A, and let the dividing lines meet the circumference at the several points D, E, F, &c Join AD, DE &c Then the fig so formed shall be the required quindecagon

Because the fifteen angles BAC, ABD, DBE, &c , are equal to one another (*Cons*) $\therefore$ the fifteen arcs BC, AD, DE &c are equal (III 26) and $\therefore$ the fifteen chords subtended by these arcs are equal (III 29). Hence the figure is equilateral And of such equal parts as the whole circumference contains fifteen, each angle of the quindecagon stands on an arc which is equal to thirteen of them Hence all the angles are equal, $\therefore$ it is equiangular Wherefore, the figure is a regular quindecagon.

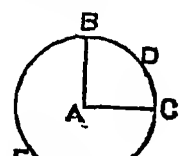
6 Let ABC be any $\triangle$, and D, E, F, the middle points of the sides AB, BC, CA respectively Join AE, BF and CD Then AB+BC+CA>AE+BF+CD

Produce AE to H, making EH=AE and join CH



$\because$ AE=EH and BE=CE (Cons)
also $\angle$AEB=$\angle$CEH (I 15)
$\therefore$ AB=CH (I 4)
Again (I, 20),
$\therefore$ AC+CH>AH
$\therefore$ AC+AB>2AE.
Similarly AB+BC>2BF
and AC+BC>2CD
$\therefore$ 2(AB+BC+AC)
>2(AE+BF+CD) (Ax 2) or AB+BC+AC>AE+BF+CD.

7 Let BDC be a quadrant or a fourth part of a circle, and BAC an angle in it Complete the circle of which BDC is a quadrant.

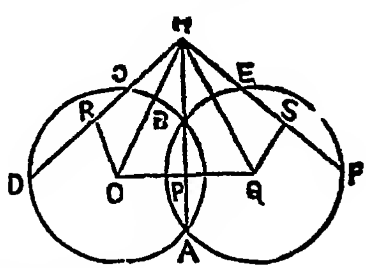


$\because$ the angles at A are 4 rt angles
and BDC is fourth part of the circumference
$\therefore$ $\angle$BAC is fourth part of 4 rt angles, i e. 1 rt angle.

8 Let O and Q be the centres of the $\odot$s ABD and ABE respectively. Join OQ cutting AB in P

Draw OR perpendicular to CD and QS to EP

Join HO, HQ, OB, BQ OA and AQ



$\because$ OB=QB and BA common, also OA=AQ } (*Hyp.*)
$\therefore$ $\angle$OBA=$\angle$QBA I 8)

Again, $\because$ OB=QB (*Hyp.*) and BP common,
also $\angle$OBP=QBP, (proved)

$\therefore$ OP=PQ and the $\angle$s at P are rt. $\angle$s (I 4)
Again, $\because$ OP=PQ (proved) and PH common
$\angle$OPH=$\angle$QPH, (Ax 11) $\therefore$ $\angle$PHO=$\angle$PHQ (I 4).
Now, $\because$ $OR^2+RH^2=OH^2=OP^2+PH^2$ (I 47)
but OR=OP (III 14) $\therefore$ RH=PH
Hence $\angle$RHO=$\angle$PHO (I 8) or $\angle$RHP=2 $\angle$PHO.
Similarly $\angle$PHS=2$\angle$PHQ
but $\angle$PHO=$\angle$PHQ (proved)
$\therefore$ $\angle$RHP=$\angle$PHS (Ax 6) or AH bisects the $\angle$CHE.

1887.—MORNING.

Head Examiner—BABU GOURI SANKAR DE, M.A

1 Simplify

(1) $(4\frac{2}{3}-1\frac{1}{3})\times(3\frac{1}{2}-\frac{3}{8})-(13\frac{1}{3}+7\frac{1}{2})$ of $\dfrac{3\frac{1}{3}}{1\frac{1}{6}}$

(2) $\dfrac{1{\cdot}83+2{\cdot}041\dot{6}+{\cdot}3-3\frac{1}{8}}{1{\cdot}0025+{\cdot}0625-1\frac{1}{16}}$

2 Express $\frac{3}{8}$ of 7*s* 6*d* +1·25 of 5*s* − ·545 of 9*s* 2*d* as a decimal fraction of £10

3 (*a*) Find by Practice the value of 5 tons 3 cwts 2 qrs. 17½ lbs at £3 6*s* 8*d* per ton.

(*b*) Find the income on which the income-tax at 5 pies per rupee is 52 Rs. 1 anna 4 pies

4. If 50 men can do a piece of work in 12 days, working 8 hours a day, how many hours a day would 60 men have to work in order to do another piece of work twice as great in 16 days?

5. If Rs 450 amount to Rs 540 in 4 years at simple interest what sum will amount to Rs 637 8 annas in 5 years at the same rate?

6. Extract the square root of 177·1561 and of $\dot{2}$ to 3 decimal places.

7 Divide $x^3+y^3-z^3+3xyz$ by $x+y-z$, and express $(x+3a)(x+5a)(x+7a)(x+9a)$ as the difference of two square quantities

8. Resolve $x^8+a^4x^4+x^8$ and x^8-16a^8 into their elementary rational factors

9 Simplify $\dfrac{a^3}{(a-b)(a-c)}+\dfrac{b^3}{(b-c)(b-a)}+\dfrac{c^3}{(c-a)(c-b)}$

10 Solve the following equations —

(1) $\dfrac{x-2}{x-3}+\dfrac{x-3}{x-4}=\dfrac{x-1}{x-2}+\dfrac{x-4}{x-5}$

(2) $\frac{2}{x}+\frac{3}{y}=2,$ (3) $x-2y+z=0,$

$\frac{5}{x}+\frac{10}{y}=5\frac{5}{8};$ $9x-8y+3z=0,$

$2x+3y+5z=36$

11 An officer can form his men into a hollow square 5 deep, and also into a hollow square 6 deep, but the front in the latter formation contains 4 men fewer than in the former, find the number of men

12 If $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$, prove that

(1) $\frac{a}{d}=\frac{a^3}{b^3}$

(2) $(ab+bc+cd)^2=(a^2+b^2+c^2)(b^2+c^2+d^2)$

1877.—AFTERNOON.

Head Examiner,—BABU GOURI SANKER DE, M A

1 If a side of a triangle be produced, the exterior angle is equal to the two interior and opposite angles, and the three angles of every triangle are together equal to two right angles

2 If the square described on one of the sides of a triangle be equal to the squares described on the other two sides of it, the angle contained by these two sides is a right angle

3 ABCD is a quadrilateral of which the sides AB and DC are parallel and E, F are the middle points of the sides BC and AD respectively, prove that the straight line EF is parallel to AB or CD and equal to half their sum

4 If a straight line be divided into two equal and also into two unequal parts, the squares on the two unequal parts are together double of the square on half the line and of the square on the line between the points of section

To what algebraical formula does it correspond, and why ?

5 If from any point without a circle two straight lines be drawn, one of which cuts the circle and the other touches it, the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, shall be equal to the square on the line which touches it

If two circles cut each other, then common chord (produced) bisects their common tangents

6 Inscribe a circle in a given triangle.

In what case does the centre of the inscribed circle coincide with that of the circumscribed circle and why ?

AB, AC are tangents to a given circle, and BC is the chord joining the points of contact From the middle point D of BC, the

straight line EDF is drawn at right angles to BC cutting the circumference of the given circle at E and F. Prove that E and F are the centres of two circles, one of which touches the three sides, and the other touches one side and two sides *produced*, of the triangle ABD

SOLUTIONS

1887.—MORNING.

1 (1) Fraction $=3\frac{1}{3}\times\left(3+\frac{4-3}{8}\right)\div\left(20+\frac{1}{3}+\frac{1}{2}\right)$ of $\frac{10}{3}\times\frac{9}{10}$

$=\frac{10}{3}\times3\frac{1}{8}\div20\frac{5}{6}$ of 3

$=\frac{10}{3}\times\frac{25}{8}\times\frac{6}{125}\times\frac{1}{3}=\frac{1}{6}.$

(2) Ans $=\dfrac{1.83333\ldots+2.0416666\ldots+.3333\ldots-3.333\ldots}{1.0025+.0625-1.0625}$

$=\dfrac{.875}{.0025}=\dfrac{8750}{25}=350.$

2 $\frac{3}{8}$ of 7*s* 6*d*. $=\frac{3}{8}\times90d=\frac{135}{4}d.=33\frac{3}{4}d=2s\ 9\frac{3}{4}d.$
1.25 of 5*s* = 6.25*s*. = 6*s*. 3*d*.

$.5\dot{4}\dot{5}$ of 9*s* 2*d* $=\dfrac{545-5}{990}$ of $110d=\frac{540}{990}$ of $110d.=60d=5s.$

∴ Value $=2s.\ 9\frac{3}{4}d.+6s\ 3d.-5s.=4s\ 0\frac{3}{4}d.$
and £10 = 10 × 240*d*

∴ Decimal reqd. $=\dfrac{48\frac{3}{4}}{10\times240}=\dfrac{195}{4\times10\times240}=\frac{13}{640}=.0203125.$

	£	s.	d.	
3. (*a*) 2 cwts. = $\frac{1}{10}$ of 1 ton	3	6	8	= value of 1 ton.
			5	
	16	13	4	= value of 5 tons.
1 cwt = $\frac{1}{2}$ of 2 cwts.		6	8	= value of 2 cwts.
2 qrs. = $\frac{1}{2}$ of 1 cwt.		3	4	= value of 1 cwt
14 lbs = $\frac{1}{4}$ of 2 qrs.		1	8	= value of 2qrs.
$3\frac{1}{2}$ lbs = $\frac{1}{4}$ of 14 lbs			5	= value of 14lbs.
			$1\frac{1}{4}$	= value of $3\frac{1}{2}$ lbs.
	£17	5	$6\frac{1}{4}d$	= value of 5 tons 3 cwt. 2 qrs $17\frac{1}{2}$ lb.

(*b*) Here Rs. 52 1 anna, 4 pie = 10000 pie.

5 : 10000 :: 1 Rupee : income reqd

∴ income reqd $=\frac{10000}{5}=$ Rs. 2000.

4 Let x denote the no of hours per day.

$50 \times 12 \times 8 \quad 60 \times 16 \times x \quad 1 \quad 2,$

$$\therefore x = \frac{50 \times 12 \times 8 \times 2}{60 \times 16} \text{ hrs} = 10 \text{ hrs}$$

5 Here Rs $(540-450) =$ Rs. 90 is the interest of Rs 450 in 4 years

$\therefore$ Yearly interest of Rs $450 =$ Rs $\frac{90}{4} =$ Rs $\frac{45}{2}$.

$\therefore$ . ... of Rs $100 =$ Rs $\frac{45}{2} \times \frac{100}{450} =$ Rs 5

Amount of Rs 100 in 5 years at 5 per cent

= Rs $100 +$ Rs. $25 =$ Rs. 125,

Rs 125 Rs $637\frac{1}{2}$ Rs 100 sum reqd.

$$\therefore \text{sum reqd} = \text{Rs. } \frac{100 \times 1275}{125 \times 2} = \text{Rs. } 510$$

6 $\sqrt{(177.15\dot{6}1)} = 13.31$

(a) $\sqrt{(22222\dot{2})} = 4714$ ..

7. $x+(y-z)\big)\, x^3 + 3xyz + y^3 - z^3 \big(\, x^2 - x(y-z) + y^2 + yz + z^2$

$x^3 + x^2(y-z)$

$-x^2(y-z) + 3xyz + y^3 - z^3$

$-x^2(y-z) - x(y^2 - 2yz + z^2)$

$x(y^2 + yz + z^2) + y^3 - z^3$

$x(y^2 + yz + z^2) + y^3 - z^3$

for $(y-z)(y^2 + yz + z^2) = y^3 - z^3$

(a) $(x+3a)(x+9a) = x^2 + 12ax + 27a^2$

$= (x^2 + 12ax + 31a^2) - 4a^2$

$(x+5a)(x+7a) = x^2 + 12ax + 35a^2$

$= (x^2 + 12ax + 31a^2) + 4a^2$

$\therefore$ Product $= (x^2 + 12ax + 31a^2)^2 - (4a^2)^2$

8 (a) $a^8 + a^4x^4 + x^8 = (a^4 + x^4)^2 - a^4x^4$

$= (a^4 - a^2x^2 + x^4)(a^4 + a^2x^2 + x^4)$

$= (a^4 - a^2x^2 + x^4)(a^2 - ax + x^2)(a^2 + ax + x^2)$.

(b) $x^8 - 16a^8 = (x^4 + 4a^4)(x^4 - 4a^4)$

$= \{x^4 + 4a^2x^2 + 4a^4) - 4a^2x^2\}(x^4 - 4a^4)$

$= \{(x^2 + 2a^2)^2 - (2ax)^2\}(x^4 - 4a^4)$

$= (x^2 + 2a^2 + 2ax)(x^2 + 2a^2 - 2ax)(x^2 + 2a^2)(x^2 - 2a^2)$

$= (x^2 + 2ax + 2a^2)(x^2 - 2ax + 2a^2)(x^2 + 2a^2)(x^2 - 2a^2)$

9 Fraction

$$= \frac{a^3}{(a-b)(a-c)} - \frac{b^3}{(b-c)(a-b)} + \frac{c^3}{(a-c)(b-c)}$$

L C M. of the denrs. $=(a-b)(b-c)(a-c)$

$$\therefore \text{Ans.} = \frac{a^3(b-c)-b^3(a-c)+c^3(a-b)}{(a-b)(b-c)(a-c)}$$

Now, Numr. $= a^3b - a^3c - ab^3 + b^3c + c^3(a-b)$

$= ab(a^2-b^2) - c(a^3-b^3) + c^3(a-b)$

$= (a-b)\{ab(a+b) - c(a^2+ab+b^2) + c^3\}$

$= (a-b)\{a^2b + ab^2 - a^2c - abc - b^2c + c^3\}$

$= (a-b)\{a^2(b-c) + ab(b-c) - c(b^2-c^2)\}$

$= (a-b)(b-c)\{a^2 + ab - c(b+c)\}$

$= (a-b)(b-c)\{a^2 + ab - bc - c^2\}$

$= (a-b)(b-c)\{(a^2-c^2) + b(a-c)\}$

$= (a-b)(b-c)(a-c)(a+c+b)$

$\therefore$ Ans $= a+b+c$

10 (1) $1+\frac{1}{x-3}+1+\frac{1}{x-4} = 1+\frac{1}{x-2}+1+\frac{1}{x-5}$

$$\therefore \frac{1}{x-3}+\frac{1}{x-4} = \frac{1}{x-2}+\frac{1}{x-5}$$

$$\therefore \frac{x-4+x-3}{(x-4)(x-3)} = \frac{x-5+x-2}{(x-2)(x-5)}$$

$$\therefore \frac{2x-7}{x^2-7x+12} = \frac{2x-7}{x^2-7x+10}$$

$\therefore 2x-7=0,\qquad \therefore 2x=7,\qquad \therefore x=3\frac{1}{2}$

(2) Multiply (1) by 5 and (2) by 2,

$$\left.\begin{array}{l} \frac{10}{x}+\frac{15}{y}=10 \\ \frac{10}{x}+\frac{20}{y}=\frac{35}{3} \end{array}\right\}$$

By subtraction,

$\frac{5}{y} = \frac{5}{3}, \quad \therefore y=3$

From (1) $\frac{2}{x} = 2 - \frac{3}{y} = 2-1=1, \quad \therefore x=2$

(3) Multiply (1) by 3,

$3x-6y+3z=0$

From (2) $\quad 9x-8y+3z=0$

By subtr $\quad 6x-2y=0$, or $3x-y=0$............(4)

Multiply (1) by 5

$$5x-10y+5z=0$$

From (2) $\quad 2x+3y+5z=36$

By subtr $\quad 3x-13y=-36 \quad \ldots\ldots \quad (5)$

From (4) $\quad 3x-y=\quad 0$

By Subtr $\quad -12y=-36 \quad \therefore y=3$

From (4) $3x-y=0 \quad \therefore x=1$

" (1) $z=2y-x=6-1=5$

11 Let $x=$ no of men in front of the first hollow square then $x-4=$ . ... second . ...

No of men in the first hollow square

$$=x^2-(x-10)^2=10(2x-10)=20(x-5)$$

No of men in the second hollow square

$$=(x-4)^2-(x-16)^2=12(2x-20)=24(x-10)$$

$\therefore$ By the question,

$$20(x-5)=24(x-10), \text{ or } 5(x-5)=6(x-10)$$

$$\therefore 5x-25=6x-60, \quad \therefore x=35$$

Hence no of men $=20\times30=600$

12 (1) Since $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$

$$\therefore \left(\frac{a}{b}\right)^3=\frac{a}{b}\times\frac{b}{c}\times\frac{c}{d}=\frac{a}{d} \text{ or } \frac{a^3}{b^3}=\frac{a}{d}$$

(2) Since $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$,

$$\therefore \frac{a^2}{ab}=\frac{b^2}{bc}=\frac{c^2}{cd}$$

$$\therefore \text{Each}=\frac{a^2+b^2+c^2}{ab+bc+cd}$$

Again, $\therefore \frac{ab}{b^2}=\frac{bc}{c^2}=\frac{cd}{d^2}$

$$\therefore \text{Each}=\frac{ab+bc+cd}{b^2+c^2+d^2}.$$

Hence $\frac{a^2+b^2+c^2}{ab+bc+cd}=\frac{ab+bc+cd}{b^2+c^2+d^2}$

or $(ab+bc+cd)^2=(a^2+b^2+c^2)(b^2+c^2+d^2)$.

1887.—AFTERNOON.

1 Euclid, Book I. Prop. 32

2 Euclid, Book I. Prop 48.

5. Add together $\frac{1}{x+3}+\frac{x+1}{x^2-3x+9}-\frac{2x^2+x+12}{x^3+27}$

6. Divide $x^{\frac{4}{3}}+x^{\frac{2}{3}}y^{\frac{1}{2}}+y$ by $x^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{4}}+y^{\frac{1}{2}}$

and simplify the expressions $\frac{a+c}{(x-a)(b-a)}+\frac{b+c}{(x-b)(a-b)}$

and $\frac{a^4-b^4}{a^2-2ab+b^2}\times\frac{a-b}{a(a+b)}$

7 Solve the following equations —

(i) $\frac{1}{x-1}+\frac{1}{x-4}=\frac{2}{x-3}$

(ii) $\frac{2x+11}{x+5}-\frac{9x-9}{3x-4}=\frac{4x+13}{x+3}-\frac{15x-47}{3x-10}$

8 A person bought a picture at a certain price and paid the same price for the frame, if the frame cost £1 less and the picture 15*s* more, the price of the frame would have been only half that of the picture Find the cost of the picture

1860.—AFTERNOON.

Examiners,—{Dr Mackay, Mr Thwaites}

1 In the figure, for the 5th proposition of the 1st Book of Euclid, where H is the intersection of BG and FC, prove that AH will bisect the angle BAC.

2 Draw a straight line through a given point parallel to a given straight line

3 From a given point draw a line making equal angles with two given straight lines

4 Describe a square that shall be equal to a given rectilineal figure

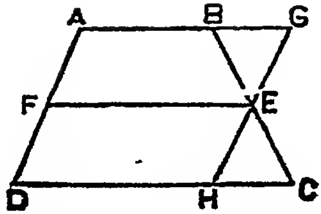
5 If on the radius of a circle a semi-circle be described, and from any point N in the diameter AO a perpendicular NPQ be drawn to meet the circles at P and Q, then if the common extremity A of their diameters be joined with these points, the square upon AQ will be double of the square upon AP

6 The angles in the same segment of a circle are equal to one another

7 Any angle of a triangle inscribed in a circle, is greater or less than a right angle by the angle contained by the side subtending the angle, and a diameter from either extremity of that side

8 From a given circle to cut off a segment which shall contain an angle equal to a given rectilineal angle

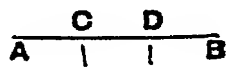
3 Through E draw HEG ∥ AD meeting DC at H and AB produced at G.



∵ BE = CE (Hyp.)
∠EBG = ∠ECH and BGE
= ∠CHE (I. 29)
∴ BG = CH and GE = HE (I. 26).
Again, ∵ AF = ½ AD (Hyp.)
= ½ GH (I. 34)
(proved = GE and also parallel
∴ AB is parallel to EF (I. 33)
Hence AB ∥ CD ∥ EF
Again, ∵ 2EF = AG + DH = AB + BG + DC − HC
= AB + DC, for BG = HC
∴ EF = ½(AB + DC) (Ax. 7)

4 Euclid, Book II 9

(a) Let AB = $2a$, and CD = b, then AC or BC = a,



and AD = $a+b$ BD = BC − CD = $a-b$
Now, $AD^2 + BD^2 = 2AC^2 + 2CD^2$ (II 9)
∴ $(a+b)^2 + (a-b)^2 = 2a^2 + 2b^2$

5 Euclid, Book III Prop 37.

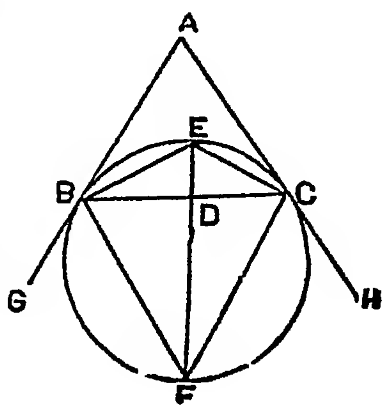
a. Let the two ⊙s cut each other at the points A and B. Join AB and produce it to meet the common tangent CD at E Then CD is bisected at E

∵ CE^2 = BE EA = ED^2 (III 37)
∴ CE = DE
Hence CD is bisected at E.

6 Euclid, Book IV

(a) In the case of an equilateral triangle, the centre of the inscribed ⊙ coincides with the centre of the circumscribed ⊙, for the lines bisecting the angles of the △ bisect also the sides opposite to them at right angles.

(b) Produce AB and AC to G and H and join BE, CE, BF and CF



∴ BD = DC (Hyp.) and DE com.
also ∠BDE = ∠CDE (Ax. 11)
∴ BE = CE and ∠EBC
= ∠ECB (I 4)
Similarly, it may be proved that
∠FBC = FCB
Again, ∵ ∠ABE = BCE
(III 32)
= ∠EBC (Proved)
∴ BE is the bisector of the ∠ABC
Similarly, CE is the bisector of the ∠ACB
Hence E is the centre of the ⊙ incribed in the △ABC (Euc. IV. 4)

Again ∵ ∠GBF=∠FCB (III 32)=∠CBE (proved)
∴BF is the bisector of the exterior angle GBC of the △ABC.
In the same manner, it may be shewn that OF is the bisector of the ext. ∠BCH
Hence F is the centre of the escribed circle of the △ABC

1888 —MORNING

Head Examiner,—Mr G W Kutchler, M A

1 Simplify

$$\frac{\frac{3}{7}(\frac{6}{19}\text{ of }3\frac{1}{3}-\frac{1}{3}\text{ of }2\frac{1}{8})}{\frac{1}{23}\times1\frac{6}{17}\times1\frac{5}{19}-\frac{7}{36}-2\frac{1}{3}}-\frac{\frac{1}{5}+\frac{6}{13}-\frac{7}{13}}{\frac{1}{3}\times\frac{1}{4}-\frac{1}{17}\times1\frac{4}{13}}.$$

2 Divide 16 016 by 00143, and extract the square root of 1440 9616.

3 Add together 55 5002, 3 17, 4 50$\dot{6}$ and 75 27$\dot{1}$, and find the value of the following —

7365 of £3 6*s* 8*d* + 504 of £15 2*s* 6*d* + 2 102083 of £5

4 Find by "Practice" the value of 2 tons 7 cwt 3 qrs 11lb. at £21 12*s* 6*d* per cwt.

5 A man can walk 600 miles in 35 days, resting 9 hours each day, how long will he take to walk 375 miles if he rests 10 hours each day, and walks $1\frac{1}{2}$ times as fast as before?

6 If the Interest on money be one pie per rupee per month, what is the rate per cent. per annum.

A man holds $15\frac{1}{4}$ shares of a bank, and receives £19 1*s* 3*d* per quarter. If the interest he receives be 5 per cent per annum find the value of a share

7 If $x=b+c$, $y=c-a$, $z=a-b$, prove that

$$x^2+y^2+z^2-2xy-2xz+2yz=4b^2$$

8 Divide

$$(ax+by)^3+(ax-by)^3-(ay-bx)^3+(ay+bx)^3 \text{ by}$$
$$(a+b)^2x^2-3ab\,x^2-y^2 .$$

9 Extract the square root of

(1) $(ab+ac+bc)^2-4abc(a+c)$,

(2) $x^4+2(y+z)x^3+(3y^2+2yz+3z^2)x^2+2(y^3+y^2z+yz^2+z^3)x+y^4+2y^2z^2+z^4.$

10 Solve the following equations —

$$(1)\ \frac{1}{3}-\frac{7x-1}{6\frac{1}{2}-3x}=\frac{8}{3}\left(\frac{x-\frac{1}{2}}{x-2}\right),$$

$$(2)\ \frac{x+2\frac{1}{3}}{15}+\frac{x+3\frac{1}{3}}{25}=\frac{x+4\frac{1}{6}}{55};$$

$$(3)\ (x+7)(y-3)+7=(y+3)(x-1)+5$$
$$5x-11y+35=0.$$

11. The dimensions of a rectangular court are such that if the length were increased by 3 yards and the breadth diminished by the same, its area would be diminished by 18 square yards, and if its length were increased by 3 yards, and its breadth increased by the same, its area would be increased by 60 square yards; find the dimensions

12 Find the G C. M. of $4x^3-8ax^2-20a^2x+24a^3$ and $6x^3+24ax^2+6a^2x-36a^3$

13. If $\frac{a}{b}=\frac{c}{d}$, prove that $\frac{(a+c)^3}{(b+d)^3}=\frac{a(a-c)^2}{b(b-d)^2}$.

1888.—AFTERNOON

Head Examiner,—Mr G W Kuchler, M A

1 Make a triangle of which the sides shall be equal to three given straight lines

Explain clearly the necessity for the limitation that any two whatever of these must be greater than the third, and show that this might be anticipated from some of the preceding propositions

2. Prove that parallelograms on the same base and between the same parallels are equal to one another

Show that the area of a triangle is equal to half the rectangle contained by its base and the perpendicular drawn to it from the opposite angle.

3. Prove that if the middle points of the sides of a quadrilateral be joined the figure thus formed is a parallelogram whose area is equal to half that of the quadrilateral

4. Prove that in every triangle the squares on the sides subtending an acute angle is less than the sum of the squares on the sides containing that angle by twice the rectangle contained by either of these sides and the straight line intercepted between the perpendicular let fall on it from the opposite angle and the acute angle

5. Prove that if a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circles, the angles which this line makes with the line touching the circle are equal to the angles of the alternate segments of the circle

Prove that if from a point two straight lines be drawn to touch a circle, these straight lines are equal.

6. Describe a circle about a given triangle

ABC is a triangle, DEF a straight line meets the sides AB at D, BC at E and AC produced at F, the point of intersection of the circles circumscribed about DBE and ECF besides E is G Prove that the circles circumscribed about ABC and ADE also pass through G.

SOLUTIONS.

1888.—MORNING.

1 1st fraction $= \dfrac{\frac{3}{7}(\frac{5}{19} \text{ of } \frac{16}{5} - \frac{1}{3} \text{ of } \frac{17}{8})}{\frac{1}{23} \times \frac{23}{17} \times \frac{34}{19} - \frac{7}{36} \times \frac{3}{7}} = \dfrac{\frac{3}{7}(\frac{16}{19} - \frac{17}{24})}{\frac{2}{19} - \frac{1}{12}}$

$$= \dfrac{\frac{3}{7}\left(\frac{384 - 323}{19 \times 24}\right)}{\frac{24 - 19}{19 \times 12}} = \frac{3}{7} \times \frac{61}{19 \times 24} \times \frac{19 \times 12}{5} = \frac{3 \times 61}{14 \times 5}$$

2nd fraction $= \dfrac{\frac{156 + 325 - 420}{5 \times 12 \times 13}}{\frac{1}{12} - \frac{1}{17} \times \frac{17}{13}} = \dfrac{\frac{61}{5 \times 12 \times 13}}{\frac{1}{12} - \frac{1}{13}}$

$$\dfrac{\frac{61}{5 \times 12 \times 13}}{\frac{13 - 12}{13 \times 12}} = \frac{61}{5 \times 12 \times 13} \times \frac{13 \times 12}{1} = \frac{61}{5}$$

$\therefore$ Ans $= \dfrac{3 \times 61}{14 \times 5} \times \dfrac{5}{61} = \dfrac{3}{14}$

2 (a) ·00143) 16·01600 (11200
143
———
171
143
———
286
286
———

(b) $\sqrt{(1440{\cdot}9616)} = 37{\cdot}96$

3 (a) 55·500222222
3·171717171
4·506506506
75·271271271
———————
138·44971717 $\therefore$ Ans. = 138·44971.

(*b*) ·7365 of £3 6*s*. 8*d*. = 7365 of 800*d*.

= 589 2*d*. = £2 9*s* 1 2*d*.

·504 of £15 12*s*. 6*d*. = ·504 of 3750*d*

= 1890 = £7 17*s*.

$$2\cdot102083 \text{ of } £5 = \frac{2102083-210208}{900000} \text{ of } £5$$

$$= \frac{1891875}{9000000} \text{ of } £5 = £10\ 10s\ \ 2\tfrac{1}{2}d.$$

	£	s.	d
∴ Ans. =	2	9	$1\frac{1}{5}$
+	7	17	6
+	10	10	$2\frac{1}{2}$
	£20	16	$9\frac{7}{10}$

	£	s	d.	
4 2 qrs = ½ of 1 cwt	21	12	6	= value of 1 cwt
			2	
	43	5	0	= value of 2 cwts.
			10	
	432	10	0	= value of 20 cwts or 1 ton
			2	
	865	0	0	= value of 2 tons
	151	7	6	= value of 7 cwts
1 qr = ½ of 2 qrs	10	16	3	= value of 2 qrs
7 lbs. = ¼ of 1 qrs	5	8	$1\frac{1}{2}$	= value of 1 qr.
	1	7	$0\frac{3}{8}$	= value of 7lbs
4lbs. = $\frac{1}{7}$ of 1 qr		15	$5\frac{5}{14}$	= value of 4 lbs
	£1034	14	$4\frac{13}{56}$	= value of 2 tons 7 cwts 3 qrs 11 lbs

5. On the first supposition, the man walks (24−9) or 15 hrs. daily, and on the second (24−10) or 14 hrs. daily

Let x = time required in days

$35 \times 15 \times 1 \quad x \times 14 \times 1\frac{1}{2} \qquad 600 \quad 375$

$$\therefore x = \frac{35 \times 15 \times 375 \times 2}{14 \times 3 \times 600} = \frac{125}{8} \text{ days}$$

$= 15\frac{5}{8}$ days.

6. (*a*) 1 × 1 100 × 12 1 (rate)

Rate reqd = 1200*p* = Rs 6 4as. = $6\frac{1}{4}$ p. c.

(b) Yearly interest $= £76\ 5s$

$£5 \quad £76\ 5s \quad \cdot £100 \cdot$ sum invested.

$\therefore$ sum invested $= £\dfrac{100 \times 76\frac{1}{4}}{5} = £1525$

Here value of a share $= £\dfrac{1525}{15\frac{1}{4}} = £100$

7 Hence, $x - (y+z) = (b+c) - \{(c-a)+(a-b)\} = 2b$.

Now, the left-hand expression

$= x^2 - 2x(y+z) + (y^2 + z^2 + 2yz)$

$= x^2 - 2x(y+z) + (y+z)^2 = \{x-(y+z)\}^2$

$= (2b)^2 = 4b^2$.

8. Dividend $= \{(ax+by)^3 + (ax-by)^3\} - \{(ay-bx)^3 - (ay+bx)^3\}$

$= 2a^3x^3 + 6axb^2y^2 + 6a^2y^2\ bx + 2b^3x^3$

$= 2x^3(a^3+b^3) + 6axby^2(b+a)$

$= 2x(a+b)\{x^2(a^2-ab+b^2) + 3aby^2\}$

$= 2x(a+b)\{(a+b)^2x^2 - 3ab(x^2-y^2)\}$

$\therefore$ Quot $= 2x(a+b)$.

9 (1) The expression $= \{b(a+c)+ac\}^2 - 4abc\,(a+c)$

$= b^2(a+c)^2 + 2abc(a+c) + a^2c^2 - 4abc(a+c)$

$= b^2(a+c)^2 - 2abc(a+c) + a^2c^2$

$= \{b(a+c) - ac\}^2$

$\therefore$ sq root $= ab + bc - ac$

(2) The expression $= x^4 + 2(y+z)x^3 + (y^2+2yz+z^2)x^2$

$+ 2(y^2+z^2)x^2 + 2y^2(y+z)x + 2z^2(y+z)x + (y^2+z^2)^2$

$= x^4 + 2(y+z)x^3 + (y+z)^2x^2 + 2(y^2+z^2)x^2$

$+ 2(y+z)(y^2+z^2)x + (y^2+z^2)^2$

$= \{x^2+(y+z)x\}^2 + 2(y^2+z^2)\{x^2+(y+z)x\}$

$+ (y^2+z^2)^2$

$= [\{x^2+(y+z)x\} + (y^2+z^2)]^2$

$\therefore$ sq root $= x^2 + (y+z)x + y^2 + z^2$.

10. (1) Multiply both sides by 3, we get

$1 - \dfrac{21x-3}{6\frac{1}{2}-3x} = \dfrac{8x-4}{x-2}, \quad \therefore \dfrac{21x-3}{6\frac{1}{2}-3x} = 1 - \dfrac{8x-4}{x-2} = \dfrac{2-7x}{x-2}$

or $(21x-3)(x-2) = (6\frac{1}{2}-3x)(2-7x)$

or $21x^2 - 45x + 6 = 13 - 51\frac{1}{2}x + 21x^2$

$\therefore 6\frac{1}{2}x = 7, \quad \therefore x = \frac{14}{13} = 1\frac{1}{13}$

(2) Clearing of fraction, we get

$$55(x+2\tfrac{1}{2})+33(x+3\tfrac{1}{3})=15(x+4\tfrac{1}{6})$$

$$\therefore 55x+137\tfrac{1}{2}+33x+110=15x+62\tfrac{1}{2}$$

$$\therefore 55x+33x-15x=62\tfrac{1}{2}-137\tfrac{1}{2}-100$$

$$73x=-175, \quad \therefore x=-\tfrac{175}{73}=-2\tfrac{29}{73}$$

(3) From (1) $xy+7y-3x-21+7=xy+3x-y-3+5$,

or, $8y-6x=16$ or $4y-3x=8$.

Now we get two equations

$$\left.\begin{array}{l} 4y-3x=8 \\ 5x-11y=-35 \end{array}\right\} \quad \left.\begin{array}{l} 20y-15x=40 \\ 15x-33y=-105 \end{array}\right\}$$

Adding we get $-13y=-65$, $\therefore y=5$

From (2) $5x=11y-35=55-35=20$ $\quad\therefore x=4$.

11 Let x = length of the court in yds
and y = breadth ... ...
then xy = area ... ...sq ys.

Now by the question

$$\left.\begin{array}{ll} (x+3)(y-3)=xy-18 & \ldots\ldots (1) \\ (x+3)(y+3)=xy+60 & \ldots\ldots (2) \end{array}\right\}$$

From (1) $xy-3x+3y-9=xy-18$ ⎫
" (2) $xy+3x+3y+9=xy+60$ ⎭

$$\left.\begin{array}{l} \therefore -3x+3y=-9 \\ \text{and } 3x+3y=51 \end{array}\right\} \text{ or } \left.\begin{array}{l} x-y=3 \\ \text{and } x+y=17 \end{array}\right\}$$

$\therefore 2x=20$, and $x=10$,
and $2y=14$, $\therefore y=7$

Hence length = 10 yds and breadth = 7 yds.

12 1st quantity $=4(x^3-2ax^2-5a^2x+6a^3)$
2nd " $=6(x^3+4ax^2+a^2x-6a^3)$

$$\begin{array}{r|l|l} x^3-2ax^2-5a^2x+6a^3 & x^3+4ax^2+a^2x-6a^3 & 1 \\ & x^3-2ax^2-5a^2x+6a^3 & \\ \hline 6a & 6ax^2+6a^2x-12a^3 & \\ \hline & x^2+ax-2a^2 & \end{array}$$

$$\begin{array}{r|l|l} x^2+ax-2a^2 & x^3-2ax^2-5a^2x+6a^3 & x-3a \\ & x^3+ax^2-2a^2x & \\ \hline & -3ax^2-3a^2x+6a^3 & \\ & -3ax^2-3a^2x+6a^3 & \\ \hline \end{array}$$

And G. C. M. of 4 and 6 is 2.

$\therefore$ G. C. M. reqd. $=2(x^2+ax-2a^2)$.

13. Since $\frac{a}{b}=\frac{c}{d}$, $\therefore \frac{a}{c}=\frac{b}{d}$,

$$\therefore \frac{a+c}{c}=\frac{b+d}{d} \text{ and } \frac{a-c}{c}=\frac{b-d}{d}$$

$$\therefore \frac{a+c}{b+d}=\frac{c}{d} \text{ and } \frac{a-c}{b-d}=\frac{c}{d}$$

$$\therefore \frac{(a+c)^3}{(b+d)^3}=\frac{c^3}{d^3}=\frac{c}{d} \quad \frac{c^2}{d^2}=\frac{c}{d}. \quad \frac{(a-c)^2}{(b-d)^2}=\frac{a(a-c)^2}{b(b-d)^2}$$

1888.—AFTERNOON

1 Euclid, Prop 22, Book I.

[See Fig of Euc I 22] The condition that B and C are greater than A, ensures that the circle described from the centre G shall not fall entirely within the circle described from centre F, the condition that A and B are greater than C, ensures that the circle described from the centre F shall not fall entirely within the circle described from the centre F, the condition that A and C are greater than B, ensures that one of the circles shall not fall entirely without the other Hence the circles must meet

This might be anticipated from Euc I. 20

2. Euclid, Prop. 35, Book I.

Let ABC be any triangle Through A draw DAE ∥ BC, and from B, C, draw ED, EC, at right angles to BC, meeting DE in D and E

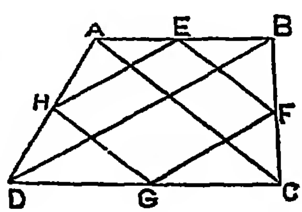
Then BDEC, is a rectangle.

From A draw AF perpendicular to BC

Now, △ABC=½ rect BDEC=½ BD.BD (I. 41)

=½ BO AF (I 34)

3 Let ABCD be any quadrilateral figure, and E, F, G, H, the middle points of its sides



Draw the diagonals AC and BD

Because E, F are the the middle points of the sides AB, BC, of the △ ABC, therefore EF ∥ AC

Similarly, HG ∥ AC, therefore EF∥HG

Likewise HE ∥ GF

∴ the figure EFGH is a parallelogram

Again, ∵ △ EBF=¼ △ ABC, (See Ques 8 of 1873)

and △HDG=¼ △ADC

∴ △EBF+∠HDG=¼ fig ABCD (Ax 2)

Similarly △AHE +△CGF=¼ fig ABCD

∴ △EBF+△HDG+△AHE+CGF=½ fig ABGD (Ax 2)

Hence rem ▭EFGH=½ fig ABCD (Ax 3).

4 Euclid, Prop 13, Book II

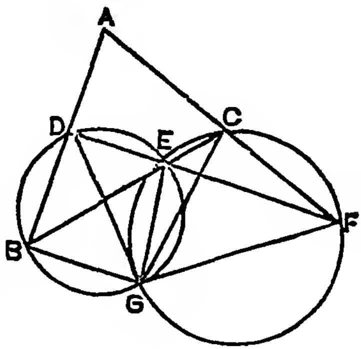
5 Euclid, Prop 32, Book III

From a point P without the circle draw PA, PB touching the circle at A and B Then PA=PB

From P draw any secant PQR cutting the circle in Q and R.

Since $PA^2=PR.PQ=PB^2$ (III 6), AP=BP

or III 17, Cor



6 Euclid, Prop 5, Book III

Join BG, DG, FG and EG

$\because \angle BGE=\angle ADF$ (III 22) and $\angle CGE=\angle CFE$ (III 21)

$\therefore \angle BGE + CGE$ or $\angle BGC=\angle ADF+\angle CFE$ (Ax 2)

To each add the $\angle$ BAC,

$\therefore \angle BGC+\angle BAC$
$=\angle ADF + CFE + \angle BAC=2$ right angles (I 32)

Hence the ⊙ circumscribing the $\triangle$ ABC passes through G (III 22, Cor)

Again, $\because$ $\angle$ DGE = $\angle$ DBE (III 21) and $\angle EGF=\angle ACB$(III 22)

$= \angle$ DGE + $\angle$ EGF or $\angle$ DGF $= \angle DBE+\angle ACB$

To each add the $\angle BAC$

$\therefore \angle DGF+\angle BAC=\angle DBE+\angle ACB+\angle BAC$,
$=2$ right angles (I 32)

Hence the ⊙ circumscribing the $\triangle$ADF passes through G (III 22, Con)

1889.—MORNING.

Head Examiner,—MR W BOOTH, B.A.

1. Multiply 0096347 by 7439 6

2. Divide 2100 006983 by 243 5846 corrent to five places of decimals

3 Find in any way the value of 1,347 cwt 3 qrs. and 21 lbs at £3 17*s* 10¼*d* per owt

4. Extract the square root of

$1+(0634)^3$ to six places of decimals

5 Find in English money the value of rupees 100,000 at 1*s*. $4\frac{3}{32}d$. per rupee.

6 Solve the equation—

$$x-\left(3x-\frac{2x-5}{10}\right)=\tfrac{1}{6}(2x-57)-\tfrac{5}{3}.$$

7 Find the square root of

$$\frac{x^4}{4}+4x^2+\frac{ax^2}{3}+\frac{a^2}{9}-2x^3-\frac{4ax}{3}.$$

8. If $\frac{x}{(b-c)(b+c-2a)}=\frac{y}{(c-a)(c+a-2b)}=\frac{z}{(a-b)(a+b-2c)}$; find the value of $x+y+z$

9 Reduce to its lowest terms

$$\frac{3x^3-27ax^2+78a^2x-72a^3}{2x^3+10ax^2-4a^2x-48a^3}.$$

10 A man rides one-third of the distance from A to B at the rate of a miles per hour and the remainder at the rate of $2b$ miles per hour If he had travelled at a uniform rate of $3c$ miles per hour he could have ridden from A to B and back again the same time. Prove that $\frac{2}{c}=\frac{1}{a}+\frac{1}{b}$.

11. Simplify the expression

$$(16x^5-20x^3+5x)^2+(1-x^2)\{16(1-x^2)^2-20(1-x^2)+5\}^2.$$

1889.—AFTERNOON.

Head Examiner,—MR W BOOTH, M A.

1 From a given point A to draw a right line equal to a given finite right line BC, the point A being in the line BC.

2 To a given right line to apply a parallelogram which shall be equal to a given triangle and have one of its angles equal to a given angle.

3. Divide the hypotenuse of a right angled triangle into two parts, such that the difference between their squares shall be equal to the square on one of the sides

4. Prove that the rectangle under the sum and the difference of two lines is equal to the difference of the squares on the lines.

5. Construct a square equal to a given equilateral triangle.

6. When do two circles touch each other? Show that if two circles touch each other at any point their centres and that point are in a line.

7. Describe a circle about a given triangle, and show that the three perpendiculars dropped from the vertices on the opposite sides of any triangle meet in a point.

SOLUTIONS.

1889.—MORNING.

1. $0096347\times7439{\cdot}6=51\ 59139412$

2 $21009{\cdot}006983000\div243\ 5846=8{\cdot}62126.$

SOLUTIONS

1860—MORNING.

1. 100br. × (16 × 8 × 10) cub in. : 921600br. × $(12 \times 6 \times \frac{15}{2})$ cub. in. : Rs 2 9as : x

$$\therefore x = \frac{921600 \times 12 \times 6 \times 15 \times 41}{100 \times 16 \times 8 \times 10 \times 2 \times 16} \text{Rs.} = \text{Rs } 9963.$$

2 See question 3 of 1858.

$\sqrt{(57214096)} = 7564.$

$\sqrt{(50000000)} = 7071\ldots$.

3 $$\left(1+\frac{5}{6}+\frac{7}{8}+\frac{11}{12}\right)-\left(\frac{3}{4}-\frac{5}{8}\right) = \left(\frac{24+20+21+22}{24}\right) \div \left(\frac{6-5}{8}\right)$$

$$= \frac{87}{24} \div \frac{1}{8} = \frac{87}{24} \times 8 = \frac{87}{3} = 29.$$

(a) $$\frac{7\frac{1}{2}}{6\frac{1}{2}} + \frac{11\frac{1}{2}-2\frac{2}{5}}{11\frac{1}{2}+2\frac{2}{5}} \times 10\tfrac{9}{13} - 6\tfrac{2}{13}$$

$$\frac{15}{13} + \frac{9\,\frac{5-4}{10}}{13\,\frac{5+4}{10}} \times 10\tfrac{9}{13} - 6\tfrac{2}{13}$$

$$= 1\tfrac{2}{13} + \frac{9\frac{1}{10}}{13\frac{9}{10}} \times 10\tfrac{9}{13} - 6\tfrac{2}{13}$$

$$1\tfrac{2}{13} + \tfrac{91}{139} \times \tfrac{139}{13} - 6\tfrac{2}{13}$$

$$1\tfrac{2}{13} + 7 - 6\tfrac{2}{13} = 2$$

4 2 maunds, 16 seers = 96 seers

The cost of 96 seers = Rs $4\frac{1}{8} \times 96$ = Rs $\frac{33}{8} \times 96$ = Rs 396

Two chests containing 3m 24 seers each = 2×144 seers = 288 seers.

The cost of 288 seers = Rs $4\frac{5}{8} \times 288$ = Rs $\frac{37}{8} \times 288$ = Rs. 1332.

∴ The cost of 3 chests of tea = Rs (396 + 1332) = Rs 1728 and he is to gain Rs 576

∴ the price at which the whole is to be sold = Rs 1728 + Rs. 576 = Rs 2304, and the total quantity of tea = (96 + 288) seers = 384 seers

∴ selling price per seer = Rs $\frac{2304}{384}$ = Rs 6

5 $$\text{Ans.} = \frac{(x^2-3x+9)+(x+1)(x+3)-(2x^2+x+12)}{x^3+27}$$

$$\text{Numr} = x^2-3x+9+x^2+4x+3-2x^2-x-12 = 0.$$

$$\therefore \text{Ans.} = 0.$$

3

	£	s	d.	
	3	17	$10\frac{1}{4}$	= value of 1 cwt.
			10	
	38	18	$6\frac{1}{2}$	= value of 10 cwt.
			10	
	389	5	5	= value of 100 cwts.
			10	
	3892	14	2	= value of 1000 cwts.
	1167	16	3	= value of 300 cwts.
	155	14	2	= value of 40 cwts
2 qrs = $\frac{1}{2}$ of 1 cwt.	27	4	$11\frac{3}{4}$	= value of 7 cwts.
1 qr. = $\frac{1}{2}$ of 2 qrs.	1	18	$11\frac{1}{8}$	= value of 2 qrs.
14 lbs. = $\frac{1}{2}$ of 1 qr		19	$5\frac{9}{16}$	= value of 1 qr.
		9	$8\frac{25}{32}$	= value of 14 lbs
7 lbs = $\frac{1}{2}$ of 14 lbs.		4	$10\frac{57}{64}$	= value of 7 lbs.
	£5247	2	$6\frac{39}{64}$	= value of 1347 cwts. 3 qrs. 21 lbs.

4. $1+(\cdot 0634)^3=1\,000254840104.$

$\sqrt{(\dot{1}\,\dot{0}0\dot{0}2\dot{5}4\dot{8}4\dot{0}1\dot{0}4)}=1\,000127.$

5. Here 1*s*. $4\frac{3}{32}d,=16\frac{3}{32}d$

$\therefore$ Ans. $=(100000)\times 16\frac{3}{32}d=1609375d.$

$=$ £6705 14*s*. 7*d*

6 $x-\left(3x-\dfrac{2x-5}{10}\right)=\frac{1}{6}(2x-57)-\frac{5}{3}.$

Multiply both sides by 30

$30x-90x+6x-15=10x-285-50$

$\therefore 36x-90x-10x=15-335.$

$\therefore 64x=320, \quad \therefore x=5.$

7 $\dfrac{x^4}{4}-2x^3+4x^2+\dfrac{ax^2}{3}-\dfrac{4ax}{3}+\dfrac{a^2}{9}\left(\dfrac{x^2}{2}-2x+\dfrac{a}{3}=\text{sq root.}\right.$

$$\begin{array}{l}
\dfrac{x^4}{4} \\
\hline
x^2-2x\,\big)\ -2x^3+4x^2 \\
\qquad\qquad -2x^3+4x^2 \\
\hline
x^2-4x+\dfrac{a}{3}\,\Big)\ \dfrac{ax^2}{3}-\dfrac{4ax}{5}+\dfrac{a^2}{9} \\
\qquad\qquad\qquad \dfrac{ax^2}{3}-\dfrac{4ax}{3}+\dfrac{a^2}{2} \\
\hline
\end{array}$$

8 Let $\dfrac{x}{(b-c)(b+c-2a)}=\dfrac{y}{(c-a)(c+a-2b)}$

$$=\frac{z}{(a-b)(a+b-2c)}=p$$

then $x=p(b-c)(b+c-2a)=p(b^2-c^2)-2ap(b-c)$

$y=p(c-a)(c+a-2b)=p(c^2-a^2)-2bp(c-a)$

$z=p(a-b)(a+b-2c)=p(a^2-b^2)-2cp(a-b)$

$\therefore x+y+z=p\times 0=0$

9 Here G C M $=x-2a$

$\therefore$ Ans $=\dfrac{3(x^3-9ax^2+26a^2x-24a^3)\div(x-2a)}{2(x^3+5ax^2-2a^2x-24a^3)\div(x-2a)}$

$$=\frac{3(x^2-7ax+12a^2)}{2(x^2+7ax+12a^2)}.$$

10 Let $3x$ = distance in miles from A to B

∴ By the question,

$\dfrac{x}{a}+\dfrac{2x}{2b}=\dfrac{6x}{3c}$, or $\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{2}{c}$

11 1st quantity $=256x^{10}-640x^8+560x^6-200x^4+25x^2$

2nd quantity $=(1-x^2)(16x^4-12x^2+1)^2$

$=(1-x^2)(256x^8-384x^6+176x^4-24x^2+1)$

$=-256x^{10}+640x^8-560x^6+200x^4-25x^2+1$

$\therefore$ Sum $=1$

1889—AFTERNOON

1 Euclid, I 2 On AB describe the equilateral $\triangle$ABD With B as centre and BC as radius, describe the $\odot$ CEG, and produce DB to meet it in E With D as centre and DE as radius, describe $\odot$EFA, and produce DA to meet it in F

Then AF is the required line

Because D is the centre of the $\odot$ EFA,

$\therefore$ DE=DF (Def 15) But DB=DA (Def 24)

$\therefore$ BE=AF (Ax 3) and BE=BC (Def 15)

$\therefore$ AF=BC.

2 Euclid, Bk I Prop. 44.

3 Let ABD be a right angled $\triangle$ having the rt. $\angle$ at A.

Draw AE$\perp$BD, and bisect DE in C.

Then C is the required point of division

$\because$ $AD^2=AB^2+BD^2-2BD\cdot BE$ (II 13)

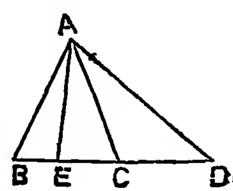
$=AB^2+AB^2+AD^2-2BD\cdot BE$ (I 47)

$\therefore 2AB^2=2BD\cdot BE$ or $AB^2=BE.BD$

Again $\because BD\cdot BE+CE^2=BC^2$ (II 6)

$\therefore BC^2-CD^2=BD\cdot BE$, for $CD=CE$

$\therefore BC^2-CD^2=AB^2$



4 See Solution of Question 5 of 1870

5 Euclid Bk II Prop 14

6 Two circles are said to touch each other when they meet at a point but do not cut
Euclid, Bk III Props 11 and 12

7 Euclid, Bk IV Prop 5
Let ABC be any $\triangle$
From B and C draw BH and CG$\perp$AC and AB respectively cutting at F
Join AF and produce it to meet BC in K
Then AK shall be $\perp$BC
Join GH
$\because$ angle BGC=a rt angle=angle BHC
$\therefore$ a $\odot$ will go round the fig BHC (III 31, Cor)
Again, $\because$ angle AGF=a rt angle=AHF
$\therefore$ a $\odot$ will go round the fig AGFH (III 22, Cor)
$\therefore$ angle GAF=angle GHT=angle BCG (III 21)
and angle AFG=angle GFK (I 15)
$\therefore$ angle AGF=angle AKC=a rt. angle (I 32)
Hence AK is $\perp$BC

1890 —MORNING.

Head Examiner,—Mr. W. Griffiths, M.A

1 Simplify $2\frac{3}{7}$ of $\dfrac{13\frac{1}{2}-9\frac{3}{4}}{15\frac{1}{5}-11\frac{7}{30}} \div 3\frac{2}{7} + \dfrac{1\frac{7}{10}}{4\frac{2}{9}-3\frac{4}{9}}$, and find by Practice the value of 3049 articles at Rs 7 13as 7p each

2 Divide 27·03 by 0037, and reduce $75 - \cdot\dot{1}02 - \cdot 2\dot{7}$ to a vulgar fraction.

3 Find the cost of putting a fence round a square field whose area is 13 225 acres at Re. 1 12as. per yard.

4 A piece of work can be done in 72 days by 17 men working together. If after 9 days of work, these are joined by 4 others, in how many days will the work be finished ?

5 Find the price of $4\frac{1}{4}$ per cent Government Promissiory Notes when an investment of Rs 59,422 8as produces a monthly income of Rs 213 12as

6 Multiply $ax^{\frac{1}{4}}+3a^{\frac{1}{2}}x^{\frac{1}{2}}+4x^{\frac{3}{4}}$ by $a-3a^{\frac{1}{2}}x^{\frac{1}{4}}+4x^{\frac{1}{2}}$, and find the greatest common measure of

$$x^6+x^4+1 \text{ and } x^6-2x^4+x^2-1.$$

7. Extract the square root of $9x^2-24x+19-\dfrac{4}{x}+\dfrac{1}{4x^2}$.

8 Solve the equations—

(i) $\frac{1}{3}(x+1)+\frac{1}{2}(x-1)-\frac{1}{4}(3x-7)=2$

(ii) $\dfrac{1}{x-a}-\dfrac{1}{x-a+c}=\dfrac{1}{x-b-c}-\dfrac{1}{x-b}$

(iii) $\dfrac{1}{3x}-\dfrac{1}{7y}=\frac{2}{3},\ \dfrac{1}{2x}-\dfrac{1}{3y}=\frac{1}{6}$

9 Of the candidates in a certain examination 45 per cent. passed If there had been 30 more candidates of whom 19 failed, the number of successful candidates would have been 44·8 per cent How many candidates were there?

10 If $a \quad b \quad c : d$, shew that (i) $a-b \quad a+b \quad c-d \quad c+d$. and (ii) $a^2d-bc^2=ac(b-d)$

1890—AFTERNOON.

Head Examiner,—Mr. W Griffiths, M A

1 Define parallel straight lines and a gnomon. When are segments of circles said to be similar, and when is a circle said to be inscribed in a rectilineal figure?

2 If two angles of a triangle be equal to one another, the sides also which subtend, or are opposite to the equal angles, shall be equal to one another

3 If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles, and three interior angles of every triangle are together equal to two right angles

In a right-angled triangle the line joining the right angle to any point (except the middle point) of the hypotenuse is greater than one segment of the hypotenuse and less than the other

4 Prove that the area of a quadrilateral is equal to the area of a triangle having two sides equal to the diagonals of the quadrilateral, and the contained angle equal to that between the diagonals

5 Divide a given straight line into two parts so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part

Prove that the rectangle contained by the two parts equal to the difference of the squares on the two parts

Draw a straight line from a given point without the circumference, which shall touch a given circle, and show that two, and only two, such straight lines can be drawn from the given point

7 In equal circles, equal straight lines cut off equal arcs, the greater equal to the greater, and the less equal to the less.

If two opposite sides of the quadrilateral inscribed in a circle are equal, prove that the other two sides are parallel.

8 P is a point in APB an arc of a circle, the tangents at P meet the chord AB produced in R and AQ, perpendicular to AB, in Q and QR is bisected in P. Prove that the angle ABP is double of the angle BAP.

9. Describe an isosceles triangle, having, each of the angles at the base double of the third angle.

SOLUTIONS.

1890.—MORNING.

1. (*a*) 1st fraction $=\frac{17}{7}$ of $\frac{4\frac{1}{2}-\frac{2}{3}}{4\frac{1}{5}-\frac{7}{30}} \div 3\frac{2}{7} = \frac{17}{7} \times \frac{\frac{9}{2}-\frac{2}{3}}{\frac{21}{5}-\frac{7}{30}} \times \frac{7}{23}$

$$= \frac{17}{7} \times \frac{\frac{27-4}{6}}{\frac{126-7}{30}} \times \frac{7}{23} = \frac{17}{7} \times \frac{23}{6} \times \frac{30}{119} \times \frac{7}{23} = \frac{5}{7}$$

and 2nd fraction $= \frac{1\frac{1}{7}}{1\frac{1}{6}-\frac{2}{3}} = \frac{1\frac{1}{7}}{\frac{7}{6}-\frac{4}{6}} = \frac{\frac{8}{7}}{\frac{1}{2}} = \frac{8}{7} \times \frac{2}{1} = \frac{16}{7}$

$\therefore$ Ans $= \frac{5}{7} + \frac{16}{7} = \frac{21}{7} = 3$

(*b*)

	Rs.	as.	p	
	3049	0	0	= value at Re 1 each
			7	
8as. = ½ of Re. 1	21343	0	0	= value at Rs 7 each.
4 as = ½ of 8 as	1524	8	0	= value at 8 as each
1 a = ¼ of 4 as	762	4	0	= value at 4 as each
6 p = ½ of 1 a	190	9	0	= value at 1 a each.
1 p. = ⅙ of 6 p	95	4	6	= value at 6 p each.
	15	14	1	= value at 1 p each
.	Rs 23931	7	7	= value of Rs 7 13as, 7p. each

2 (*a*) $27\cdot0300000 \div \cdot0037 = 7305\cdot\dot{4}0\dot{5}$

(*b*) Ans $= \frac{75-7}{90} - \frac{102}{999} - \frac{27-2}{90}$

$$= \frac{68}{90} - \frac{102}{999} - \frac{25}{90} = \frac{43}{90} - \frac{102}{999}$$

$$= \frac{4773-1020}{9990} = \frac{3753}{9990} = \frac{139 \times 27}{370 \times 27} = \frac{139}{370}.$$

3. 13·225 acres $=$ 13·225 $\times$ 4840 sq yds. $=$ 64009 sq yds

Hence a side of the square field $= \sqrt{64009}$ yds. $=$ 253 yds.

and $\therefore$ boundary $= 253 \times 4$ yds. $=$ 1012 yds

$\therefore$ cost $= 1012 \times 1\frac{3}{4}$ Rs. $=$ Rs. 1771

4. 17 men do $\frac{1}{72}$ of the work in one day.

$\therefore$ ......$\frac{9}{72}$ or $\frac{1}{8}$...... ...... 9 days

Hence work remaining to be done $= 1 - \frac{1}{8} = \frac{7}{8}$

The no. of men at work $= 17 + 4 = 21$.

Let x denote the required no. of days.

$\therefore 17 \times 9 : 21 \times x :: \frac{1}{8} : \frac{7}{8}$

$$\therefore x = \frac{17 \times 9 \times 7 \times 8}{21 \times 8} = 51 \text{ days}$$

5. Annual income $=$ Rs $213\frac{3}{4} \times 12 =$ Rs. 2565.

Rs. 2565 : Rs $4\frac{1}{2}$ :: Rs $59422\frac{1}{2}$: x

$$x = \frac{9 \times 118845}{2565 \times 2 \times 2} = \frac{417}{4} = \text{Rs } 104\tfrac{1}{4}.$$

6. (a)

$$\begin{array}{l} ax^{\frac{1}{4}} + 3a^{\frac{1}{2}}x^{\frac{1}{2}} + 4x^{\frac{3}{4}} \\ a - 3a^{\frac{1}{2}}x^{\frac{1}{4}} + 4x^{\frac{1}{2}} \\ \hline a^2x^{\frac{1}{4}} + 3a^{\frac{3}{2}}x^{\frac{1}{2}} + 4ax^{\frac{3}{4}} \\ \quad - 3a^{\frac{3}{2}}x^{\frac{1}{2}} - 9ax^{\frac{3}{4}} - 12a^{\frac{1}{2}}x \\ \qquad\qquad 4ax^{\frac{3}{4}} + 12a^{\frac{1}{2}}x + 16x^{\frac{5}{4}} \\ \hline a^2x^{\frac{1}{4}} \qquad - ax^{\frac{3}{4}} \qquad + 16x^{\frac{5}{4}} = \text{Product.} \end{array}$$

(b)

$$\begin{array}{l} x^5 + x^4 + 1 \,)\, x^6 - 2x^4 + x^2 - 1 \,(\, x - 1 \\ \qquad\qquad\quad x^6 + x^5 + x \\ \hline \qquad\qquad\quad -x^5 - 2x^4 + x^2 - x - 1 \\ \qquad\qquad\quad -x^5 - x^4 - 1 \\ \hline \qquad\qquad -x \,|\, -x^4 + x^2 - x \\ \qquad\qquad\qquad x^3 - x + 1 \,)\, x^5 + x^4 + 1 \,(\, x^2 + x + 1 \\ \qquad\qquad\qquad\qquad\qquad x^5 - x^3 + x^2 \\ \hline \qquad\qquad\qquad\qquad\qquad x^4 + x^3 - x^2 + 1 \\ \qquad\qquad\qquad\qquad\qquad x^4 - x^2 + x \\ \hline \qquad\qquad\qquad\qquad\qquad\quad x^3 - x + 1 \\ \qquad\qquad\qquad\qquad\qquad\quad x^3 - x + 1 \\ \hline \end{array}$$

$\therefore$ G. C. M. $= x^3 - x + 1$.

7. $9x^2-24x+19-\frac{4}{x}+\frac{1}{4x^2}\Big(3x-4+\frac{1}{2x}$ = sq. root

$$
\begin{array}{r|l}
 & 9x^2 \\
\hline
6x-4 & -24x+19 \\
 & -24x+16 \\
\hline
6x-8+\frac{1}{2x} & 3-\frac{4}{x}+\frac{1}{4x^2} \\
 & 3-\frac{4}{x}+\frac{1}{4x^2} \\
\hline
\end{array}
$$

8. (1) $\frac{1}{3}(x+1)+\frac{1}{2}(x-1)-\frac{1}{4}(3x-7)=2$
clearing of fractions, we get
$4x+4+6x-6-9x+21=24$
or $10x-9x=24-19$, $\therefore x=5$.

(ii) $\frac{1}{x-a}-\frac{1}{x-a+c}=\frac{1}{x-b-c}-\frac{1}{x-b}$

Reduce each side and divide by c, thus

$$\frac{1}{(x-a)(x-a+c)}=\frac{1}{(x-b-c)(x-b)}$$

whence $(x-a)^2+c(x-a)=(x-b)^2-c(x-b)$,
or $(x-a)^2-(x-b)^2=-c\{(x-a)+(x-b)\}$
$\therefore 2x-a-b=0$, $\therefore 2x=a+b$, $\therefore x=\frac{1}{2}(a+b)$

(iii) $\frac{1}{3x}-\frac{1}{7y}=\frac{2}{3}$ . . (1) $\frac{1}{2x}-\frac{1}{3y}=\frac{1}{6}$ ....(2)

Multiply (1) by 3, and (2) by 2,

$$\left.\begin{array}{l}\frac{1}{x}-\frac{3}{7y}=2\\[4pt] \frac{1}{x}-\frac{2}{3y}=\frac{1}{3}\end{array}\right\}\quad \begin{array}{l}\text{By subtraction,}\\[4pt] \frac{5}{21y}=\frac{5}{3},\quad \therefore \frac{1}{7y}=1,\quad \therefore y=\frac{1}{7}\end{array}$$

Also from (2) $\frac{1}{2x}=\frac{1}{6}+\frac{1}{3y}=\frac{1}{6}+\frac{7}{3}=\frac{15}{6}$, $\therefore x=\frac{1}{5}$

9 Let x be the no of candidates.

Then no passed $=\frac{45}{100}x=\frac{9x}{20}$.

Of the additional candidates, no passed $=11$ and the whole no of candidates, is $x+30$,

$\therefore$ By the question

$$\frac{9x}{20}+11=\frac{44\cdot8}{100}(x+30) \text{ or } \frac{9x}{20}+11=\frac{112}{250}(x+30)$$

Clearing of fractions we get

$225x+5500=224x+6720 \quad \therefore x=1220.$

Hence no. of candidates is 1220

10. (i) Since $\frac{a}{b}=\frac{c}{d}$, $\therefore \frac{a}{b}+1=\frac{c}{d}+1$, $\therefore \frac{a+b}{b}=\frac{c+d}{d}$ ... (1)

also $\frac{a}{b}-1=\frac{c}{d}-1$, $\therefore \frac{a-b}{b}=\frac{c-d}{d}$ ....(2)

Dividing (2) by (1), we get

$\frac{a-b}{a+b}=\frac{c-d}{c+d}$ or $a-b \quad a+b \,.\, c-d \quad c+d$

(ii) Since $\frac{a}{b}=\frac{c}{d}$, $\therefore ad=bc$

Hence $a^2d-bc^2=a\,ad-bc\,c=abc-adc=ac(b-d)$

1890.—AFTERNOON.

1. Euclid, I. Def 35; Euc II. Def. 2, Euc. III Def. 11; Euc. IV. Def. 5.
2. Euclid, I. 6.
3. Euclid, II. 32.

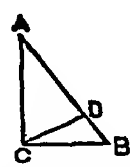
(a) Let ABC be a right angled $\triangle$, having the rt $\angle$ at C. At C in CA make the $\angle$ACD<$\angle$CAD (I. 23), and let CD meet the hypotenuse at D

Since $\angle$CAD>$\angle$ACD (Constr.), $\therefore$ CD>AD (I. 19)

Again $\because$ $\angle$CAB + $\angle$CBA=a rt. $\angle$ = $\angle$ACB (32) of which $\angle$CAD>$\angle$ACD (Constr)

$\therefore$ $\angle$BCD>$\angle$CBA, and $\therefore$ BD>CD (I 19)

But if D be the middle point of AB, then AD, CD, DB are all equal See solution of Ques 8 of 1864



4 Let ABCD be a quadrilateral, of which the diagonals AC and BD are intersected at O

Through the angular points A, C and B, D of the quadrilateral draw str lines ∥ to BD and AC respectively, intersecting each other in E, F, G and H. Join FH

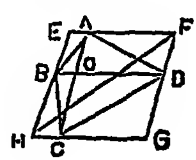
Then the $\triangle$EFH has its sides EH, EF respectively equal to BD, AC and the contained $\angle$ at E=$\angle$AOB (I. 34)

Since EFGH is a ▭ (by construction)

$\therefore$ $\triangle$EFH=½ ▭ EFGH (I 34)

For the same reasons the $\triangle$ AOB=½▭OE, $\triangle$BOC = ½▭HO, $\triangle$ DOC = ½▭OG and $\triangle$AOD = ½▭OF

Adding these, we get fig. ABCD=½▭EFGH =$\triangle$EFH (Proved).



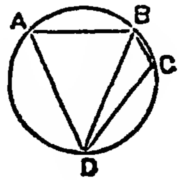
5 Euclid, II 11.

(a) Since $AB.BH=AH^2$ (II. 11), and $AB\ BH=AH\ HB+HB^2$ (II. 3.) $\therefore AH\ HB+HB^2=AH^2$, or $AH\ HB=AH^2-HB^2$

6. Euclid, III 17.

(a) Since the ⊙ of construction cut the line DF′ (see Fig. 2 of III. 17) in two points (III 2), ∴ two tangents only can be drawn from the given point.

7 Euclid, III. 27.



(a) Let ABCD be a quadrilateral inscribed in a ⊙, of which the opp sides AB, DC are equal

Then shall AD be parallel to BC

Join BD

Because AB = DC (Hyp) ∴ arc AB = arc DC (III 28)

Hence ∠ADB = ∠CBD (III 27) and they are alternate ∠s, ∴ AD ∥ BC (I 27).

8 Join PA, PB

Since QAR is a right angled △, of which P is the middle point of the hypotenuse (Hyp) ∴ AP = PR (see Ques 8 of 1864)

Hence ∠PAR = ∠PRA (I 5) But ∠QPA = ∠PAR + ∠PRA (I. 32) ∴ ∠QPA = 2 ∠PAR

Now ∠QPA = ∠PBA (III 32). ∴ ∠PBA = 2∠PAB.

9 Euclid IV. 10

1891.—MORNING.

Paper set by—Mr. G W. Kutchler, M A

Head Examiner,—Mr W. Booth, M A.

1. Simplify the following expressions —

(a) $\dfrac{\frac{3}{5}-\frac{2}{9}}{\frac{5}{8}-\frac{1}{6}}-\dfrac{6-\frac{13}{45}}{\frac{1}{3}-\frac{1}{11}}$

(b) $\dfrac{1}{4-\dfrac{1}{2-\dfrac{1}{1-\frac{4}{15}}}}$

2 Find the value of 2·4607 × ·06 − 3·75 × ·012 + 2·163 − 1·03.

3 Find the value of 15 cwts 3 qrs 9 lbs. at Rs 25 12as 7p. per cwt

4. If a man, walking at the rate of $3\frac{1}{2}$ miles an hour, walks to a place in 4 hours 30 minutes, how long will it take a man, walking at the rate of $3\frac{1}{4}$ miles an hour, to walk there and back?

5. A man invests a certain sum in $4\frac{1}{2}$ per cent Government Paper at 104 The price falling to 101 he sells out and loses Rs 600 by the transaction, exclusive of brokerage. Find the sum invested.

6 A gives B 10 yards start and C 15 yards in race of 100 yards, how much should B give C in 150 yards?

7. Divide $x+6a^{\frac{1}{5}}x^{\frac{4}{5}}+6a^{\frac{1}{5}}x^{\frac{2}{5}}+a+5a^{\frac{2}{5}}x^{\frac{3}{5}}+7a^{\frac{4}{5}}x^{\frac{1}{5}}$ by $x^{\frac{1}{5}}+a^{\frac{1}{5}}$.

8 Solve the following equations —

(a) $x-\frac{3-x}{5}=3 \quad \frac{x-1}{2}+\frac{x+1}{5}-\frac{3}{10}$

(b) $\frac{1}{(a-b)(x-a)}-\frac{1}{(c-d\,(x-c)}=\frac{1}{(a-b)(x-b)}=\frac{1}{(c-d)(x-d)}$

9 A tradesman sells two articles together for 46 rupees, making 10 per cent profit on one and 20 per cent on the other If he had sold each article at 15 per cent profit, the result would have been the same At what price does he sell each article?

10 Prove the rule for finding the greatest common measure of two numbers, a and b

Find the greatest common measure of

$20a^4-3x^3b+b^4$ and $64a^4-3ab^3+5b^4$.

11. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\dots$, each of these ratios will be

$$=\left\{\frac{pa^n+qc^n+re^n+\dots}{pb^n+qd^n+rf^n+\dots\dots}\right\}^{\frac{1}{n}}$$

If $a+b \quad b+c=c+d \quad d+a$, prove that $a=c$, or $a+b+c+d=0$.

1891.—AFTERNOON.

Paper set by—BABU ASHUTOSH MUKHOPADHAY, M A, F R A S, F R S E.

Head Examiner,—MR BOOTH, M A.

1 Define a plane angle, the centre of a circle, parallel straight lines, the angle *of* a segment and an angle *in* a segment.

2 If from the ends of a side of a triangle there be drawn two straight lines to a point within the triangle, these shall be less than the other two sides of the triangle, but shall contain a greater angle.

Prove the necessity of the condition that the lines are to be drawn from the *ends* of the side

3. In a right angled triangle the square described on the hypotenuse is equal to the sum of the squares described on the other two sides

4. In every triangle, the square on the side subtending an acute angle, is less than the squares on the sides containing that angle, by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall on it from the opposite angle, and the acute angle

5 Let B and C be two fixed points, and PQ a straight line in the same plane as B, C Find the position of the point A on the

6

$$
\begin{array}{l}
x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{4}} + y^{\frac{1}{2}}\big)\, x^{\frac{4}{3}} + x^{\frac{2}{3}}y^{\frac{1}{2}} + y \quad \big(x^{\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{1}{4}} + y^{\frac{1}{2}} \\
\qquad\qquad\qquad\quad \underline{x^{\frac{4}{3}} - xy^{\frac{1}{4}} + x^{\frac{2}{3}}y^{\frac{1}{2}}} \\
\qquad\qquad\qquad\quad xy^{\frac{1}{4}} + y \\
\qquad\qquad\qquad\quad \underline{xy^{\frac{1}{4}} - x^{\frac{2}{3}}y^{\frac{1}{2}} + x^{\frac{1}{3}}y^{\frac{3}{4}}} \\
\qquad\qquad\qquad\quad x^{\frac{2}{3}}y^{\frac{1}{2}} - x^{\frac{1}{3}}y^{\frac{3}{4}} + y \\
\qquad\qquad\qquad\quad \underline{x^{\frac{2}{3}}y^{\frac{1}{2}} - x^{\frac{1}{3}}y^{\frac{3}{4}} + y}
\end{array}
$$

(a) Ans $= \dfrac{a+c}{(x-a)(b-a)} - \dfrac{b+c}{(b-a)(x-b)}$

$$= \frac{(x-b)(a+c)-(b+c)(x-a)}{(b-a)(x-a)(x-b)}$$

Numr. $= ax - bc - bx + ac$

$$= -x(b-a) - c(b-a) = -(b-a)(x+c)$$

$\therefore$ Ans $= \dfrac{x+c}{(a-x)(x-b)}$

(b) Ans $= \dfrac{(a^2+b^2)(a+b)(a-b)^2}{(a-b)^2 a(a+b)} = \dfrac{a^2+b^2}{a}.$

7. (1) $\dfrac{1}{x-1} + \dfrac{1}{x-4} = \dfrac{2}{x-3}$

Multiplying both sides by $x-3$, we get

$$\frac{x-3}{x-1} + \frac{x-3}{x-4} = 2$$

or $1 - \dfrac{2}{x-1} + 1 + \dfrac{1}{x-4} = 2$

$\therefore \dfrac{2}{x-1} = \dfrac{1}{x-4}$, or $2x - 8 = x - 1.$

$$\therefore x = 7.$$

(2) $\dfrac{2x+11}{x+5} - \dfrac{9x-9}{3x-4} = \dfrac{4x+13}{x+3} - \dfrac{15x-47}{3x-10}$

or $\left(2 + \dfrac{1}{x+5}\right) - \left(3 + \dfrac{3}{3x-4}\right) = \left(4 + \dfrac{1}{x+3}\right) - \left(5 + \dfrac{3}{3x-10}\right)$

$\therefore \dfrac{1}{x+5} - \dfrac{3}{3x-4} = \dfrac{1}{x+3} - \dfrac{3}{3x-10}$

$\therefore \dfrac{3x-4-3x-15}{(x+5)(3x-4)} = \dfrac{3x-10-3x-9}{(x+3)(3x-10)}$

straight line PQ, which is such that the sum of the squares on AB, AC is least

6 If a straight line drawn through the centre of a circle, bisect a straight line in it which does not pass through the centre, it shall cut it at right angles, and if it cut it at right angles, it shall bisect it

7 If a straight line touch a circle, and from the point of contact a chord be drawn, the angles which this chord makes with the tangent shall be equal to the angles in the alternate segments of the circle

8 Draw a common tangent to two circles, and show that, in general, four common tangents may be drawn to two given circles

9 Give *only* the constructions of—

(*a*) IV 4 To inscribe a circle in a given triangle.

(*b* IV 10 To describe an isosceles triangle having each of the angles at the base double of the third angle.

10 In the triangle ABC, O is the centre of the inscribed circle, and O_1, O_2, O_3, the centres of the escribed circle (that is, circles touching any side and the other two sides produced). Show that the four circles, each of which passes through three of the points O, O_1, O_2, O_3, are all equal.

SOLUTIONS

1891.—MORNING.

1. (*a*) $\text{Ans} = \dfrac{\dfrac{21-10}{35}}{\dfrac{25-18}{30}} \div \dfrac{\dfrac{35-13}{49}}{\dfrac{11-5}{55}} = \left(\dfrac{11}{35}\times\dfrac{30}{7}\right) \div \left(\dfrac{22}{49}\times\dfrac{55}{6}\right)$

$= \frac{11}{35}\times\frac{30}{7}\times\frac{49}{22}\times\frac{6}{55} = \frac{18}{55}$

(*b*) $\text{Ans} = \dfrac{1}{4-\dfrac{1}{2-\dfrac{1}{\frac{8}{13}}}} = \dfrac{1}{4-\dfrac{1}{2-\frac{13}{8}}} = \dfrac{1}{4-\dfrac{1}{\frac{3}{8}}}$

$= \dfrac{1}{4-\frac{8}{3}} = \dfrac{1}{\frac{4}{3}} = \frac{3}{4}$

2 Ans = 147642 − 045 + 2 1 = 2 202642

3

	Rs	as.	p	
2 qrs $= \frac{1}{2}$ of 1 cwt	25	12	7	= value of 1 cwt.
			15	
	386	12	9	= value of 15 cwts
1 qr $= \frac{1}{2}$ of 2 qrs.	12	14	$3\frac{1}{2}$	= value of 2 qrs
7 lb $= \frac{1}{4}$ of 1 qr	6	7	$1\frac{3}{4}$	= value of 1 qr.
2 lb $= \frac{1}{14}$ of 1 qr.	1	9	$9\frac{7}{16}$	= value of 7 lbs.
		7	$4\frac{23}{56}$	= value of 2 lbs.
	Rs 408	3	$4\frac{11}{112}$	= value of 15 cwts. 3 qrs 9 lbs.

4. In $4\frac{1}{2}$ hrs. the 1st man walks $(4\frac{1}{2} \times 3\frac{1}{2})$ miles.

Hence time taken by 2nd man $= \dfrac{2 \times 4\frac{1}{2} \times 3\frac{1}{2}}{3\frac{1}{4}}$

$$= \frac{2 \times 9 \times 7 \times 4}{2 \times 2 \times 13} \text{ hrs} = \frac{126}{13} \text{ hrs} = 9\tfrac{9}{13} \text{ hrs.}$$

5. In every Rs. 104, he loses Rs $(104 - 101) =$ Rs 3.
∴ 3 Rs : 600 Rs. :: Rs 104 : sum invested,
∴ sum invested $=$ Rs $104 \times 200 =$ Rs 20800

6. While A runs 100 yds, B runs 90 yds
and .. .. C runs 85 yds.
∴ While B runs 90 yds, C runs 85 yds.
∴ .. .. 30 yds, C .. $28\frac{1}{3}$ yds
∴ .. .. 150 yds, C runs $28\frac{1}{3} \times 5$ yds or $141\frac{2}{3}$ yds.
Hence B can give C a start of $(150 - 141\frac{2}{3})$ yds, i.e. $8\frac{1}{3}$ yds.

7. $x^{\frac{1}{5}} + a^{\frac{1}{5}}$) $x + 6a^{\frac{1}{5}}x^{\frac{4}{5}} + 5a^{\frac{2}{5}}x^{\frac{3}{5}} + 6a^{\frac{3}{5}}x^{\frac{2}{5}} + 7a^{\frac{4}{5}}x^{\frac{1}{5}} + a$ ($x^{\frac{4}{5}} + 5a^{\frac{1}{5}}x^{\frac{3}{5}} + 6a^{\frac{3}{5}}x^{\frac{1}{5}} + a^{\frac{4}{5}}$ = quotient.

$$\begin{array}{l} x + a^{\frac{1}{5}}x^{\frac{4}{5}} \\ \hline 5a^{\frac{1}{5}}x^{\frac{4}{5}} + 5a^{\frac{2}{5}}x^{\frac{3}{5}} \\ 5a^{\frac{1}{5}}x^{\frac{4}{5}} + 5a^{\frac{2}{5}}x^{\frac{3}{5}} \\ \hline 6a^{\frac{3}{5}}x^{\frac{2}{5}} + 7a^{\frac{4}{5}}x^{\frac{1}{5}} \\ 6a^{\frac{3}{5}}x^{\frac{2}{5}} + 6a^{\frac{4}{5}}x^{\frac{1}{5}} \\ \hline a^{\frac{4}{5}}x^{\frac{1}{5}} + a \\ a^{\frac{4}{5}}x^{\frac{1}{5}} + a \\ \hline \end{array}$$

8. (a) $x - \dfrac{3-x}{5} = 3\dfrac{x-1}{2} + \dfrac{x+1}{5} - \dfrac{3}{10}$

Clearing of fractions, we get
$10x - 6 + 2x = 15x - 15 + 2x + 2 - 3$,
or $12x - 6 = 17x - 16$, ∴ $12x - 17x = 6 - 16$,
or $-5x = -10$, ∴ $x = 2$

(b) $$\frac{1}{(a-b)(x-a)} - \frac{1}{(c-d)(x-c)} = \frac{1}{(a-b)(x-b)} - \frac{1}{(c-d)(x-d)}$$

Transposing, we get

$$\frac{1}{(a-b)}\left\{\frac{1}{x-a} - \frac{1}{x-b}\right\} = \frac{1}{(c-d)}\left\{\frac{1}{x-c} - \frac{1}{x-d}\right\}$$

or $$\frac{a-b}{(a-b)(x-a)(x-b)} = \frac{c-d}{(c-d)(x-c)(x-d)}$$

Hence $(x-a)(x-b)=(x-c)(x-d)$

or $x^2-(a+b)x+ab=x^2-(c+d)x+cd$.

$\therefore \{(a+b)-(c+d)\}x=ab-cd$.

$$\therefore x=\frac{ab-cd}{(a+b)-(c+d)}.$$

9. Let x be the selling price of 1st article in Rs.
and y.................. ... .. 2nd....... ....

Then, their cost prices are respectively $\frac{10}{11}x$ and $\frac{5}{6}y$ Rs.

Also the cost price of both together $=\frac{20}{23}(x+y)$ Rs

$\therefore$ By the question,

$$\left.\begin{array}{l} x+y=46 \text{.........} \quad \text{.... ....(1)} \\ \frac{10}{11}x+\frac{5}{6}y=\frac{20}{23}(x+y) \quad \text{. .. ... (2)} \end{array}\right\}$$

From (2), $1380x+1265y=1320x-1320y$

$\therefore 60x=55y$ or $12x=11y$ . ...(3)

Substitute this in (1)

$\therefore \frac{11}{12}y+y+46$, or $\frac{23}{12}y=46$, $\therefore y=24$ Rs

Hence $x=46-y=22$ Rs. Thus the prices are 22 and 24 Rs.

10. Todhunter's Algebra, Art 110, or any Algebra.

(a) $20a^4-3a^3b+b^4)\ 64a^4-3ab^3+5b^4\ (16$

$$\begin{array}{r} 5 \\ \hline 320a^4-15ab^3+25b^4 \\ 320a^4-48a^3b+16b^4 \\ \hline 3b\,|\,48a^3b-15ab^3+9b^4 \\ \hline 16a^3-5ab^2+3b^3 \end{array}$$

$16a^3-5ab^2+3b^3)\ 20a^4-3a^3b+b^4\ (5a-3b$

$$\begin{array}{l} \qquad\qquad 4 \\ \hline 80a^4-12a^3b+4b^4 \\ 80a^4-25a^2b^2+15ab^3 \\ \hline -12a^3b+25a^2a^2-15ab^3+4b^4 \\ \quad 4 \\ \hline -48a^3b+100a^2b^2-60ab^3+16b^4 \\ -48a^3b+15ab^3-9b^4 \\ \hline 25b^2|100a^2b^2-75ab^3+25b^4 \\ \hline 4a^2-3ab+b^2 \end{array}$$

$$4a^2-3ab+b^2)\;16a^3-5ab^2+3b^3\;(4a+3b$$
$$16a^3-12a^2b+4ab^2$$
$$\overline{12a^2b-9ab^2+3b^3}$$
$$12a^2b-9ab^2+3b^3$$

$\therefore$ G C M $=4a^2-3ab+b^2$.

11. Let $\frac{a}{b}=\frac{a}{d}=\frac{e}{f}=\quad .\quad ..\quad k$ (suppose)

the $a=bk$, $c=dk$, $e=fk$, &c $=$ &

$\therefore pa^n+qc^n+re^n+\&. \quad =(pb^n+rf^n+\&c.\;)k^n$

$\therefore k^n=\dfrac{pa^n+qc^n+re^n+\&c}{pb^n+qd^n+rf^n+\&c}$

Hence each fraction $=k=\left(\dfrac{pa^n+pc^n+re^n+\quad ...}{pb^n+pd^n+rf^n+\quad ...}\right)^{\frac{1}{n}}$

(a) Since $\dfrac{a+b}{b+c}=\dfrac{c+d}{d+a}$

$\therefore \dfrac{a+b}{c+d}=\dfrac{b+c}{d+a}$ (alternately)

$\therefore \dfrac{a+b+c+d}{c+d}=\dfrac{a+b+c+d}{d+a}$ (componenao)

$\therefore$ either $a+b+c+d=0$, or $c+d=d+a$, $\therefore c=a$

1891.—AFTERNOON.

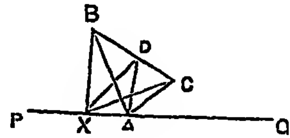
1 Euclid, Book I Def 8, Def 35, Euclid, Book III Def 7, 8
The centre of a circle is the point within a circle from which al straight lines drawn to the circumference are equal

2 Euclid, Book I Prop 21
(a) If this condition be omitted the two straight lines will no necessarily be less than two sides of the triangle

3. Euclid, Book I Prop 47

4. Euclid, Book I Prop. 47

5 Join BC and bisect it at D From D drawn DA$\perp$ to PQ to meet PQ in A Then A shall be the required pt Join AB, AC



For take any other point X is PQ and join BX, DX, and CX

Since DAX is a right angled $\triangle$, having the rt $\angle$ at A,

$\therefore DX>DA$ (1 19) and $\therefore DX^2>DA^2$

To each add BD^2, $\therefore DX^2+BD^2>DA^2+BD^2$

$\therefore 2DX^2+2BD^2>2DA^2+2BD^2$,

$\left.\begin{array}{l}\text{But } 2DX^2+2BD^2=BX^2+CX^2\\ \text{and } 2DA^2+2BD^2=AB^2+AC^2\end{array}\right\}$ Todhunter's Euclid, App Ex I.

$\therefore BX^2+CX^2>AB^2+AC^2$

And this may be shown of all other points in PQ except A, $\therefore$ A is the required point

6 Euclid, Book III. 3

7 Euclid, Book III. 32

8 See Hall and Stevens' Euclid, Ex 17, Page 218.

9 (*a*) Euclid, B IV 4 (*b*) Euclid, B IV 10.

10 Join O_3O_2, and it will pass through A Similarly $O_1 O_3$ passes through B and O_1O_2 passes through C

Join AO_1 and it passes through O Likewise O_2B and O_3C passes through O

Produce BA and CA to G and H respectively

Since the $\angle CAO_2 = \frac{1}{2} \angle$ *Hyp* $CAG = \frac{1}{2} \angle BAH$ (I 15) $= BAO_2$ and the $\angle OAC = \angle OAB$ (Hyp) $\therefore$ the $\angle OAO_2 = \angle OAO_3$ Hence O_1A is $\perp$ to O_2O_3

Similarly, it may be shewn that O_2B is $\perp$ to O_1O_3 and $O_3C \perp$ to O_1O_2 Thus O is the orthocentre of the $\triangle O_1O_2O_3$.

(i) Consider the $\odot$s circumscribed about the $\triangle$s $O_1O_2O_3$ and OO_2O_3

Produce OA to meet the circumf of the $\odot$ described about OO_2O_3 in X and join O_3X, O_2X

Since the $\angle$s OBO_1, OCO_1 are right angles (Proved), $\therefore$ the $\angle$s $BO_1C + BOC = 2$ rt $\angle$s (I 32 *Cor*) But the $\angle BOC = \angle O_3OO_2$ (I. 15), $\therefore$ the $\angle BO_1C + \angle O_3OO_2 = 2$ rt $\angle$s

Again since OO_3XO_2 is a quad inscribed in a $\odot$, $\therefore$ the $\angle O_3OO_2 + \angle O_3XO_2 = 2$ rt $\angle$s (III 22) Hence the $\angle BO_1C + \angle O_3OO_2 = \angle O_3OO_2 + \angle O_3XO_2$ (*Ax* 1). Taking away equals the $\angle BO_1C = \angle O_3XO_2$ Now since the segments $O_3O_1O_2$ and O_3XO_2 contain equal angles, and are on the same base O_2O_3, $\therefore$ they are equal, (III 24). In like manner it may be shewn that segments on the other side O_2O_3 are also equal Hence the $\odot$s are equal

(ii) Similarly it may be shewn that the $\odot$s about the $\triangle$s O_3OO_1 and O_1OO_2 are each = the $\odot$ about $\triangle O_1O_2O_3$ Thus the four $\odot$s are equal [For figure see Hall and Stevens' Euclid IV Book 2nd additional Prop]

1892 —MORNING.

Head Examiner,—MR. W BOOTH, M A.

1 Simplify $\dfrac{3\frac{5}{9} - 1\frac{7}{8} \text{ of } \frac{24}{35}}{11\frac{1}{3} \text{ of } \frac{9}{14} \text{ of } \frac{4}{7}} - \dfrac{4\frac{1}{3} - 7\frac{5}{6} + 3\frac{7}{8}}{\frac{5}{9} \text{ of } 12}$.

2 Find, to the nearest integer, the value of—

$$\frac{39\,37 \times 760 \times 13\,596}{1\,293 \times 12}$$

3. Find the square roots of 097344, of 009604, and of 996004.

4. Find the interest on 10 lakhs of rupees for 10 days at $4\frac{1}{2}$ per cent. per annum.

5. £3,000, which I held in the four per cents, was sold for me when they were at $82\frac{3}{4}$ by a broker whose commission is $\frac{1}{4}$ per cent: and the proceeds were re-invested by him in the four and a half per cents at $93\frac{3}{4}$. What amount of the latter stock did he purchase

6 Solve the equations—

(i) $\frac{1}{5}(x-8)+\frac{4+x}{4}+\frac{x-1}{7}=7-\frac{23-x}{5}$.

(ii) $\frac{x+a}{x+b}=\frac{x+3a}{x+a+b}$

7 The express leaves Bristol at 3 p.m and reaches London at 6 p m the ordinary train leaves London at 1-30 p m, and arrives at Bristol at 6 p m. If both trains travel uniformly, find the time when they will meet

8 Find the value of—

$$\frac{4y}{5}(y-x)-35\left[\frac{3x-4y}{5}-\frac{1}{10}\{3x-\tfrac{5}{7}(7x-4y)\}\right]$$

when $x=-\frac{1}{2}$ and $y=2$.

9 Find the square root of—

$$\frac{4a^2-12ab-6bc+4ac+9b^2+c^2}{4a^2+9c^2-12ac}.$$

10 If $x \quad y \quad\quad y \quad z$, find the simplest value of—

$$\frac{xyz(x+y+z)^3}{(xy+yz+xz)^2}.$$

1892—AFTERNOON.

Head Examiner,—MR W BOOTH, M A

1 Define a plane surface, rhombus and an axiom What axiom affords the ultimate test of equality of two geometrical magnitudes?

2 Prove that on the same base and on the same side of it, there cannot be two triangles having their sides which are terminated at one extremity of the base equal to one another, and likewise those which are terminated at the other extremity equal to one another

3 At a given point in a given straight line you are required to make an angle equal to a given angle

4. If there are two straight lines, one of which is divided into any number of parts, the rectangle contained by the two straight lines is equal to the sum of the rectangles contained by the undivided straight line and the several parts of the divided line

5 You are required to find the centre of a given circle

6 Prove that one circle cannot cut another at more than two points

7 You are required to inscribe a regular quindecagon in a given circle

(a) Show that if a polygon inscribed in a circle is equilateral, it is also equiangular

8. Bisect a quadrilateral figure by a straight line drawn through an angular point

9 Describe a circle to touch a given circle and also to touch a given straight line at a given point

10. Prove that of all triangles of given base and area the isosceles is that which has the least perimeter.

SOLUTIONS

1892.—MORNING.

1 1st fraction $=\dfrac{\frac{32}{9}-\frac{15}{8}\times\frac{24}{25}}{\frac{35}{3}\times\frac{9}{14}\times\frac{2}{5}}=\dfrac{\frac{32}{9}-\frac{9}{5}}{\frac{7}{3}\times\frac{9}{7}}=\dfrac{\frac{160-81}{45}}{3}=\dfrac{\frac{79}{45}}{3}$

$=\frac{79}{45}\times\frac{1}{3}=\frac{79}{135}$

2nd fraction $=\dfrac{4+3-7+\frac{3-5+2}{9}}{\frac{5}{9}\times 12}=\dfrac{0}{\frac{5}{3}\times 4}=0$

$\therefore$ Ans $=\frac{79}{135}$

2. Ans $=\dfrac{3937\times 760\times 13596}{1293\times 12\times 100}=\dfrac{3937\times 19\times 2266}{1293\times 5}=\dfrac{169503598}{6465}$

$=26218\frac{4228}{6465}=26219$, nearly

3 (a) $\sqrt{\cdot 097344}=\cdot 312$; (b) $\sqrt{\cdot 009604}=\cdot 098$;

(c) $\sqrt{\cdot 996004}=\cdot 998$

4. Interest $=1000000\times\frac{10}{365}\times\frac{9}{100}$ Rs

$=\frac{90000}{73}=$ Rs 1232 14as nearly.

5 Price exclusive of brokerage $=(82\frac{3}{4}-\frac{1}{4})£=82\frac{1}{2}£$

$\therefore$ £100 : £3000 :: £$82\frac{1}{2}$: proceeds of sale

$\therefore$ proceeds of sale $=£30\times 82\frac{1}{2}=£2475$.

Price including brokerage $=(98\frac{3}{4}+\frac{1}{4})£=99£$.

$\therefore$ £99 : £2475 :: £100 : stock bought.

$\therefore$ Stock $=£\dfrac{100\times 2475}{99}=£2500$

6 (a) $\frac{1}{5}(x-8)+\dfrac{4+x}{4}+\dfrac{x-1}{7}=7-\dfrac{23-x}{5}$

Clearing of fractions, we get,

$28x-224+140+35x+20x-20=980-644+28x.$

or $28x+35x+20x-28x=224-140+20+980-644$

or $55x=440 \qquad \therefore x=8$

(b) Multiplying across we get

$(x+a)^2+b(x+a)=(x+3a)(x+b),$

or $x^2+2ax+a^2+bx+ab=x^2+3ax+bx+3ab,$

or $2ax-3ax=-a^2+2ab$, or $-ax=-a(a-2b)$

$\therefore x=a-2b$

7 Let x denote the no of hrs after 3 p m when they will meet.

The express takes 3 hrs to travel the distance from B to L

The ordinary takes $4\frac{1}{2}$ hrs ....L to B

Take l = distance from B to L in miles

Then express rate $=\frac{l}{3}$ miles, and ordinary $=\frac{l}{4\frac{1}{2}}$ miles,

the ordinary started $1\frac{1}{2}$ hrs before the express

$\therefore$ By the question,

$$(x+1\tfrac{1}{2})\times\frac{l}{4\frac{1}{2}}+x\times\frac{l}{3}=l,$$

$$\left(x+\frac{3}{2}\right)\times\frac{2}{9}+\frac{x}{3}=1 \text{ or } \frac{2x}{9}+\tfrac{1}{3}+\frac{x}{3}=1 \text{ or } 5x=6$$

$\therefore x=1\frac{1}{5}$ hrs $=1$ hr 12 min

Hence they will meet 12 min past 4

8 Ans $=\frac{4y}{5}(y-x)-35\left[\frac{3x-4y}{5}-\frac{1}{10}\{3x-\frac{5}{7}(7x-4y)\}\right]$

$=\frac{4y}{5}(y-x)-7(3x-4y)+\frac{7}{2}\{3x-\frac{5}{7}(7x-4y)\},$

$=\frac{4y}{5}(y-x)-21x+28y+\frac{21x}{2}-\frac{5}{2}(7x-4y),$

$=\frac{4y}{5}(y-x)-21x+28y+\frac{21x}{2}-\frac{35x}{2}+10y,$

$=\frac{4y}{5}\times\frac{5}{2}-21x+28y-7x+10y$

$=2y-28x+38y=40y-28x$

$=40\times2+28\times\frac{1}{2}=80+14=94$

9. $4a^2-12ab+4ac+9b^2-6bc+c^2\ (2a-3b+c$

$4a^2$

$4a-3b \mid -12ab+4ac+9b^2$

$\qquad\quad -12ab+9b^2$

$4a-6b+c \mid 4ac-6bc+c^2$

$\qquad\qquad\quad 4ac-6bc+c^2$

and $\sqrt{(4c^2-12ac+9c^2)}=2a-3c$

$\therefore$ Ans $=\dfrac{2a-3b+c}{2a-3c}$

10 Here $y^2=xz$

Now, $\dfrac{xyz(x+y+z)^3}{(xy+yz+zx)^3}=\dfrac{xyz(x+y+z)^3}{(xy+yz+y^2)^3}=\dfrac{xyz(x+y+z)^3}{y^3(x+y+z)^3}$

$=\dfrac{xz}{y^2}=\dfrac{y^2}{y^2}=1$

1892.—AFTERNOON

1 Euclid Bk I Def 32

An anxiom is a self evident truth, or more properly *Common Notion* Ax 8

2 Euclid, Book I Prop 7

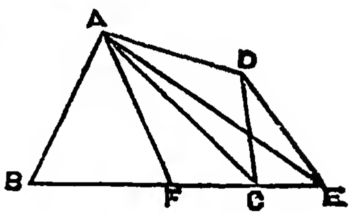
3. Euclid, Book I Prop 23

4. Euclid, Book II Prop 1

5 Euclid, Book III Prop 1

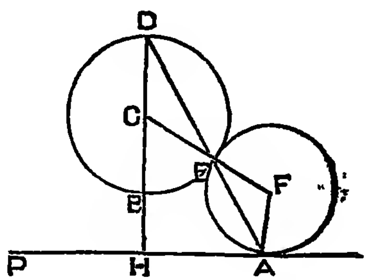
7 Euclid, Book IV Prop. 16

(a) See Hall and Stevens' Euclid, Ex 2, Page 275.



8 Let ABCD be a quadrilateral figure, and A the angular point.

Through D draw DE ‖ to AC to meet BC produced at E and join AE Bisect BE in F and join AF. Then shall AF bisect the figure. Because AC is ‖ to DE (*Constr*) $\therefore$ the $\triangle$ ADC $=\triangle$ACE (I 37) To each add the $\triangle$ ABC $\therefore$ the quad EADC $=\triangle$ ABE Again, $\because$ BF = BF (*Constr*), $\therefore$ the $\triangle$ ABF $=\frac{1}{2}$ $\triangle$ ABE (I 39) $\frac{1}{2}$ = fig ABCD (*Proved*). Hence AF bisects the figure



9 Let DEB be the given ⊙, PQ the given str line, and A the given point in it It is required to describe a ⊙ to touch the ⊙ DEB and also to touch PQ at A

At A draw AF ⊥ to PQ (I 11) Find C the centre of the ⊙ DEB (III 1) and draw the diameter BD ⊥ to PQ Join A to one extremity D of the diam, cutting the circumf at E Join GE, and produce it to cut AF at F Then F is the centre and FA the radius of the required ⊙

Because DB and AF are perp to PQ (*Constr*) $\therefore$ DB is ‖ to AF (I 21) $\therefore$ the ∠CDE = ∠EAF (I 29); but the ∠CDE=∠GED (I. 5) for CD=CE, (being radius. and the ∠CED=∠AEF I. 15) ;

∴ the ∠EAF=∠AEF, and ∴ AF=EF (I 6) Hence the ⊙ with centre F and radius FA will pass through A and F and will touch PQ at A, *for the* ∠s at A are rt ∠s (III 16), and also touch the given ⊙ at E, for the pts C, E, F are collinear (III 12)

[For figure see Hall and Stevens' Book III. additional proposition]

10 Let ABC and DBC be two △s on the given base BC and having a given area, of which the △ABC is isosceles, having AB=AC Then shall AB+AC<BD+DC

Produce BA to E, making AE=BA [I 3] and join ED

In the △s ADE, ADC, since AE=AC constr for AB=AC (Hyp) and AD common, also the ∠ EAD=ABC for AD ∥ to BC Proved=the ∠ABC (I 5)=∠CAD (I 29), ∴ ED=DC (I 4)

Now, since BD+DE > BE (I 20); but DE=DC (Proved) and AE=AC (*Constr*), ∴ BD+DC>AB+AC

1893.—MORNING

Paper set by—Mr W Griffiths, M A

Head Examiner,—Mr W Booth, M A

1. Simplify —

(1) $1+\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$

(2) $\dfrac{8\frac{1}{7}-1\frac{1}{6}}{\frac{1}{3}\div 1\frac{1}{2}}-\dfrac{1}{5\frac{3}{4}-1\frac{1}{6}}$

2 Divide 1 84626 by 23 4

Express 456 and 654 as vulgar fractions reduced to their lowest terms, and their sums as a circulating decimal

3. Find the cost of 73 cwt. 3 qrs 14 lbs at 4*l* 13*s* 6*d* per cwt.

4. Distinguish between true discount and banker's discount

Find the former in the case of a bill for Rs 3486 6as 8 p due 16 months hence, the rate of interest being $5\frac{1}{2}$ per cent per annum.

5 A man invests Rs 163,000, part in government 4 per cent. stock at 108, and the remainder in municipal 5 per cent debenture stock at $109\frac{1}{2}$ Find how much he must invest in each in order that he may have an equal income from the two sources

6 Find the highest common factor of—

$$3x^3-5x^2+5x-2, \text{ and } 2x^4-2x^3+3x^2-x+1$$

7 Extract the square root of—

$$4x^6-12x^5+13x^4-22x^3+25x^2-8x+16.$$

8 Solve the equations —

(1) $120x-4[5x-2\{6x+7(x-8)\}]$
$$=16-4[3x-2\{x-6(x-1)\}]$$

$$\therefore \frac{-19}{(x+5)(3x-4)} = \frac{-19}{(x+3)(3x-10)}$$

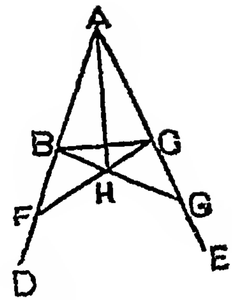
$\therefore (x+5)(3x-4) = (x+3)(3x-10)$
$\therefore 3x^2+11x-20 = 3x^2-x-30$
$\therefore 12x = -10 \qquad \therefore x = -\frac{10}{12} = -\frac{5}{6}$

8. Let x be the cost of the picture in shilling.
Then x is also. .. ..the frame ...

By the question, $x-20 = \frac{1}{2}(x+15)$
$\therefore 2x-40 = x+15 \qquad \therefore x = 55s. = £2\ 15s.$

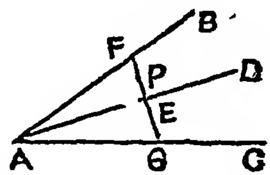
1860.—AFTERNOON.

1 $\because \angle CBG = \angle BCF$ (Proved in I 5)
$\therefore BH = CH$ (1 6);
Again $\because BA = AC$ and AH com.
and $BH = CH$
$\therefore \angle BAH = \angle CAH$ (I. 8).



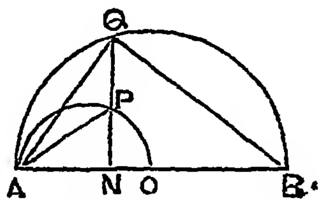
2. Euclid I. 31.

3. Let AB and AC be the given str. lines and P the given point.
Bisect the $\angle BAC$ by AD;
From P draw $PE \perp AD$
Prod. PE both ways to meet AB in F and AC in G
$\because \angle FAE = \angle GAE$ (Cons)
and $\angle FEA = \angle AEG$ (Ax 11) also AE com.
$\therefore \angle AFE = \angle AGE$. (I. 26)



4 Euclid II 14.

5. Join QB
$\because$ The angle AQB is a rt angle (III 31)
$\therefore AB^2 = AQ^2 + BQ^2$ (I 47),
$= AN^2 + NB^2 + 2\,NQ^2$
Also $AB^2 = AN^2 + NB^2 + 2AN\ NB$
(II 4).
$\therefore 2NQ^2 = 2AN\ NB$, or $NQ^2 = AN\ NB$
Add to each of those equals AN^2,
then $AN^2 + NQ^2 = AN^2 + AN\ NB$.
But $AN^2 + NQ^2 = AQ^2$ (I 47)
and $AN^2 + AN\ NB = AB\ AN$ (II 3).
$\therefore AQ^2 = AB\ AN = 2AO\ AN$
If PO be joined, we can likewise prove that $AP^2 = AO.AN$.
Wherefore $AQ^2 = 2AP^2$.



$$(2)\ \frac{x+b}{a-b} = \frac{x-b}{a+b}, \qquad (3)\ \left.\begin{array}{l}\dfrac{6}{x}+\dfrac{4}{y}=3. \\ \dfrac{9}{a}-\dfrac{1}{y}=2\frac{3}{4}\end{array}\right\}$$

9 Divide the number 834 into two parts such that 30 per cent. of one part exceeds 40 per cent of the other part by 6

10 If $a : b :: c : d$, prove that
$ma-nb : a+b :: mc-nd : c+d$

What number must be added to each of the numbers 3, 5, 7, 10 to give four numbers in proportion ?

1893.—AFTERNOON.

Paper set by,—Mr W Griffiths, M A

Head Examiner,—Mr W Booth, M A.

1 Define a right angle, a rectangle, a tangent to a circle, and a regular polygon

2 If two triangles have two sides of the one equal to two sides of the other, each to each, and have also the angles contained by those sides equal, then shall their bases or third sides be equal, and the triangles shall be equal in area, and their remaining angles shall be equal each to each, namely, those to which the equal sides are opposite ; that is to say, the triangles shall be equal in all respects. Prove this proposition

3. ABCD is a parallelogram, and KC, KD, are the complements of the parallelograms EH, GF, about the diagonal AC, EKF being parallel to AHD, and GKH to AEB show that the complement BK will be equal to the complement KD

Show also that the gnomon BHF will be double the triangle CFH

4 In an obtuse-angled triangle, if a perpendicular is drawn from either of the acute angles to the opposite side produced, show that the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side on which, when produced, the perpendicular falls, and the line intercepted without the triangle, between the perpendicular and the obtuse angle

5 Prove that angle at the centre of a circle is double of an angle at the circumferance, standing on the same arc

5 If from any point without a circle a tangent and a secant be drawn, prove that the rectangle contained by the whole secant and the part of it without the circle will be equal to the square on the tangent

7 In a given circle inscribe a triangle equiangular to a given triangle

8 Bisect a triangle by a straight line drawn through a given point in one of its sides

9 Two circles touch each other externally in A, and a straight line touches them in B and C respectively Prove that BAC is a right angle

10 Given the base and vertical angle of a triangle, find the locus of the centre of the inscribed circle

SOLUTIONS

1893 —MORNING

1. (1) Ans $=1+\dfrac{210+280+315+336+350+360}{420}$

$=1+\frac{1851}{420}=1+4\frac{171}{420}=5\frac{57}{140}$

(2) Ans $=\dfrac{7\frac{1}{7}-\frac{1}{6}}{\frac{2}{3}+\frac{1}{2}}-\dfrac{1}{4\frac{3}{4}-\frac{1}{5}}=\dfrac{\frac{50}{7}-\frac{1}{6}}{\frac{2}{3}+\frac{1}{2}}-\dfrac{1}{4+\frac{3}{4}-\frac{1}{5}}$

$=\dfrac{\frac{300-7}{42}}{\frac{4+9}{6}}-\dfrac{1}{4+\frac{15-4}{20}}=\dfrac{\frac{293}{42}}{\frac{13}{6}}-\dfrac{1}{4\frac{11}{20}}$

$=\dfrac{293}{42}\times\dfrac{6}{13}-\dfrac{1}{\frac{91}{20}}=\dfrac{293}{91}-\dfrac{20}{91}=\dfrac{273}{91}=3$

2 (a) 23 4) 1 84626 (·0789

(b) $456=\dfrac{456-4}{990}=\dfrac{452}{990}=\dfrac{226}{495}$.

(c) $654=\dfrac{654-6}{990}=\dfrac{648}{990}=\dfrac{18\times36}{18\times55}=\dfrac{36}{55}$

(d)
```
  4565656
  6545454
 11111110
```
$\therefore$ Ans $=1\dot{1}$

3

	£	s	d	
	4	13	6	= cost of 1 cwt
			10	
	46	15	0	= cost of 10 cwts
			7	
	327	5	0	= cost of 70 cwts
	14	0	6	= cost of 3 cwts
2 qrs $=\frac{1}{2}$ of 1 cwt	2	6	9	= cost of 2 qrs
1 qr $=\frac{1}{2}$ of 2 qrs	1	3	$4\frac{1}{2}$	= cost of 1 qr
14 lbs $=\frac{1}{2}$ of 1 qr		11	$8\frac{1}{4}$	= cost of 14 lbs
	£345	7	$3\frac{3}{4}$	= cost of 73 cwts. 3 qrs 14lbs

4. See Banker's Discount = Interest

(a) Int of Rs 100 for 16 mo @ $5\frac{1}{2}$ p ct $=\frac{16}{12}\times\frac{11}{2}$ Rs = Rs $\frac{22}{3}$
∴ Rs $(100+\frac{22}{3})$: Rs 3486, 6 as 8 p :: Rs $\frac{22}{3}$: true disc.
or Rs. $\frac{322}{3}$: Rs $3486\frac{5}{12}$ :: Rs. $\frac{22}{3}$: true disc.

$\therefore$ true disc $= \frac{22}{3} \times \frac{41837}{12} \times \frac{3}{731}$ Rs.

$= \text{Rs } \frac{20000}{84} = \text{Rs } 238 \text{ 3as } 2\frac{4}{7} \text{ p}$

6 $4 \cdot 5 \;.\; 108 \quad x \qquad x = \frac{5 \times 108}{4} = 135 \quad 135 + 109\frac{1}{2} = 244\frac{1}{2}$

$244\frac{1}{2} : 163000 \quad 135 \quad$ 1st invest $= \frac{163000 \times 135 \times 2}{489}$

$=$ Rs. 90,000, 2nd invest $= 163000 - 90,000 =$ Rs 73000.

7. $3x^3 - 5x^2 + 5x - 2)\, 2x^4 - 2x^3 + 3x^2 - x + 1 \,(2x + 4$

$$3$$

$$6x^4 - 6x^3 + 9x^2 - 3x + 3$$
$$6x^4 - 10x^3 + 10x^2 - 4x$$

$$4x^3 - x^2 + x + 3$$
$$3$$

$$12x^3 - 3x^2 + 3x + 9$$
$$12x^3 - 20x^2 + 20x - 8$$

$$17)\, 17x^2 - 17x + 17$$

$$x^2 - x + 1$$

$x^2 - x + 1)\, 3x^3 - 5x^2 + 5x - 2 \,(3x - 2$

$$3x^3 - 3x^2 + 3x$$

$$-2x^2 + 2x - 2$$
$$-2x^2 + 2x - 2$$

$\therefore$ G. C M $= x^2 - x + 1$.

7. $4x^6 - 12x^5 + 13x^4 - 22x^3 + 25x^2 - 8x + 16 \,(2x^3 - 3x^2 + x - 4$ $=$ s root.

$$4x^6$$

$4x^3 - 3x^2 \mid -12x^5 + 13x^4$

$\qquad\qquad \mid -12x^5 + 9x^4$

$4x^3 - 6x^2 + x \mid 4x^4 - 22x^3 + 25x^2$

$\qquad\qquad \mid 4x^4 - 6x^3 + x^2$

$4x^3 - 6x^2 + 2x - 4 \mid -16x^3 + 24x^2 - 8x + 16$

$\qquad\qquad \mid -16x^3 + 24x^2 - 8x + 16$

8. (1) $120x - 4[5x - 2\{6x + 7x - 56\}]$

$= 16 - 4[3x - 2\{x - 6x + 6\}]$

or $120x - 4[5x - 2(13x - 56)] = 16 - 4[3x - 2(-5x + 6)]$

or $120x - 4[5x - 26x + 112] = 16 - 4(3x + 10x - 12]$

or $120x - 4[-21x + 112] = 16 - 4[13x - 12]$

or $120x + 84x - 448 = 16 - 52x + 48$

or $204x-448=64-52x$, or $204x+52x=64+448$

or $256x=512$ $\therefore x=2$

(2) $\dfrac{x+b}{a-b}=\dfrac{x-b}{a+b}$

or $\dfrac{x+b}{x-b}=\dfrac{a-b}{a+b}$. By Comp and Divd, $\dfrac{x}{b}=\dfrac{a}{-b}$ $\therefore x=-a$.

(3) $\dfrac{6}{x}+\dfrac{4}{y}=3$ (1) $\dfrac{9}{x}-\dfrac{1}{y}=2\frac{3}{4}$.... (2)

Multiply (2) by 4 and add (1)

$\dfrac{6}{x}+\dfrac{4}{y}+\dfrac{36}{x}-\dfrac{4}{y}=3+11$, or $\dfrac{42}{x}=14$ $\therefore x=3$

And $-\dfrac{1}{y}=\dfrac{11}{4}-\dfrac{9}{x}=\dfrac{11}{4}-3=-\frac{1}{4}$, $\therefore y=4$

9 Let x be the greater part,

then $834-x$ is the lesser part,

Then by the question,

$\frac{40}{100}x-\frac{30}{100}(834-x)=6$

or $\dfrac{4x}{10}-\frac{3}{10}(834-x)=6$,

$\therefore 4x-2502+3x=60$, $\therefore 7x=2562$, $\therefore x=366$.

Hence the parts are 366 and 468.

10 Since $\dfrac{a}{b}=\dfrac{c}{d}$, $\therefore \dfrac{ma}{nb}=\dfrac{mc}{nd}$, $\therefore \dfrac{ma-nb}{nb}=\dfrac{mc-nd}{nd}$

$\therefore \dfrac{ma-nb}{mc-nd}=\dfrac{nb}{nd}=\dfrac{b}{d}$.

Also $\dfrac{a+b}{b}=\dfrac{c+d}{d}$, $\therefore \dfrac{a+b}{c+d}=\dfrac{b}{d}$

Hence $\dfrac{ma-nb}{mc-nd}=\dfrac{a+b}{c+d}$, or $\dfrac{ma-nb}{a+b}=\dfrac{mc-nd}{c+d}$

$\therefore ma-nb : a+b :: mc-nd : c+d$.

(a) Let x be the no to be added,

then by the question,

$3+x : 5+x = 7+x : 10+x$

$\therefore (3+x)(10+x)=(5+x)(7+x)$,

or $30+13x+x^2=35+12x+x^2$,

$\therefore x=5$

Hence the required no to be added is 5

1893.—AFTERNOON.

1. Euclid, Book I Def. 10, Def. 31, Book III Def 2

A polygon which has all its sides equal and all its angles equal is called a *regular* polygon

2. Euclid, Book I Prop 4

3. Euclid, Book I Prop 43

(a) Since AK is a ▭ and EH its diagonal

∴ ▭AK=2△EHK (I 34)

Again, ∵ the ▭ BK and △EKC are on the same base EK and bet. the same ∥s EK and BC

∴ ▭ BK=2△EKC (I 4)

Similarly, ▭ KD=2△HKC

Hence, adding we get the gnomon BHE=2△CEH

4. Euclid, Book II Prop 12

5. Euclid, Book III Prop 20

6. Euclid Book III Prop. 36

7. Euclid, Book IV. Prop 2

8. Let ABC be the given △, and P the given point in the side AB

Bisect AB at Z, and join CZ, CP

Through Z draw ZQ ∥ to CP

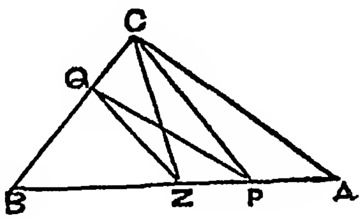
Join PQ

Then shall PQ bisect the △

Because BZ=AZ (constr),

∴ △BZC=½△ABC (I 38)

Again, ∵ △s PQZ ZCQ are on the same base ZQ and bet the same ∥s ZQ and PC (*Proved*)



∴ △PQZ=△ZCQ (I 13) to each add the △BZQ

∴ △BPQ=△BZC=½△ABC

9. At A draw the common tangent AP to meet BC at P

Since BP = AP (III 17, *Cor*),

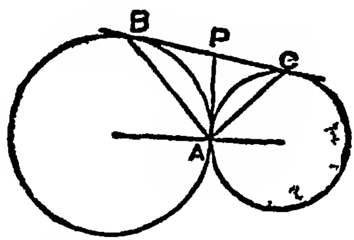
∴ the ∠PAB=∠PBA (I 5)

Again, ∵ AP=PC (III 17. *Cor*);

∴ the ∠PAC=∠PCA (I 5)

∴ the ∠BAC=∠s PBA+PCA (*Ax* 2)

Hence the ∠BAC is a rt ∠ (I. 32)



10. Let BAC be any △ on the given base BC, having its vertical ∠=the given ∠X, and let AO, BO, CO be the bisectors of its angles. It is required to find the locus of O

Then from △BOC O+½B+½C=2 rt ∠s (I 32)

Also from the △ABC. A+B+C=2 rt ∠s (I 32)

$\therefore \frac{1}{2}A+\frac{1}{2}B+\frac{1}{2}C=$ a rt $\angle$

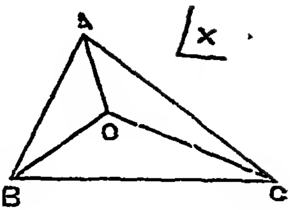
Taking the difference of these equals, we get $O-\frac{1}{2}A=$ one rt $\angle$

$\therefore O=$ a rt $\angle+\frac{1}{2}\angle A$

But A is constant, being always $=\angle X$,

$\therefore O$ is constant

Hence since the base BC is fixed, the locus of O is the arc of a segment of which BC is the chord



1894.—MORNING.

Head Examineo,—MR W BOOTH, M A

1 In a compound metal containing tin and copper only, the proportion of tin to copper is 7 75 to 92 25 Find to the nearest penny the value of 8 cwt 3 qrs of it. Tin costs 140*l* , coppei 80*l* per ton.

2 A rectangular court is 50 yards long and 30 yards broad It has paths joining the middle points of the opposite sides of 6 feet in breadth and also paths of the same breadth running all round it The remainder is covered with grass If the cost oft he pavement be 1s. 8*d* per square foot and the turf 3s per square yard, find the cost of laying out the court

3 Find the value of $\cdot 267\dot{1}875$ of 3*l* in shillings, pence, and decimal of a penny

4 Find the square root of $1-(0678)^3$ to four places of decimals.

3 At a cricket match a contractor provided luncheon for 24 and fixed the price to gain 12½ per cent on his outlay Three persons were absent The remaining 21 paid the fixed price and the contractor lost 2 rupees What was the charge ?

6 If $a \quad b=c \quad d$, prove that

$$a^2+ab+b^2 \quad a^2-ab+b^2=c^2+cd+d^2 \quad c^2-cd+d^2$$

7. Find the G C M of

$$x^4-2x^3-4x^2-55x \text{ and } x^5+8x^4+25x^3+52x^2-11x.$$

8 Solve the equation —

$$\frac{x+5}{6}+\frac{1}{9}\left(\frac{x}{2}+\frac{2}{5}\right)-\frac{2}{3}(3+2x)=\frac{4x-14}{3}+\frac{x+10}{10},$$

$$(2)\quad \frac{3-x}{2}-\frac{1}{3}\left(\frac{3-2x}{4}\right)=\frac{2x+3}{7}+\left(\frac{1}{4}-\frac{1+3x}{2}\right).$$

9 Find the simple interest on Rs 12345 for 134 days at $2\frac{3}{4}$ per cent per annum

10 Simplify

$$\frac{(2k-3l)^2-k^2}{4k^2-(3l+k)^2}+\frac{4k^2-(3l-k)^2}{9(k^2-l^2)}+\frac{9l^2-k^2}{(2k+3l)^2-k^2}$$

1894.—AFTERNOON.

Head Examiner,—Mr W. Booth, M.A.

1. From a given point in a given straight line you are required to draw a straight line equal to the given straight line.

2. Define parallel lines. Define a plane.
Prove that a straight line falling on two other straight lines makes the alternate angles equal to one another, then the straight lines shall be parallel.

3. You are required to construct a square equal to a regular pentagon.

4. Prove that if any two points are taken in the circumference of a circle the chord which joins them falls within the circle.

5. Prove that similar segments of circles on equal chords are equal to one another.

6. From a given circle you are required to cut off a segment which shall contain an angle equal to a given angle.

7. By the fourth Book of Euclid you are required to construct an angle equal to one-thirtieth part of a right angle.

8. Trisect a right angle.

9. Describe a circle passing through two given points and touching a given straight line.

10. Given two points A and B and a straight line L, find a point P in L such that AP plus B shall be a minimum.

SOLUTIONS

1894.—MORNING.

1. Since $7 \cdot 75 + 92 \cdot 25 = 100$

$\therefore$ In the compound $\dfrac{7 \cdot 75}{100} \times 8\frac{3}{4}$ cwts is tin, or $\frac{217}{6400}$ tons

. . . . . . $\dfrac{52 \cdot 25}{100} \times 8\frac{3}{4}$ cwts is copper, or $\frac{2583}{6400}$ tons

$\therefore$ value reqd. $= \left(\frac{217}{6400} \times 140 + \frac{2583}{6400} \times 80\right)$£
$= £4\ 14s.\ 11\frac{1}{4}d + £32\ 5s.\ 9d = £37\ 0s.\ 8d$

2. Area of paths surrounding the court

$= 2\{(150-6) + (90-6)\} \times 6$ sq. ft. $= 2 \times 228 \times 6$ sq. ft.

Area of paths joining the middle points of opposite sides

$= 6\{(150-12) + 90-12)\} - 6 \times 6$ sq. ft. $= 210 \times 6$ sq. ft.

$\therefore$ Area of pavement $= (2 \times 228 \times 6 + 210 \times 6)$ sq. ft.
$= 666 \times 6$ sq. ft. $= 3996$ sq. ft.

Hence area of ground covered with turf

$= 50 \times 30$ sq. yds. $- 222 \times 2$ sq. yds. $= 1056$ sq. yds.

$\therefore$ cost $= (3996 \times 1\frac{2}{3} + 1056 \times 3)s = 9828s = £491\ 8s.$

3 Ans $= \dfrac{2671875-267}{9999000}$ of $£3 = \dfrac{2671608}{9999000} \times 3 \times 20s.$

$= 16\frac{2604}{83325}s = 16s\ \ 0\ 37\dot{5}013d$

4 $1-(0678)^2 = 1-0\ 00311665762 = 999688334248$

$\sqrt{(}\ 9\dot{9}9688\dot{3}34248 = 9998$.

5 Here gain $= \dfrac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$ of the outlay

Suppose Re. 1 to be the cost, ∴ charge is $\frac{9}{8}$

. Loss $= 24 - 21 \times \frac{9}{8} = \dfrac{192-189}{8} = \frac{3}{8}$

$\frac{3}{8}$ 2 $\frac{9}{8}$ charge, charge $=$ Rs $\dfrac{2\times 9\times 8}{3\times 8} =$ Rs 6

6 Let $\dfrac{a}{b} = \dfrac{c}{d} = k$, then $a = bk$ and $c = dk$

Now $\dfrac{a^2+ab+b^2}{a^2-ab+b^2} = \dfrac{b^2k^2+b^2k+b^2}{b^2k^2-b^2k+b^2} = \dfrac{b^2(k^2+k+1)}{b^2(k^2-k+1)} = \dfrac{k^2+k+1}{k^2-k+1}$,

and $\dfrac{c^2+cd+d^2}{c^2-cd+d^2} = \dfrac{d^2k^2+d^2k+d^2}{d^2k^2+d^2k+d^2} = \dfrac{d^2(k^2+k+1)}{d^2(k^2-k+1)} = \dfrac{k^2+k+1}{k^2-k+1}$

. $\dfrac{a^2+ab+b^2}{a^2-ab+b^2} = \dfrac{c^2+cd+d^2}{c^2-cd+d^2}$,

or $a^2+ab+b^2 \ .\ a^2-ab+b^2 = c^2+cd+d^2 \quad c^2-cd+d^2.$

7 1st quantity $= x(x^3-2x^2-4x-55)$

2nd . ... $= x(x^4+8x^3+25x^2+52x-11)$

$$
\begin{array}{r|l|l}
\text{Now, } x^3-2x^2-4x-55 & x^4+8x^3+25x^2+52x-11 & x+10 \\
 & x^4-2x^3-4x^2-55x & \\
\hline
 & 10x^3+29x^2+107x-11 & \\
 & 10x^3-20x^2-40x-550 & \\
\hline
49 & 49x^2+147x+539 & \\
\hline
 & x^2+3x+11 &
\end{array}
$$

$$
\begin{array}{r|l|l}
x^2+3x+11 & x^3-2x^2-4x-55 & x-5 \\
 & x^3+3x^2+11x & \\
\hline
 & -5x^2-15x-55 & \\
 & -5x^2-15x-55 & \\
\hline
\end{array}
$$

∴ G C M required $= x(x^2+3x+11)$

8 (1) $\dfrac{x+5}{6} + \frac{1}{9}\left(\dfrac{x}{2}+\dfrac{2}{5}\right) - \frac{2}{3}\,3+2x) = \dfrac{4x-14}{3} + \dfrac{x+10}{10}$

Clearing of fractions, we get

$$15(x+5)+10\left(\frac{x}{2}+\frac{2}{5}\right)-60(3+2x)=30(4x-14)-9(x+10),$$

or $15x+75+5x+4-180-120x=120x-420+9x+90$,

or $-100x-101=129x-330$, or $229x=229$, $\therefore x=1$.

(2) $\frac{3-x}{2}-\frac{1}{3}\left(\frac{3-2x}{4}\right)=\frac{2x+3}{7}+\left(\frac{1}{4}-\frac{1+3x}{2}\right)$

Clearing of fractions, we get

$42(3-x)-7(3-2x)=12(2x+3)+21-42(1+3x)$,

or $126-42x-21+14x=24x+36+21-42-126x$,

or $105-28x=-102x+15$, or $74x=-90$, $\therefore x=-\frac{45}{37}$.

9. Interest $=$ Rs. $12345\times\frac{134}{365}\times\frac{2\frac{3}{4}}{100}=$ Rs $12345\times\frac{134}{365}\times\frac{11}{400}$

$=$ Rs $\frac{3639306}{29200}=$ Rs $124\frac{18506}{29200}$

$=$ Rs. 124 10as. $1\frac{1247}{1825}$ p

10. 1st quantity $=\frac{(2k-3l+k)(2k-3l-k)}{(2k+3l+k)(2k-3l-k)}=\frac{3(k-l)(k-3l)}{3(k+l)(k-3l)}$

$=\frac{k-l}{k+l}$.

2nd.... ... $=\frac{(2k+3l-k)(2k-3l+k)}{9(k+l)(k-l)}=\frac{(k+3l)\times 3(k-l)}{9(k+l)(k-l)}$

$=\frac{k+3l}{3(k+l)}$.

3rd .. .... $=\frac{(3l+k)(3l-k)}{(2k+3l+k)(2k+3l-k)}$

$=\frac{3l-k}{3(k+l)}$.

$\therefore$ Ans. $=\frac{k-l}{k+l}+\frac{k+3l}{3(k+l)}+\frac{3l-k}{3(k+l)}=\frac{3k-3l+k+3l+3l-k}{3(k+l)}$

$=\frac{3(k+l)}{3(k+l)}=1$.

1894—AFTERNOON

1. Euclid, Book I, Prop. 2. The given point being situated in the given straight line

2. Euclid, Def 35, Book I. Def 7, Book I.
   (a) Euclid, Book I Prop 27.

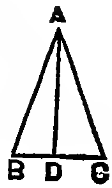
3. Euclid, Book II. Prop 14

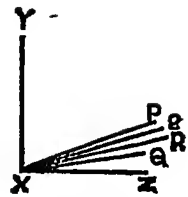
4 Euclid, Book III Prop 2.

5. Euclid, Book III Prop 24

6 Euclid, Book III Prop 34

7 Describe an isosceles $\triangle ABC$ having each of its angles at the base BC double of the vertical $\angle A$ (IV 10) Bisect the $\angle A$ by the straight line AD (I 9)





Since $\angle A + \angle B + \angle C = 2$ rt $\angle$s (I 32),
but $\angle B = \angle C = 2\angle A$ (*Constr.*),
$\therefore 5\angle A = 2$ rt. $\angle$s and $\therefore \angle A = \frac{1}{5}$ of 2 rt. $\angle$s

Thus the $\angle BAD = \frac{1}{5}$ one rt $\angle$

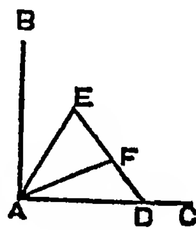
Take a right $\angle YXZ$ and trisect it Let ZXP be one part At X in XZ make the $\angle ZXQ = \frac{1}{5}$ of a rt. $\angle$ (I 23)

Then the $\angle PXQ = (\frac{1}{3} - \frac{1}{5})$ of a rt $\angle = \frac{2}{15}$ of a rt $\angle$ Bisect the $\angle PXQ$ by XR (I 9) and again bisect the $\angle PXR$ by XS (I. 9) Thus the $\angle PXS = \frac{1}{30}$ of a rt $\angle$

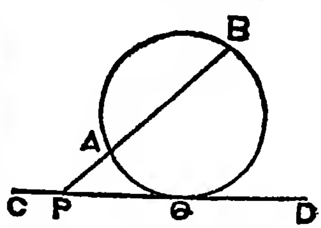
8 Let BAC be the right angle

In AC take any point D and on AD describe an equilateral $\triangle AED$ (I 1). Bisect the $\angle EAD$ by AF (I 9). Then AE, AF trisect the $\angle A$

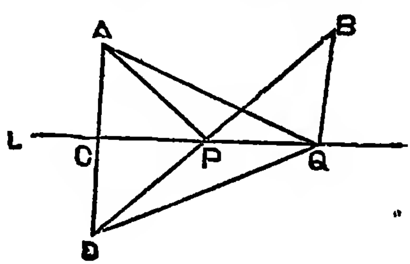
Since the $\angle EAD = \frac{2}{3}$ of a rt $\angle$ (I 32) $\therefore$ the $\angle BAE = 1 - \frac{2}{3}$ rt $\angle = \frac{1}{3}$ rt $\angle$, and the $\angle DAE$ is bisect by AF, $\therefore$ each of the parts EAF, DAF is $\frac{1}{3}$ of a rt $\angle$



9 Let A and B be the given points and CD the given str line Join BA and produce it to meet CD at P Describe a square = the rect PA, PB (II 14), and from PC PD cut off PQ = a side of this square Through A, B and Q describe a ⊙ (IV 5) Then ABQ is the required ⊙ Since $PA \cdot PB = PQ^2$ (*constr*), $\therefore$ CD touches the ⊙ ABQ at Q (III 37).

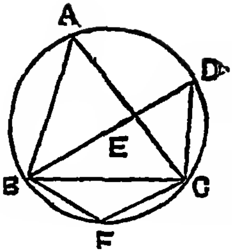


10 Draw AO perp to L, and produce it to D, making $CD = AC$ Join BD, cutting L at P Then P is the reqd. pt Join AP Then $AP + BP$ shall be a minimum For, take Q any other pt in L and join AQ, BQ, DQ



In the $\triangle$s ACP, DCP, because $AC = DC$ (*Ax* 11), $\therefore AP = DP$ (I 4) Similarly, it may be shewn that $AQ = DQ$

6 Euclid III. 21.



7 Let the $\triangle ABC$ be inscribed within the $\odot ABC$, also let BED be the diameter.
Join DC
The angle DCB is a rt angle (III 31)
Then $\angle BAC = \angle BDC$ (III 21)
To each add the angle DBC
$\therefore \angle BAC + \angle DBC = \angle BDC + \angle DBC$
$= 1$ rt angle
(because BCD is a rt angle)
i e $\angle BAC$ is less than a rt angle by $\angle DBC$

8 Euclid III 34

1861.—MORNING.

Examiner,—H Scott Smith, B A

1. Express as a decimal fraction $\dfrac{4\frac{3}{4}\times 8\frac{6}{11}}{\frac{5}{8}-10\frac{1}{2}}\times\dfrac{6\frac{1}{4}\text{ of }4\frac{4}{9}}{4+2\frac{1}{3}}$.

2 Reduce *3s 6d* to the decimal of £5 and 0234 to a vulgar fraction

3. If an estate be worth £2374 16*s* per annum, and the land-tax be assessed at 1*s* 11½*d* in the £, what will be the net annual income?

4 How much land may be rented for £1716 10*s* 6*d*. if 3 acres are rented for £4 13*s* 4*d* ?

5 Extract the square root of 00099856

6 Divide $28x^4+13x^2y^2-xy^3+15y^4$ by $4x^2+4xy+3y^2$.

7. Reducee $\dfrac{1}{4a^3(a+x)}+\dfrac{1}{4a^3(a-x)}+\dfrac{-1}{2a^2(a^2+x^2)}$

to the form $\dfrac{1}{a^4-x^4}$.

8 Multiply $x-x^{\frac{1}{2}}y^{\frac{1}{2}}+y$ by $x^{\frac{1}{2}}-y^{\frac{1}{2}}$.

9 Solve the following equations —

(i) $6\frac{1}{3}-\dfrac{x-7}{3}=\dfrac{4x-2}{5}$

(ii) $4x+3=8x-9$

(iii) $\sqrt{x+9}=1+\sqrt{x}$

(iv) $\left.\begin{array}{l} x+3y=10 \\ 3x+2y=9 \end{array}\right\}$

Now, since $DQ+BQ>BD$ (I 20), but $DQ=AQ$ [*Proved*], and $BD=BP+PD=BP+AP$ (*Proved*), $\therefore AQ+BQ>BP+AP$.

And this may be proved of all str lines drawn from A and B to any other pt in L

$\therefore AP+BP$ is a minimum

1895.—MORNING.

Paper set by—MR W BOOTH, M A.

Head Examiner,—MR J H GILLILAND, M A

1. Find the square root of $1+\frac{1}{2}(.0345)^3$ correctly in four places of decimals.

2 Find the sum of money which put out at simple interest at $2\frac{3}{4}$ per cent per annum will in 134 days exactly produce 124 Rs. 10as $1\frac{1247}{1825}$ p [A year contains 365 days]

3. If one pound sterling be worth twenty-five francs and sixty centimes and also worth sixth-thalers and twenty silber groschen; how many francs and centimes is one thaler worth?

[N B —One thaler = 30 silber groschen, one franc = 100 centimes.]

4 Simplify—

$$\frac{1\frac{1}{4}-1\frac{5}{12}}{1\frac{1}{4}+1\frac{5}{12}}+\frac{7}{8}\text{ of }\frac{9\times5}{14\times3}-\frac{11\frac{1}{4}}{15}.$$

5. I invest 12805 Rs. in the 4 per cents at $98\frac{1}{4}$, and when they have risen to $102\frac{3}{8}$ I sell out and invest in the $4\frac{1}{2}$ per cents. at $105\frac{3}{8}$: what is the change in my income? (Brokerage $\frac{1}{4}$ per cent on all transactions)

Or convert $\frac{1247}{1825}$ into a decimal fraction, pointing out accurately the recurring portion (if any).

6. Multiply together—

x^3-99x^2+x-29 and $x^5-17x^4+105x^3-19x^2+23x-41$, and arrange the product in descending powers of x.

7. Find the Greatest Common measure (if any) of

$x^5+3x^4+46x^3+89x^2+127x+164$

and $x^6+3x^5+46x^4+89x^3+132x^2+169x+205$.

8 Solve the simultaneous equations—

$$\left.\begin{aligned}3x+20&=4y-10\\4(x-1)&=3(y-3)\end{aligned}\right\}$$

9. If $a : b = c : d$ you are required to prove that

$(a^{\frac{1}{2}}+b^{\frac{1}{2}})^2 : (c^{\frac{1}{2}}+d^{\frac{1}{2}})^2 = a-b : c-d$.

10 Simplify the expression—

$$\left\{\frac{1}{\dfrac{4}{(1+t^2)^{\frac{3}{2}}}-\dfrac{3}{(1+t^2)^{\frac{1}{2}}}}\right\}^2-\left(\frac{3t-t^3}{1-3t^2}\right)^2+(1+t^2)^2-2t^2-2.$$

1895.—AFTERNOON.

Paper set by,—MR W. BOOTH, M.A.

Head Examiner,—MR J H GILLILAND, M A

1 On the same base and on the same side of it there cannot be two different equilateral triangles constructed prove this and quote accurately the enunciation of the proposition of which this is a particular case

2 Being given two straight lines meeting in a point you are required to draw through the point a straight line which shall make with one of the straight lines an angle equal to the angle it makes with the other

3 If two squares are equal their sides are also equal. prove this State and prove the propositions in the first book of Euclid in which the preliminary of this question is used.

4 Being given a circle you are required to find a point such that all straight lines drawn from it to the circumference are equal

5 Being given a circle and a point outside it you are required to draw a straight line through the point to meet the circle, and which on being further produced in the same direction will not cut the circle.

6 To a given straight line you are required to apply a parallelogram equal in area to a given equilateral triangle, and containing an angle equal to an angle of an equilateral triangle

7 Being given a triangle you are required to find a point equidistant from the three vertices quote the enunciation of the proposition of which this is a part

8. Being given a triangle, find a point such that the perpendiculars from it on the sides are equal quote the enunciation of the proposition of which this is a part

9 Prove that the perpendiculars dropped from the vertices on the opposite sides of a triangle are concurrent.

10 Being given two intersecting lines and a point O, you are required to draw through O a straight line meeting the given lines in P and Q so that the rectangle OP OQ may be given

SOLUTIONS.

1895.—MORNING.

1 $1+\frac{1}{2}(.0345)^3=1.0000205318125$

$\therefore$ Ans. $=\sqrt{(1.00002053181250)}=1.00001\ldots$

2 Int of 100 Rs for 134 days at $2\frac{1}{4}$ per cent.

$=(\frac{134}{365}\times 2\frac{1}{4})$ Rs $=\frac{737}{730}$ Rs

and Rs 124 10*a* $1\frac{1247}{1825}$p $=$ Rs $\frac{3639306}{29200}$

$\therefore$ Rs $\frac{737}{730}$: Rs $\frac{3639306}{29200}$ :: Rs. 100 : sum

$$\therefore \text{sum}=\text{Rs}\ \frac{3639306\times 100\times 730}{737\times 29200}=\text{Rs}\ 12345.$$

3 £1 = 25fr. 60c = 2560c.

Also £1 = 6 thalers 20s. gros = $6\frac{2}{3}$ thalers

$\therefore$ $6\frac{2}{3}$ thalers = 2560c

$$1 \text{ thaler} = \frac{2560}{6\frac{2}{3}}c = \frac{2560 \times 3}{20}c = 384c = 3\text{fr } 84c.$$

4 Ans. $= \dfrac{\frac{3}{4} - \frac{5}{12}}{\frac{3}{4} + \frac{5}{12}} + \frac{7}{6} \times \dfrac{9 \times 5}{14 \times 3} - \dfrac{45}{4 \times 15}$

$$= \frac{\frac{15-5}{12}}{\frac{15+5}{12}} + \frac{7}{6} \times \frac{3 \times 5}{14} - \frac{3}{4} = \frac{10}{20} + \frac{5}{4} - \frac{3}{4} = \frac{1}{2} + \frac{1}{2} = 1$$

5 Here $(98\frac{1}{4} + \frac{1}{4}) = 98\frac{1}{2}$

$\therefore$ Rs $98\frac{1}{2}$: Rs 12805 :: Rs 100 : stock

$\therefore$ stock = Rs. $\dfrac{12805 \times 100 \times 2}{197}$ = Rs 13000

Again $(102\frac{5}{8} - \frac{1}{4}) = 102\frac{3}{8}$ and $(105\frac{3}{8} + \frac{1}{4}) = 105\frac{5}{8}$

$\therefore$ Rs. $105\frac{5}{8}$: Rs $102\frac{3}{8}$ :: Rs 13000 : stock by transfer

$\therefore$ stock reqd. = Rs. $\dfrac{13000 \times 819 \times 8}{845 \times 8}$ = Rs. 12600

Income in 1st case = Rs 130×4 = Rs 520

....2nd case = Rs. $126 \times 4\frac{1}{2}$ = Rs 567

$\therefore$ Increase in income = Rs. $(567 - 520)$ = Rs 47

```
(a) 1825) 12470 ( 683287672̇
          10950
          -----
           15200                14000
           14600                12775
           -----                -----
            6000                12250
            5475                10950
            ----                -----
             5250                13000
             3650                12775
             ----                -----
             16000                2250
             14600                1825
             -----                ----
              14000               4250
                                  3650
                                  ----
                                  6000
```

6 $$x^5-17x^4+105x^3-19x^2+23x-41$$
$$x^3-99x^2+\quad x\ -29$$

$$x^8-17x^7+105x^6-\quad 19x^5+\quad 23x^4-\quad 41x^3$$
$$-99x^7+1683x^6-10395x^5+1881x^4-2277x^3+4059x^2$$
$$+\quad x^6-\quad 17x^5+\ 105x^4-\quad 19x^3+\quad 23x^2-\ 41x$$
$$-\quad 29x^5+\ 493x^4-3045x^3+\ 551x^2-667x$$
$$+1189$$

$$x^8+116x^7+1789x^6-10460x^5+2502x^4-5382x^3+4633x^2-$$
$$708x+1189=\text{Product.}$$

7 $$x^5+3x^4+46x^3+89x^2+127x+164)$$
$$x^6+3x^5+46x^4+89x^3+132x^2+169x+205(x$$
$$x^6+3x^5+46x^4+89x^3+127x^2+164x$$

$$5)\,5x^2+5x+205$$
$$x^2+x+41$$

$$x^2+x+41)\ x^5+3x^4+46x^3+89x^2+127x+164\ (x^3+2x^2+3x+4$$
$$x^5+x^4+41x^3$$

$$2x^4+5x^3+89x^2$$
$$2x^4+2x^3+82x^2$$

$$3x^3+7x^2+127x$$
$$3x^3+3x^2+123x$$

$$4x^2+4x+164$$
$$4x^2+4x+164$$

$\therefore$ G C M $=x^2+x+41.$

8. $3x+20=4y-10 \quad \ldots \quad (1)$
$4(x-1)=3(y-3) \quad \ldots \quad (2)$

From (1) $3x-4y=-30$

" (2) $4x-3y=-5$

Adding and subtracting, we get

$7x-7y=-35$, or $x-y=-5$

and $-x-y=-25$, or $x+y=25$

Hence $2x=20$, and $x=10$

and $2y=30$ and $y=15$

9. Since $\frac{a}{b}=\frac{c}{d}$, $\therefore \frac{a-b}{b}=\frac{c-d}{d}$, and $\therefore \frac{a-b}{c-d}=\frac{b}{d}$

Also $\frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}} = \frac{c^{\frac{1}{2}}}{d^{\frac{1}{2}}}$, $\therefore \frac{a^{\frac{1}{2}}+b^{\frac{1}{2}}}{b^{\frac{1}{2}}} = \frac{c^{\frac{1}{2}}+d^{\frac{1}{2}}}{d^{\frac{1}{2}}}$;

$\therefore \frac{a^{\frac{1}{2}}-b^{\frac{1}{2}}}{c^{\frac{1}{2}}+d^{\frac{1}{2}}} = \frac{b^{\frac{1}{2}}}{d^{\frac{1}{2}}}$, and $\therefore \left(\frac{a^{\frac{1}{2}}+b^{\frac{1}{2}}}{c^{\frac{1}{2}}+d^{\frac{1}{2}}}\right)^2 = \frac{b}{d}$,

Hence $\left(\frac{a^{\frac{1}{2}}+b^{\frac{1}{2}}}{c^{\frac{1}{2}}+d^{\frac{1}{2}}}\right)^2 = \frac{a-b}{c-d}$;

or $(a^{\frac{1}{2}}+b^{\frac{1}{2}})^2 : (c^{\frac{1}{2}}+d^{\frac{1}{2}})^2 = a-b : c-d$

10 The Denr of 1st quantity

$$= \left[\frac{1}{(1+t^2)^{\frac{1}{2}}} \times \left\{\frac{4}{(1+t^2)} - \frac{3}{1}\right\}\right]^2$$

$$= \frac{1}{1+t^2} \times \left(\frac{1-3t^2}{1+t^2}\right)^2 = \frac{(1-3t^2)^2}{(1+t^2)^3}$$

$$\therefore \text{1st quantity} = \frac{(1+t^2)^3}{(1-3t^2)^2}$$

$$\therefore \text{Ans} = \frac{(1+t^2)^3}{(1-3t^2)^2} - \frac{t^2(3-t^2)^2}{(1-3t^2)^2} + (1+t^2)^2 - 2t^2 - 2$$

$$= \frac{(1+t^2)^3 - t^2(3-t^2)^2}{(1-3t^2)^2} + (1+t^2)^2 - 2t^2 - 2$$

$$= \frac{(1-3t^2)^2}{(1-3t^2)^2} + 1 + 2t^2 + t^4 - 2t^2 - 2$$

$$= 1 + 1 + t^4 - 2 = t^4,$$

1895.—AFTERNOON.

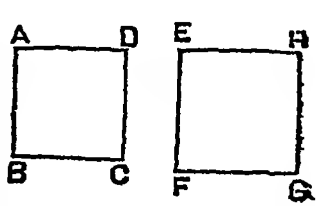
1. If possible, on the same base BC and on the same side of it let there be two equilateral Δs ABC and DBC

Then since AB=BC (I Def 19)=BD, ∴ the sides AB and DB of the two Δs constructed on the same base BC and terminated at B are equal. In like manner, AC and DC terminated at C equal. But this is impossible (I 7). Therefore there cannot exist two such Δs

(a) Euc I 7

2. Euclid, I 9.

3 Let ABCD and EFGH be two equal squares For, if the square EFGH be applied to the square ABCD, so that the point E may be on the point A and the str line EF along the str line AB, the str line EH will fall along the str line AD for the $\angle$A=$\angle$E, being rt $\angle$s (*Hyp*) and since the areas of the two squares are



equal (*Hyp*), ∴ the point G will coincide with point C and GF, GH will fall along CB, CD respectively, for the ∠G=∠C, being rt ∠s. (*Hyp*) Thus the points H and F will coincide with D and B respectively. Hence the sides of the squares are equal (i e) AB=EF

(*a*) Euclid, I 48

4 Euclid, III 1

5 Euclid, III 17

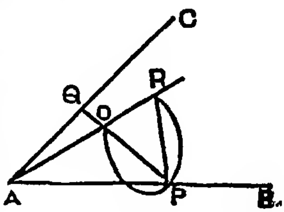
6 Euclid I 44 A particular case

7 Euclid, IV 5

8 Euclid, IV 4.

9 See Hall and Steven's Euclid, Ex 6, page 106

10 Let AB and AC be the intersecting str lines Join AO and produce it to R, so that the AO.OR=the given rectangle On OR describe a segment of a ⊙ containing an ∠=∠RAC and the segment intersect AB at P Join OP and produce it to meet AC at Q Then shall POQ be the required line. Join PR.



Since the ∠RAQ=∠OPR (*Constr.*), ∴ a ⊙ will go round the fig. PAQR (III. 21, *Conv*) Hence PO.OQ=AO.OR (III. 35) =the given rectangle

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1896.—MORNING.

Head Examiner,—BABU GOURI SANKER DE, M A

1 What greatest number and what least number can be subtracted from 23759143 that the remainders may be divisible by 24 35, 91, 130 and 150?

2 (1) Simplify—

$$\frac{5\frac{5}{8}}{6\frac{3}{7}} \text{ of } \frac{6\frac{7}{11}}{9\frac{1}{8}} \div \frac{8}{9}\left(2\frac{3}{11}+\frac{13}{22}\right) \text{ of } \frac{7s\ 6d}{12s\ 6d}$$

(2) Divide 0023465 by 03125

3 Extract the square rooot of $5\frac{1}{7}$ correct to 4 places of decimals.

4 Find the simple interest on Rs 4235 12as $9\frac{3}{5}$ pies for 3 years and 7 months at $3\frac{1}{2}$ per cent. per anuum.

5 If by selling a horse for Rs 1100, I lose 18 per cent. how much per cent should I have gained, or lost, had it been sold for Rs. 1320?

6. A man invested the same sum in two different stocks, $3\frac{1}{2}$ per cent Government Securities at $103\frac{1}{2}$, and 4 per cent Municipal Debentures at 105, his income from one is Rs 93 more than from the other, what sum was invested in each stock?

7. Find the G C M of

$x^4+ax^3+a^2x^2+2a^4$ and $x^5+a^2x^3+a^3x^2+a^5$.

8 Simplify—

$$\frac{bc(x-a)}{(a-b)(a-c)}+\frac{ca(x-b)}{(b-a)(b-c)}+\frac{ab(x-c)}{(c-a)(c-b)}.$$

9. Prove that

$$2\{(b+c-2a)^4+(c+a-2b)^4+(a+b-2c)^4\}$$
$$=\{(b+c-2a)^2+(c+a-2b)^2+(a+b-2c)^2\}^2.$$

10. Solve the following equations —

(1) $\dfrac{x-a}{3b+5c}+\dfrac{x-3b}{5c+a}+\dfrac{x-5c}{a+3b}=3$,

(2) $\begin{cases} x+y+z=0, \\ bcx+cay+abz=0, \\ ax+by+cz+(b-c)(c-a)(a-b)=0. \end{cases}$

11. A number consists of two digits of which the digit in the unit's place is double of the other if the digits be inverted, the new number exceeds the original number by 18, find the number.

12 If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then each of these ratios is equal to

$$\left(\frac{pa^n+qc^n+re^n}{pb^n+qd^n+rf^n}\right)^{\frac{1}{n}}$$

1896—AFTERNOON.

Head Examiner,—Babu Gouri Sankar De, M A

1. Enunciate Proposition 16, Book I

The side CA of the triangle ABC is produced to D, prove that the angle BAD is greater than the angle ACB

2 If the square described on one side of a triangle be equal to the sum of the squares described on the other two sides, then the angle contained by these two sides shall be a right angle

3 Describe a square that shall be equal to a given rectilineal figure.

4. If in a circle two chords cut one another, which do not both pass through the centre, they cannot both be bisected at their point of intersection

5. Enunciate and prove Proposition 31, Book III (The angle in a semicircle .....).

6 Inscribe a circle in a given triangle

7 From the ends of the base of a triangle perpendiculars are drawn to the bisector of the vertical angle prove that the feet of the perpendiculars are equally distant from the middle point of the base.

8 Find the locus of a point such that the sum of the squares on its distances from two given points may be equal to a given square.

9 In any triangle, if the perpendiculars drawn from the vertices on the opposite sides are produced to meet the circumscribed circle, then each side bisects that portion of the line perpendicular to it which lies between the orthocentre and the circumference

10 From an external point P two tangents are drawn to a given circle whose centre is O, and OP meets the chord of contact at Q: if R be the middle point of PQ, prove that RP is equal to a tangent from R to the given circle

SOLUTIONS.

1896.—MORNING

1. The L C. M of 24, 35, 91, 130, and 150 = 54600

The greatest number required = 23759143 — 54600 = 23704543.

The least number required is the remainder left after dividing 23759143 by 54600, which = 8143

2 (1) Ans. $= 4\frac{5}{8} \times \frac{7}{16}$ of $7\frac{3}{11} \times \frac{9}{73} + \frac{8}{9}(2 + \frac{19}{22})$ of $\frac{90d}{150d}$.

$$= \frac{7}{8} \text{ of } \frac{9}{11} - (\frac{8}{9} \times \frac{63}{22} \text{ of } \frac{3}{5}) = \frac{7}{11} - \frac{4 \times 7 \times 3}{11 \times 5}$$

$$= \frac{7}{11} \times \frac{11 \times 5}{4 \times 7 \times 3} = \frac{5}{12}.$$

(2) *03125) 00234650000 (075088 Ans.*

3 $5\frac{1}{7} = \frac{36}{7} = \frac{36 \times 7}{7 \times}$.

$$\therefore \sqrt{(5\frac{1}{7})} = \frac{6\sqrt{(7)}}{7} = \frac{6 \times 2\ 645751}{7} = \frac{15\ 874506}{7} = 2\ 2678\ldots$$

4 Rs 4235 12as. $9\frac{3}{5}$ p. = Rs $4235\frac{4}{5}$

Rs 100 Rs $4235\frac{4}{5}$ Rs $3\frac{1}{2} \times 3\frac{7}{12}$ Int

Interest = Rs $\frac{21179 \times 7 \times 43}{100 \times 5 \times 2 \times 12}$ = Rs $\frac{6374879}{12000}$

= Rs 531 3as $10\frac{8}{125}$ p.

5 Here 100 − 18 = 82

1100 : 1320 82 x

$$x = \frac{1320 \times 82}{1100} = 98\frac{2}{5}$$

Hence Loss = $100 - 98\frac{2}{5} = 1\frac{3}{5}$

6. Int per Rupee of Govt Securities $=$ Rs. $\dfrac{3\frac{1}{2}}{103\frac{1}{2}}=$ Rs $\frac{7}{207}$

Int per Rupee of Muncipal Debentures $=$ Rs $\frac{4}{105}$.

∴ difference in income per Rupee $=$ R $\left(\frac{4}{105}-\frac{7}{207}\right)$

$$=\text{Rs } \frac{93}{105\times 207}$$

Now, R $\dfrac{93}{105\times 207}$: Rs 93 (1 R sum invested in each)

∴ sum invested in each $=$ Rs $\dfrac{93\times 105\times 207}{93}=$ Rs 21735.

7. 1st quantity $=(x^4+a^2x^2+a^4)+a(x^3+a^3)$

$$=(x^2+ax+a^2)(x^2-ax+a^2)+a(x+a)(x^2-ax+a^2)$$
$$=(x^2-ax+a^2)\{x^2+ax+a^2+a(x+a)\}$$
$$=(x^2-ax+a^2)(x^2+2ax+2a^2)$$

2nd quantity $=x^3(x^2+a^2)+a^3(x^2+a^2)$

$$=(x^2+a^2)(x^3+a^3)=(x^2+a^2)(x+a)(x^2-ax+a^2)$$

∴ G. C. M. $=x^2-ax+a^2$.

8. The Exp $=\dfrac{bc(x-a)}{(a-b)(a-c)}-\dfrac{ac(x-b)}{(a-b)(b-c)}+\dfrac{ab(x-c)}{(a-c)(b-c)}$.

The L. C M of the Denrs $=(a-b)(b-c)(a-c)$

∴ Ans $=\dfrac{bc(x-a)(b-c)-ac(x-b)(a-c)+ab(x-c)(a-b)}{(a-b)(b-c)(a-c)}$.

The Numr. of which

$$=bc(b-c)x-abc(b-c)-ca(a-c)x+abc(a-c)$$
$$+ab(a-b)x-abc(a-b)$$
$$=x\{bc(b-c)-ca(a-c)+ab(a-b)\}$$
$$=x\{b^2c-bc^2-a^2c+ac^2+ab(a-b)\}$$
$$=x\{-c(a^2-b^2)+c^2(a-b)+ab(a-b)\}$$
$$=x(a-b)\{-c(a+b)+c^2+ab\}=x(a-b)\,(a-c)(b-c)$$

∴ Ans. $=x$

9. Let $b+c-2a=x$, $c+a-2b=y$ and $a+b-2c=z$,
then $x+y+z=0$, and ∴ $x+y=-z$

Squaring, $x^2+2xy+y^2=z^2$, and ∴ $x^2+y^2-z^2=-2xy$

Squaring again, $x^4+y^4+z^4+2x^2y^2-2x^2z^2-2y^2z^2=4x^2y^2$

transposing, $x^4+y^4+z^4=2x^2y^2+2x^2z^2+2y^2z^2$

Add to each side $x^4+y^4+z^4$,

$\therefore 2(x^4+y^4+z^4)=x^4+y^4+z^4+2x^2y^2+2y^2z^2+2y^2z^2$
$=(x^2+y^2+z^2)^2$

Restoring values, we get

$2\{(b+c-2a)^4+(c+a-2b)^4+(a+b-2c)^4$
$=\{(b+c-2a)^2+(c+a-2b)^2+(a+b-2c)^2\}^2$

10 (1) $\dfrac{x-a}{3b+5c}+\dfrac{x-3b}{5c+a}+\dfrac{x-5c}{a+3b}=3.$

Transposing, we get

$\left(\dfrac{x-a}{3b+5c}-1\right)+\left(\dfrac{x-3b}{5c+a}-1\right)+\left(\dfrac{x-5c}{a+3b}-1\right)=0$

or $\dfrac{x-a-3b-5c}{3b+5c}+\dfrac{x-a-3b-5c}{5c+a}+\dfrac{x-a-3b-5c}{a+3b}=0$

$\therefore x-a-3b-5c=0$, and $\therefore x=a+3b+5c$

(2) $x+y+z=0$ . ...(1)

$bcx+cay+abz=0$ .. .. (2)

$ax+by+cz+(b-c)(c-a)(a-b)=0$ (3)

From (1) and (2), by the *Rule of Cross Multiplication*, we get

$\dfrac{x}{ab-ca}=\dfrac{y}{bc-ab}=\dfrac{z}{ca-bc}=\lambda$, suppose

$\therefore x=\lambda a(b-c),\ y=\lambda b(c-a),\ z=\lambda c(a-b)$

Substituting these values of x, y, z in (3), we have

$\lambda a^2(b-c)+\lambda b^2(c-a)+\lambda c^2(a-b)+(b-c)(c-a)(a-b)=0$

$\therefore \lambda\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}=-(b-c)(c-a)(a-b)$

$\therefore \lambda(a-b)(b-c)(a-c)=(b-c)(a-c)(a-b)$

$\therefore \lambda=1$

Hence $x=a(b-c)$, $y=b(c-a)$ and $z=c(a-b)$

11 Let x be the digit in the ten's place,

then $2x$ is the digit in the unit's place,

$\therefore$ Number $=10\times x+2x=12x$.

The number formed by inverting the digits is $10\times 2x+x=21x$

$\therefore$ By the question,

$21x-12x=18$, $\therefore 9x=18$, and $\therefore x=2$

Hence the number required is 24

1861.—AFTERNOON

Examiner,—W SAMPSON

1. In any right-angled triangle, the square which is described upon the side subtending the right angle, is equal to the sum of the squares described upon the sides which contain the right angle.

2 Through a given point draw a straight line which shall make equal angles with two straight lines given in position.

3 If the straight line bisecting the vertical angle of a triangle also bisect the base, the triangle is isosceles

4 If a straight line be bisected and produced to any point, the square of the whole line thus produced and the square of the part of it produced are together double of the square of half the line bisected and of the square of the line made up of the half and the part produced

5. If a straight line be divided into two equal parts, and also into two unequal parts, the rectangle contained by the unequal parts together with the square of the line between the points of section is equal to the square of half the line

6 The sum of the squares of the sides of a parallelogram is equal to the sum of the squares of the diagonals

7. In a circle, the angle in a semicircle is a right angle; the angle in a segment greater than a semicircle is less than a right angle; and the angle in a segment less than a semicircle is greater than a right angle.

8 In equal circles, equal angles stand upon equal arcs whether they be at the centres or circumferences.

9 Given the angle at the base of an isosceles triangle and the perpendicular from it on the opposite side, construct the triangle.

SOLUTIONS

1861.—MORNING.

1. $$\frac{4\frac{3}{4}\times 8\frac{2}{11}}{\frac{5}{6}\div 10\frac{1}{2}}\times\frac{6\frac{1}{4}\text{ of }4\frac{2}{7}}{4+2\frac{1}{3}}=\frac{\frac{19}{4}\times\frac{90}{11}}{\frac{5}{6}\times\frac{2}{21}}\times\frac{\frac{25}{4}\times\frac{40}{7}}{\frac{19}{3}}$$

$$=\frac{19}{4}\times\frac{90}{11}\times\frac{6}{5}\times\frac{21}{2}\times\frac{25}{4}\times\frac{40}{9}\times\frac{3}{19}=\frac{24675}{11}=2243\,\dot{1}\dot{8}.$$

2. $3s.\ 6d = 3\ 5s$

$$\begin{array}{l} 20\,|\,3\ 5s. \\ \ \ 5\,|\,£\ 175 \\ \hline \ \ \ \ 035 \text{ Ans.} \end{array}$$

$$0234=\frac{234-2}{9900}=\frac{232}{9900}=\frac{58}{2475}$$

3. £1 : £2374 16s : 1s. $11\frac{1}{4}d$: land tax
or £1 : £$2374\frac{4}{5}$: $23\frac{1}{4}d$: land tax

$$\therefore \text{land tax}=\frac{47\times 11874}{5\times 2}d=55807\tfrac{4}{5}d$$

$$=£232\ 10s\ 7\tfrac{4}{5}d.$$

12 Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$,

then $a = bk$, $c = dk$, and $e = fk$

$\therefore a^n = b^n k^n$, $c^n = d^n k^n$ and $e^n = f^n k^n$

Hence $pa^n + qc^n + re^n = (pb^n + qd^n + rf^n)k^n$

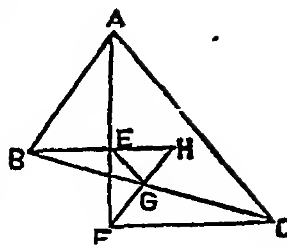
$$\therefore k^n = \frac{pa^n + qc^n + re^n}{pb^n + qd^n + rf^n}$$

$$k = \left(\frac{pa^n + qc^n + re^n}{pb^n + qd^n + rf^n}\right)^{\frac{1}{n}}$$

1896 —AFTERNOON

1 See Euc I 16
(a) See Solution of Ex. 2, 1880
2 See Euc I 48
3 See Euc II 14
4 See Euc III 4
5 See Euc. III 31
6. See Euc. IV. 4.

7 Let ABC be any $\triangle$ and AD the bisector of its vertical $\angle$BAC From B and C draw BE and CF perp to AD Bisect BC in G, and join EG, GF. Then shall EG be=GF



Produce BE, FG to meet in H

Since the $\angle$s BEF, CFE are rt $\angle$s (*Constr*)

$\therefore$ BE is $\parallel$ to CF (I 28)

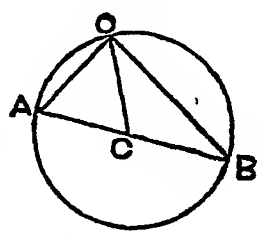
Hence the $\angle$EBG=$\angle$FCG (I 27)

Then, in the $\triangle$s BHG, CFG, since the $\angle$HBG=$\angle$FCG

(*Proved*), the $\angle$BGH=CGF (I 15) and BG=CG (*Hyp*); $\therefore$ GH=GF (I 26) Therefore G is the middle pt of the hypot FH of the rt $\angle$d $\triangle$HFE.

Hence EG=FG (see Solution of Ex 8, 1864) Q E D.

8 Let A and B be the two given points, and O any other point, such that OA, OB being joined, OA^2+OB^2 is equal to a given square. It is required to find the locus of O



Join AB and bisect it at C Join OC

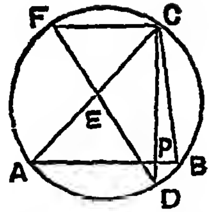
Since C is the middle pt of AB,

$\therefore 2\,AC^2+2CO^2=AO^2+BO^2$ [see Solution of Ex. 6 (b) 1868]=a given square (*Hyp*) =a constant

Hence AC^2+CO^2 is constant: but AC is constant,

(*Hyp*) $\therefore$ CO is constant, and C a fixed pt

Therefore the locus of O is a circle whose centre is C and radius OC Q E F.



9 Let ABC be any $\triangle$ inscribed in a $\odot$, and AD, BE, CF perps from the vertices on the opp side intersecting at the ortho-centre O Produce AD to meet the $\odot$ in G The OG shall be bisected at O

Join BG

Because in the $\triangle$s BOD, AOE, since the $\angle BDO = \angle AEO$,

(Ax 11) and the $\angle BOD = \angle AOE$ (I 15),

$\therefore$ the $\angle DBO = \angle EAO$ (I 32) $= \angle DBG$ (III 27)

Thus in the $\triangle$s BDO, BDG, since the $\angle BDO = \angle BDG$ (Ax 11)

and the $\angle DBO = \angle DBG$ (*Proved*),

and BD common, $\therefore$ DO=DG (I 26)

Similarly the other cases may be treated

10 Let PA, PB be tangents to the $\odot$ and AB the chord of contact. From R draw the tangent RD to the $\odot$

Join AO, DO

Since PA=PB (III 17 *Cor*) and the $\angle APO = \angle BPO$ (III. 17, *Cor*): also PQ common: $\therefore$ the $\triangle$s APQ, BPQ are equal in all respects (I 26), so that the $\angle AQP = \angle BQP$ and they are adjacent angles, $\therefore$ each = a rt $\angle$

Again, since PQ is bisected at R and produced to O,

$\therefore$ PO OQ+QR^2=OR^2 (II 6)

=OD^2+RD^2 (I 47)

for the $\angle RDO$ is a rt $\angle$ (III 18)

=AO^2+RD^2 (III *Def* 1)

But since the $\angle PAO$ is a rt $\angle$ (III 18) AQ $\perp$ to PO,

$\therefore$ AO^2=PO OQ (see Euc page 353, Ex 159)

Hence PO OQ+QR^2=PO OQ+RD^2

$\therefore$ QR^2=RD^2 (*Ax* 3), and $\therefore$ QR=RD

But QR=PR (*Hyp*); $\therefore$ PR=RD Q E D

1897.—MORNING.

Paper set by—BABU MAHENDRANATH RAY, M A, B L.
Head Examiner,—W. BOOTH, ESQ, M A

1 Reduce $\dfrac{2\frac{1}{3}-1\frac{1}{4}}{2\frac{1}{3}+1\frac{1}{4}} \times 15\frac{17}{35} \div \dfrac{3\frac{1}{3}\times 3\frac{1}{3}\times 3\frac{1}{3}-1}{3\frac{2}{3}\times 3\frac{1}{3}+3\frac{1}{3}+1}$ of 1 cwt 3 qrs 7lbs. to the decimal of $2\frac{1}{2}$ tons.

(*a*) Find the vulgar fraction equivalent to the recurring decimal $\cdot 133$, without assuming any rule

2 What do you understand by an *aliquot* part of a quantity? Is an area equal to $15\frac{1}{8}$ sq yards an aliquot part of an acre? Find by Practice the income-tax on Rs 1250 10as 8 pies at the rate of 5 pies per Rupee

3 What is a meant by the *ratio* of one quantity to another. What is a *proportion*?

320 people dine together 4 days a week, but on the remaining 3 days some are absent, the consumption of food is thus reduced, for the whole week in the ratio of 109 to 112 Find the number of absentees

4 In what time will Rs 3546 amount to Rs 7683 at $3\frac{1}{2}$ per cent.? simple interest?

5 A person has stock in the $3\frac{1}{2}$ per cent Government securities, which yields Rs 2856 a year He sells ont half of the stock at $109\frac{7}{8}$, and invests the proceeds in Howrah Mills shares at 153 What dividend ought the latter to pay that he may thereby increase his annual income by Rs 330

6 Extract the square root of 3 14159 to 4 decimal places

7. Distinguish between *term* and *expression*, *power* and *index* *measure* and *multiple*, *equation* and *identity*

(*a*) Write down the values of the following —

$$A \times 0,\ 0 \times A,\ \frac{A}{0},\quad \frac{0}{A} \text{ and } \frac{0}{0}$$

8 Resolve the following expressions into factors :—

$$x^2-7x+12,\ x^3-8 \text{ and } a^3+b^3+c^3-3abc.$$

9. Find the G C M of

$$x^4+3x+20 \text{ and } 3x^5-2x^4+81x-85$$

10 Show that

$$\frac{a-b}{1+ab}+\frac{b-c}{1+bc}+\frac{c-a}{1+ca}=\frac{(a-b)(b-c)(c-a)}{(1+ab)(1+bc)(1+ca)}.$$

11 Solve the equations :—

(1) $(x-a)(x-b)=(x-c)(x-d)$

(2) $\left.\begin{array}{l} ax+by=c \\ a'x+b'y=c \end{array}\right\}$

*12 If $3a+4b : 5a+6b=3c+4d : 5c+6d$, then will $a : b=c : d$.

1897 —AFTERNOON.

Paper set by—BABU MOHENDRANATH RAY, M A, B L
Head Examiner,—W BOOTH ESQ, M A

1 Prove that any two sides of a triangle are together greater than the third, giving the construction for each of the three cases

Prove that the sum of the two sides of a triangle is greater than twice the straight line drawn from the vertex to the middle point of the base

2 Prove that in any right-angled triangle, the square which is described on the side subtending the right angle is equal to the squares described on the sides containing the right angle

Give a proof of the proposition by showing how two squares may be cut into pieces and put together so as to form a third square

3 Divide a given straight line into two parts, so that the rectangle contained by the whole line and one part shall be equal to the square on the other part

In what proposition in the first four Books of Euclid is this proposition used? Explain briefly how the construction enables us to describe a regular pentagon on a given straight line

4 Prove that if two circles touch each other externally, the straight line which joins their centres shall pass through the point of contact

In the enunciation of this proposition, is it strictly correct to speak of *the* point of contact?

5 If from a point without a circle two straight lines be drawn one of which cuts the circle and the other touches it, the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, shall be equal to the square on the line which touches it Prove this.

Describe a circle which shall touch a given straight line and pass through two given points.

6 Describe a circle about a given triangle, giving the figures for all the cases that may arise, and showing from the construction that the perpendiculars at the middle points of the sides of a triangle meet at the same point

With the help of the rider above mentioned, prove that the three perpendiculars of a triangle drawn from the vertices to the opposite sides meet at the same point

7 Why does Euclid define a *point* as having no *magnitude* and a *straight line* as having no *breadth*?

8 Write a short essay on Euclid's theory of parallel straight lines

9 What are *converse* propositions? Enumerate all the instances of converse propositions in the first four Books of Euclid How does Euclid generally prove converse propositions? Do you know of any exceptions to this general rule?

SOLUTIONS.

1897.—MORNING.

1. $\dfrac{2\frac{1}{3}-1\frac{1}{4}}{2\frac{1}{3}+1\frac{1}{4}}\times 15\frac{37}{39}+\dfrac{3\frac{1}{3}\times 3\frac{1}{3}\times 3\frac{1}{3}-1}{3\frac{1}{3}\times 3\frac{1}{3}+3\frac{1}{3}+1}$ of 1 cwt 3 qrs 7lbs

$=\dfrac{1\frac{1}{12}}{3\frac{7}{12}}\times 15\frac{37}{39}-\dfrac{(3\frac{1}{3}^3-1)}{(3\frac{1}{3})^2+3\frac{1}{3}\times 1+1}$ of 1 cwt 3 qrs 7lbs

$=\frac{13}{43}\times\frac{622}{39}-(3\frac{1}{3}-1)$ of 1 cwt 3 qrs. 7lbs.

$=\frac{13}{43}\times\frac{622}{39}\times\frac{3}{7}$ of $7\frac{1}{4}$ qrs $=\frac{29}{2}$ qrs

$2\frac{1}{2}$ tons = 50 cwts = 200 qrs.

$\therefore \frac{\frac{29}{2}}{200}$ or $\frac{29}{400}$ or $\frac{725}{10000}$ or $\cdot 0725$ is the reqd. decimal

(a) $\cdot 13\dot{3}$ is the same as $\cdot 1\dot{3}$

Let $S = \cdot 1\dot{3}$ or $\cdot 13333$.

Then $100s = 13 \cdot 3333$

and $10s = 1 \cdot 3333$

$\therefore 90s = 12$, $S = \frac{12}{90} = \frac{2}{15}$.

2 An aliquot part of any quantity is such a part of it as being taken a certain number of times will exactly make up that quantity. As, 5 seers is aliquot part of a maund, for 5 seers being taken 8 times will give us a maund while 6 or 7 seers is not an aliquot part of a maund

An acre = 4840 sq yds.

and $15\frac{1}{8}$ sq. yds = $\frac{121}{8}$ sq yds

Now, $4840 \div \frac{121}{8} = \frac{4840 \times 8}{121} = 320$, an exact no

$\therefore 15\frac{1}{8}$ sq yds is an aliquot part of an acre.

	Rs	as	p	
1 a = $\frac{1}{16}$ of Re 1	1250	10	8	at Re 1
4 p. = $\frac{1}{3}$ of 1 a	78	2	8	at 1 a
1 p = $\frac{1}{4}$ of 4 p	26	0	$10\frac{2}{3}$	at 4 p
	6	8	$2\frac{2}{3}$	at 1 p
	Rs 32	9	$1\frac{1}{3}$	at 5 p *Ans*

3 Ratio is the relation which one quantity bears to another of the same kind in respect of magnitude, the comparison being made by considering what multiple, part or parts, one is of the other

Proportion is the equality of two ratios

By "in the ratio of 109 to 112," the Examiner means from 1 to $\frac{109}{112}$

Taking the whole food as 112
we find that 16 is the food of 320 people per day.
$\therefore$ 1 is the food of 20 people per day
and the reduction of 3 means the absence of 20 people per day for 3 days

And so if no of persons absent is to be taken to be the same per day, the answer is 20

4. Rs 7683—amount
Rs 3546—principal

∴ Rs. 4137—Interest on Rs 3546 @ $3\frac{1}{2}$ per cent.
Let x be the no of years
then $100 \times 1 \quad 3546 \times x \quad 3\frac{1}{2} \quad 4137$

$$\therefore\ x = \frac{100 \times 4137 \times 2}{3546 \times 7} = \frac{100}{3} = 33\tfrac{1}{3} \text{ years}$$

5. Rs $3\frac{1}{2}$ 2856 Rs 100 stock he had

$$\cdot\ \text{Stock} = \text{Rs. } \frac{2856 \times 100 \times 2}{7} = \text{Rs. } 81600$$

Half the stock, i e, Rs. 40800 is sold at $109\frac{7}{8}$
∴ Money received = Rs. $408 \times 109\frac{7}{8}$
Half the income in stock = Rs 1428
and that is increased by Rs 330

(for the other half remains the same)
∴ Rs. 1758 is the income for the Howrah Mills shares.

Rs $408 \times 109\frac{7}{8}$ Rs 153 : Rs. 1758 Reqd. dividend

$$\therefore\ \text{Dividend} = \text{Rs. } \frac{153 \times 1758 \times 8}{408 \times 879} = \text{Rs. } 6.$$

6.

```
        3 14159000 ( 1 7724
        1
       ----------
   27 ) 214
        189
       ------
  347 ) 2515
        2429
       ------
 3542 ) 8690
        7084
       -------
35444 ) 160600
        141776
       --------
         18824
```

7 An algebraical expression is that which consists of two or more single terms

Thus, $5a+6b-9ab$ is an algebraical expression

while $9ac$ is a simple term.

$a^4 = a \times a \times a \times a$.

Here the no 4 is the index and $a \times a \times a \times a$ is a form of a power

$\left.\begin{array}{l} 2 \times 3 = 6 \\ 6 \text{ is a multiple of } 2 \\ \text{and } 2 \text{ is a measure of } 6 \end{array}\right\}$ For difr see page 38, P. Ghose's Arithmetic.

An equality which is true for all values of the symbol cotained in it is an identity, while that which is true for some particular value or values is known as an equation, as,

$(x+2)(x-2) = x^2-4$ is an identity

While $x+2=3$ is an equation,

holding true for the particular value $x=1$.

(a) $A \times 0 = 0$

$0 \times A = 0$

$\frac{A}{0} = \infty$(infinity)

$\frac{0}{A} = 0$

$\frac{0}{0}$ is indeterminate ; i e. it may have any finite value.

8. $x^2-7x+12 = x^2-3x-4x+12 = x(x-3)-4(x-3)$
$= (x-4)(x-3)$

$x^3-8 = x^3-2^3 = (x-2)(x^2+2x+2^2) = (x-2)(x^2+2x+4)$

$a^3+b^3+c^3-3abc = (a+b)^3-3ab(a+b)+c^3-3abc$

$= (a+b)^3+c^3-3ab(a+b)-3abc$

$= (a+b+c)\{(a+b)^2-c(a+b)+c^2\}-3ab(a+b+c)$

$= (a+b+c)\{(a^2+2ab+b^2-ac-bc+c^2)-3ab\}$

$= (a+b+c)(a^2+b^2+c^2-ab-ac-bc)$

9 $x^4+3x+20 \,)\, 3x^5-2x^4+81x-85 \,(\, 3x-2$

$$\begin{array}{r} 3x^5+9x^2+60x \\ \hline -2x^4-9x^2+21x-85 \\ -2x^4 \qquad\quad -6x-40 \\ \hline -9x^2+27x-45 \\ = -9(x^2-3x+5) \end{array}$$

Now, rejecting the factor -9, which cannot be a factor of any of the given expressions

$$
\begin{array}{r|l|l}
x^2-3x+5 & x^4+3x+20 & x^2+3x+4 \\
 & x^4-3x^3+5x^2 & \\
\hline
 & 3x^3-5x^2+3x+20 & \\
 & 3x^3-9x^2+15x & \\
\hline
 & 4x^2-12x+20 & \\
 & 4x^2-12x+20 & \\
\hline
\end{array}
$$

$\therefore x^2-3x+5$ is the required H. C F.

10. $\dfrac{a-b}{1+ab}+\dfrac{b-c}{1+bc}+\dfrac{c-a}{1+ca}$

$$=\frac{(a-b)(1+bc)(1+ca)+(b-c)(1+ab)(1+ca)+(c-a)(1+ab)(1+bc)}{(1+ab)(1+bc)(1+ca)}$$

Now, the numerator

$$=(a-b)\{1+c(a+b)+abc^2\}$$
$$+(b-c)\{1+a(b+c)+a^2bc\}$$
$$+(c-a)\{1+b(c+a)+ab^2c\}$$

$$=(a-b+b-c+c-a)+c(a^2-b^2)+a(b^2-c^2)+b(c^2-a^2)$$
$$+abc\{c(a-b)+a(b-c)+b(c-a)\}$$
$$=c(a^2-b^2)+ab^2-a^2b-ac^2+bc^2$$
$$=c(a^2-b^2)-ab(a-b)-c^2(a-b)$$
$$=(a-b)\{c(a+b)-ab-c^2\}=(a-b)\{ac+bc-ab-c^2\}$$
$$=(a-b)\{ac-c^2)+b(c-a)\}=(a-b)\{-c(c-a)+b(c-a)\}$$
$$=(a-b)(b-c)(c-a)$$

$$\text{ex}=\frac{(a-b)(b-c)(c-a)}{(1+ab)(1+bc)(1+ca)}$$ Q E D.

11 (1) $(x-a)(x-b)=(x-c)(x-d)$

or $x^2-x(a+b)+ab=x^2-x(c+d)+cd$

$\therefore x(c+d-a-b)=cd-ab$

$$\therefore x=\frac{cd-ab}{(c+d)-(a+b)} \text{ Ans}$$

(2) $ax+by=c$ (1)

$a'x+b'y=c'$ (2)

Multiplying (1) by a' and (2) by a $\Big\}$ we get $aa'x+a'by=a'c$

$aa'x+ab'y=ac$

$\therefore y(a'b-ab')=a'c-ac'$

$$\therefore y=\frac{a'c-ac'}{a'b-ab'}$$

Again multiplying (1) by b' and (2) by b } we get $ab'x + bb'y = b'c$ and $a'b'x + bb'y = bc'$

$$\therefore x(ab' - a'b) = b'c - bc'$$

$$\therefore x = \frac{b'c - bc'}{ab' - a'b}$$

$$\left.\begin{aligned} \therefore x &= \frac{b'c - bc'}{ab' - a'b} \\ \text{and } y &= \frac{ac' - a'c}{ab' - a'b} \end{aligned}\right\}$$

12 $\quad \dfrac{3a + 4b}{5a + 6b} = \dfrac{3c + 4d}{5c + 6d}$

$\therefore (3a + 4b)(5c + 6d) = (5a + 6b)(3c + 4d)$

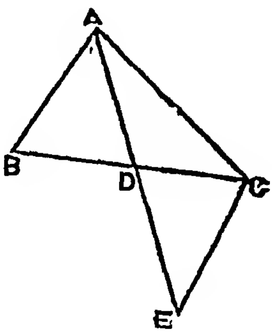
or $\underline{15ac} + 20bc + 18ad + \underline{24bd} = \underline{15ac} + 18bc + 20ad + \underline{24bd}$

$\therefore 2bc = 2ad$ or $bc = ad$

$\therefore a \quad b \quad c \quad d$ Q.E.D.

1897.—AFTERNOON

1 Proposition 20, Book 1



Produce BC to D, making CD=AD.
Join AD
Produce AB to D making BD equal to BC
Join CD.

(a) D is the middle pt of BC
We are to prove that AB+AC>2AD.
Produce AD to E making DE=AD.
Join EC
Then in the △s ABD, CDE

$$\left\{\begin{aligned} &BD=CD \ (\textit{Hyp}) \\ &AD=DE \ (\textit{Constr}) \\ &\angle ADB=\angle CDE \ (\text{I } 15) \end{aligned}\right.$$

∴ CE=AB (I 4)
But AC+CE > AE (I 20)
or, AC+AB > 2AD

Q E D.

2 Proposition 47, Book I

(a) Let ABCD and BEFG be any two squares, let them be so placed that their bases AB and BE may be in the same straight line
Make AH and CK each equal to BE
Join DK, KF, FH and HD
Because AH is equal to BE, therefore AB =HE (Ax 3)
CK=AH=BE=BG
∴ GK=CB=AB=DC
∴ GK=DC=DA=HE
∴ GF, CK, AH EF and the angles KGF, DCK, DAH HEF are equal
∴ the △s KGF, DCK, DAH, HEF are equal (I 4)

and FK, KD, DH, HF are equal
and by the help of (I 32) the angles of the quadrilateral DHFK are also equal

∴ DHFK is a square

∴ DCK and KGF are together equal to DAH and HEF; take these equals from the whole figure AEFKD, then DB and GE are together equal to DF

But DB is the square on AD,
GE is equal to the square on AH
and DF is the square on DH
Therefore &c., &c.

3 Proposition 11, Book II

This proposition is used in proposition 10, Book IV

∵ each of the base angles is double the vertical angle by inscribing an issosceles triangle similar to the triangles described in proposition 10, Book IV and bisecting the base angles by straight lines which meet the circumference in two other points we get the circumference divided into five equal parts and the arcs being joined gives us a regular pentagon

4 Proposition 12, Book III

That two circles touch each other at one pt only is proved in proposition 13, Book III and so we may say that only one point is assumed in this and the previous proposition.

5 Proposition 36, Book III

(a) See Euclid by Hall and Stevens.
Problems on tangency Deduction 21, page 235

6 Proposition 5, Book IV.

If S is joined with F the middle pt. of BC

then ∵ BF=CF (*Constr*)

and SF is common to the two $\triangle$s BFS and CFS

and also BS=CS ∴ $\angle$ BFS=$\angle$ CFS

∴ SF is at $\perp$s to BC

∴ The three perpendiculars at the three mid. pts meet at S, the circumcentre

Through A, B, C draw

KAH, KBG and HCG ∥ to BC, AC and AB respectively

Then A, B and C are mid. pts of KH, KG and GH respectively

$$\because \left.\begin{array}{l} KA=BC \\ \text{and also } AH=BC \end{array}\right\} \text{(I. 34)}$$

∴ AD, BE and CF are the perpendiculars at the mid. pts. of the $\triangle$ KGH

∴ They are concurrent (by the help of the above rider).

∴ The perpendiculars from A, B, C to the opposite sides of the triangle ABC must meet at the same pt Q E D.

7 "A point is that which has no parts or no magnitude"

This merely expresses a negative property and uniting the positive

Net income = £2374 16s 0d
− 231 10 7½

= £2143 5s 4½d

4 £4 13s. 4d £1716 10s 6d 3 ac x
or 1120d 411966d 3 ac x

$$\therefore x = \frac{411966 \times 3}{1120} \text{ ac} = 1103\tfrac{269}{560} \text{ acres}$$

5 $\sqrt{(\,00099856)} = \,0316$

6
$$4x^2+4xy+3y^2 \,)\, 28x^4+13x^2y^2-xy^3+15y^4 \,(\, 7x^2-7xy+5y^2 = \text{Quot}$$
$$28x^4 - 28x^3y + 21x^2y^2$$
$$-28x^3y - 8x^2y^2 - xy^3$$
$$-28x^3y - 28x^2y^2 - 21xy^3$$
$$20x^2y^2+20xy^3+15y^4$$
$$20x^2y^2+20xy^3+15y^4$$

7. Ans $= \frac{1}{4a^3}\left\{\frac{1}{a+x}+\frac{1}{a-x}\right\}+\frac{1}{2a^2(a^2+x^2)}$

$$= \frac{1}{4a^3}\,\frac{2a}{a^2-x^2}+\frac{1}{2a^2(a^2+x^2)} = \frac{1}{2a^2}\left\{\frac{1}{a^2-x^2}+\frac{1}{a^2+x^2}\right\}$$

$$= \frac{1}{2a^2}\,\frac{2a^2}{a^4-x^4} = \frac{1}{a^4-x^4}$$

8.
$$x - x^{\frac{1}{2}}y^{\frac{1}{2}} + y$$
$$x^{\frac{1}{2}} - y^{\frac{1}{2}}$$
$$x^{\frac{3}{2}} - xy^{\frac{1}{2}} + x^{\frac{1}{2}}y$$
$$-xy^{\frac{1}{2}} + x^{\frac{1}{2}}y - y^{\frac{3}{2}}$$
$$x^{\frac{3}{2}} - 2xy^{\frac{1}{2}} + 2x^{\frac{1}{2}}y - y^{\frac{3}{2}}.$$

9 (1) $6\frac{1}{3} - \frac{x-7}{3} - \frac{4x-2}{5} = 0$

Clearing of fractions, we get

$$95 - 5x + 35 - 12x + 6 = 0$$

$\therefore 17x = 136,$ $\therefore x = 8.$

(2) $4x+3 = 8x-9$ $\therefore 4x = 12,$ $\therefore x = 3.$

(3) $\sqrt{(x+9)} = (1+\sqrt{x})$

or $x+9 = 1+x+2\sqrt{x}$

$\therefore 8 = 2\sqrt{x}$ $\sqrt{x} = 4$ $\therefore x = 16$

(4) $\left.\begin{array}{l} x+3y=10 \;\ldots(1) \\ 3x+2y=9 \;\ldots(2) \end{array}\right\}$

idea of position, the conception of a point in Geometry is rendered more intelligible. When Euclid defines a point, he does not want us to form any conception of magnitude but only of position Then the next step is a straight line or simply a line to which he assigns only length The definition of a line requires the conception of the length only independently of all idea of its breadth though, of course, it is impossible to draw any line whatever which shall have no breadth So by a point Euclid wants to convey the idea of position only and by a line the idea of length only—a point is thus of no dimension, a line of one dimension (the length only).

8 Parallel straight lines are such as are in the same plane and which being produced ever so far both ways do not meet That the straight lines should be in the same plane is essentially necessary, for it is possible for two right lines never to meet when produced, and not be parallel. The properties of parallel straight lines are proved in proposition 29, Book I. and the conditions for the parallelism of two straight lines are expressed in their ways and proved in propositions 27 and 28 Book I which together form the converse of proposition 29 The 12th axiom is the basis of the theory of parallel straight lines But as it is rather difficult and not self evident, many substitutes have been proposed of which Playfair's Axiom is the one most approved —

'Two intersecting straight lines cannot be both parallel to a third straight line'

9 Each of two theorems is said to be the converse of the other, when the hypothesis of one is the conclusion of the other.

Instance of converse propositions .—

(1) I. 5 and 6.	(5) I. 27, 28 and I 29
(2) I 4 and 1st part of prop. 26, Book I.	(6) I 47 and I 48
(3) I 13 and I. 14	(7) III. 16 and III 18
(4) I. 18 and I. 19	(8) III 36 and III 37

Besides these, if we include another kind of converse in the list, we get the following as well in the list of converse propositions —

(9) I. 4 and I 8	(13) III 3 and Cor III. 1
(10) I. 24 and 25	(14) III 26 and 27
(11) I. 37 and 39	(15) III 28 and 29

Exhaustive list cannot be given.

Euclid generally proves the converse of some foregoing theorem by the indirect method of proof, in some cases, however, the method of proof by exhaustions is employed as in I 19 and I 25.

But the exceptions to the general rule are to be found in the proof of the propositions I. 48 and III. 37, where the proof is by means of a construction.

1898 —MORNING

Paper set by,—BABU GOUREE SANKER DEY, M.A

1. What is that least number, which, being divided by 48, 64, 72, 80, 120 and 140, leaves the remainders 38, 54, 62, 70, 110 and 130 respectively?

2 (*a*) Simplify

$$\frac{2\frac{1}{4}}{5\frac{1}{6}} \text{ of } \frac{1}{4}(7+\tfrac{1}{12}) - \frac{5\frac{7}{8}}{7\frac{1}{4}} \text{ of } \frac{2s.\ 5d.}{3s.\ 11d}$$

(*b*) What decimal of 2*l* 13*s*. 4*d* is 0625 of $2\dot{6}$ of 1*l* 6*s*. 8*d* ?

3 Extract the square root of 54756; also of $(4\cdot02)^3$ to 4 places of decimals.

4 What sum will amount to Rs 300 in $3\frac{1}{5}$ years at $6\frac{1}{4}$ per cent per annum simple interest?

5. A grocer buys 480 mds of sugar for Rs 6135 payable at the end of 3 months, and on the same day sells them at Rs 12 11ans. per maund ready money, what per cent does he gain or lose by the transaction, reckoning interest at 9 per cent per annum?

6. One-third of a certain capital is invested in the $3\frac{1}{2}$ per cent. Government Securities at 105, one fourth in the 3 per cent Government Securities at $97\frac{1}{2}$, and the remainder in $4\frac{1}{2}$ per cent Calcutta Municipal Debentures of $112\frac{1}{2}$. If the total annual income is Rs. 830, what is the capital?

7 Resolve the following expressions into elementary factors —

(1) $81a^4+64b^4$,

(2) $a^3(b-c)+b^3(c-a)+c^3(a-b)$,

(3) $2b^2c^2+2c^2a^2+2a^2b^2-a^4-b^4-c^4$

8. If $2s=a+b+c$, prove that

$$2(s-a)(s-b)(s-c)+a(s-b)(s-c)+b(s-c)(s-a)+c(s-a)(s-b)=abc$$

9. If $x+y+z=xyz$, prove that

$$\frac{x}{1-x^2}+\frac{y}{1-y^2}+\frac{z}{1-z^2}=\frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}$$

10. Solve the following equations —

(1) $\dfrac{x-a}{b+c+2a}+\dfrac{x-b}{c+a+2b}+\dfrac{x-c}{a+b+2c}+3=0$,

(2) $\begin{cases} x+2y+3z=20, \\ 2x+3y-5z=-7, \\ 4x-5y+7z=21 \end{cases}$

11 A farmer bought equal numbers of two kinds of sheep, one kinds at Rs 6 each, the other at Rs 8 each if he had expended his money equally in the two kinds, he would have had three sheep more than he did How many of each kind did he buy?

12 If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$ prove that

$$\frac{a}{d} = \frac{pa^3+qb^3+rc^3}{px^3+qc^3+rd^3}.$$

1898.—AFTERNOON.

Paper set by,—Babu Gouri Sankar Dey, M A

1. If two triangles have two angles of the one equal to two angles of the other, each, to each and a side of one equal to a side of the other these sides being either adjacent to the equal angles, or opposite to equal angles in each, then shall the triangles be equal in all respects

2. Describe a parallelogram that shall be equal to a given triangle, and have one of its angles equal to a given angle.

3 If a straight line is divided into any two parts the sum of the squares on the whole line and on one of the parts is equal to twice the rectangle contained by the whole and that part, together with the square on the other part. Prove this and write down the corresponding algebraical formula.

4. The opposite angles of any quadrilateral inscribed in a circle are together equal to two right angles.

5 Enunciate and prove proposition 32, Book III.—If a straight line touch a circle &c

6 Inscribe a regular hexagon in a given circle.

7. The point of intersection of the diagonals of the square described on the hypotenuse of a right-angled triangle is equally distant from the sides containing the right angle

8. If P is the orthocentre of the triangle ABC, prove that the rectangle contained by AP, BC, by BP, CA and by CP, AB are together equal to four times the triangle ABC.

9 If from any point on the circumference of the circle circumscribed about a triangle, perpendiculars are drawn to the three sides the feet of these perpendiculars are collinear.

10. Find the locus of a point from which tangents drawn to a given circle shall include an angle equal to the angle of an equilateral triangle.

SOLUTIONS

1898 —MORNING.

1 The least no exactly divisible by 48, 64, 72, 80, 120 and 140 is their L C M

2	48,	64,	72,	80,	120,	140
2	24,	32,	36,	40,	60,	70
2	12,	16,	18,	20,	30,	35
2	6,	8,	9,	10,	15,	35
3	3,	4,	9,	5,	15,	35
5	1,	4,	3,	5,	5,	35
	1,	4,	3,	1,	1,	7

L C. M $= 2\times2\times2\times2\times3\times5\times4\times3\times7 = 20160$

∴ The least no required is 20160 − 10 = 20150 for in each case there should be a remainder which is 10 less than the divisor

2 (a) $\frac{2\frac{3}{4}}{5\frac{1}{6}}$ of $\frac{3}{4}(\frac{7}{9}+\frac{1}{12})-\frac{5\frac{7}{8}}{7\frac{1}{4}}$ of $\frac{2s\ 5d}{3s\ 11d.}$

$= \frac{11\times6}{4\times31}$ of $\frac{3}{4}(\frac{31}{36})-\frac{47\times4}{8\times29}$ of $\frac{29}{47}$

$= \frac{11}{32}\div\frac{1}{2}=\frac{11}{32}\times2=\frac{11}{16}$

(b) 0625 of 2·6 of £1 6s 8d

$\frac{625}{10000}$ of $\frac{24}{9}$ of £$1\frac{1}{3}$ = £$\frac{2}{9}$

£2 13s 4d = $2\frac{2}{3}$

∴ Decimal reqd $=\frac{\frac{2}{9}}{\frac{8}{3}}=\frac{2}{9}\times\frac{3}{8}=\frac{1}{12}=$ 083.

3 (a)

```
   54756 ( 234
   4
   ----
43) 147
    129
   ----
464) 1856
     1856
   ----
```

(b) $(4\ 02)^3 = 64\ 964808$

```
     4 02
     4 02
    -----
      804
   1608
  -------
  16 1604
     4 02
  -------
   323208
 646416
 --------
 64 964808
```

```
      64 9648080000 ( 8·06007
      64
     ------
1606) 9648
      9636
     ------
1612007) 12080000
         11284049
        ---------
```

∴ 8 06007 is the approximate value

4. Rs 100 in $3\frac{1}{3}$ years at $6\frac{1}{4}$ per cent gives Rs $\frac{10}{3}\times\frac{25}{4}$ = Rs 20

∴ Rs 100 amounts to Rs 120 in $3\frac{1}{3}$ yrs $6\frac{1}{4}$ per cent

Rs. 120 : Rs 300 :: Rs 100 : reqd principal.

∴ Principal = Rs $\frac{100\times300}{120}$ = Rs. 250

5. The present worth of Rs 6135 payable at the end of 3 months at 9 per cent

$$= \text{Rs. } \frac{6135\times100}{102\frac{1}{4}} = \text{Rs } \frac{6135\times100\times4}{409} = \text{Rs } 6{,}000$$

Selling price of 480 mds at Rs. 12 11as per maund

= (Rs 12 11as) × 480 = Rs 6090

∴ He gains Rs 90 on Rs 6,000

∴ The profit per cent $= \frac{90\times100}{6000} = 1\frac{1}{2}$

6 Take the capital to be Rs. 1200

Then $\frac{1}{3}$rd of Rs 1200 *i e* Rs 400 is invested in the $3\frac{1}{2}$ per cent. Government Securities at 105

∴ The income from this investment = Rs $\frac{400\times3\frac{1}{2}}{105}$ = Rs $\frac{40}{3}$ = Rs. $13\frac{1}{3}$

$\frac{1}{4}$th of Rs 1200 *i e* Rs 300 invested in the 3 per cent. Government Securities at $97\frac{1}{2}$

The income from this = Rs. $\frac{300\times3}{97\frac{1}{2}}$ = Rs $\frac{1800}{195}$ = Rs $\frac{120}{13}$ = Rs $9\frac{3}{13}$

And the remainder *i e* Rs (1200 − 700) = Rs 500 in the $4\frac{1}{2}$ per cent Calcutta Mun Deb. at $112\frac{1}{2}$

$$\text{Income from this} = \text{Rs } \frac{500\times4\frac{1}{2}}{112\frac{1}{2}} = \text{Rs. } 20$$

∴ The total income on Rs 1200 = Rs. $(13\frac{1}{3}+9\frac{3}{13}+20)$

= Rs. $42\frac{22}{39}$

Rs $42\frac{22}{39}$: Rs. 830 :: Rs 1200 : reqd. capital.

∴ Capital = Rs $\frac{1200\times830\times39}{1660}$ = Rs. 23,400

7 (1) $81a^4+64b^4=(9a^2)^2+(8b^2)^2=(9a^2+8b^2)^2-144a^2b^2$

$=(9a^2+8b^2)^2-(12ab)^2=(9a^2+12ab+8b^2)(9a^2-12ab+8b^2)$

(2) $a^3(b-c)+b^3(c-a)+c^3(a-b)$

$=a^3(b-c)+b^3c-b^3a+c^3a-c^3b$

$$=a^2(b-c)+bc(b^2-c^2)-a(b^3-c^3)$$
$$=(b-c)\{a^2+bc(b+c)-a(b^2+bc+c^2)\}$$
$$=(b-c)\{a(a^2-b^2)-bc(a-b)-c^2(a-b)\}$$
$$=(a-b)(b-c)\{a(a+b)-bc-c^2\}$$
$$=(a-b)(b-c)\{(a+c)(a-c)+b(a-c)\}$$
$$=(a-b)(b-c)(a-c)(a+b+c)$$

(3) $2b^2c^2+2c^2a^2+2a^2b^2-a^4-b^4-c^4$
$$=4b^2c^2-(a^4+b^4+c^4+2b^2c^2-2a^2b^2-2a^2c^2)$$
$$=4b^2c^2-(b^2+c^2-a^2)^2=(2bc)^2-(b^2+c^2-a^2)^2$$
$$=\{2bc+(b^2+c^2-a^2)\}\{2bc-(b^2+c^2-a^2)\}$$
$$=\{(b+c)^2-a^2\}\{a^2-(b-c)^2\}$$
$$=(b+c+a)(b+c-a)(a+b-c)(a-b+c)$$
$$=(a+b+c)(b+c-a)(a+c-b)(a+b-c)$$

8. $2(s-a)(s-b)(s-c)+a(s-b)(s-c)+b(s-c)(s-a)$
$$+c(s-a)(s-b)$$
$$=(s-b)(s-c)(2s-2a+a)+(s-a)\{b(s-c)+c(s-b)\}$$
$$=(s-b)(s-c)(a+b+c-a)+(s-a)\{s(b+c)-2bc\}$$
$$=(b+c)(s-b)(s-c)+s(s-a)(b+c)-2bc(s-a)$$
$$=(b+c)\{(s-b)(s-c)+s(s-a)\}-2bc(s-a)$$
$$=(b+c)\{2s^2-s(a+b+c)+bc\}-2bc(s-a)$$
$$\because a+b+c=2s$$
$$=bc(b+c)-2bc(s-a)=bc\{b+c-2s+2a\}=abc$$
$$\because 2s=a+b+c$$

9 $\dfrac{x}{1-x^2}+\dfrac{y}{1-y^2}+\dfrac{z}{1-z^2}$
$$=\frac{x(1-y^2)(1-z^2)+y(1-x^2)(1-z^2)+z(1-x^2)(1-y^2)}{(1-x^2)(1-y^2)(1-z^2)}$$

Numerator $=\{x-x(y^2+z^2)+xy^2z^2\}$
$$+\{y-y(x^2+z^2)+yx^2z^2\}$$
$$+\{z-z(x^2+y^2)+zx^2y^2\}$$
$$=(x+y+z)-xy(x+y)-yz(y+z)$$
$$-xz(x+z)+xyz(yz+xz+xy)$$

Now $x+y=xyz-z=z(xy-1)$

and $y+z=x(yz-1)$

$x+z=y(xz-1)$

$\therefore$ Numerator

$=(x+y+z)-xyz(xy-1)-yzx(yz-1)$

$\quad -xzy(xz-1)+xyz(yz+xz+xy)$

$=xyz+xyz+xyz+xyz \qquad \because x+y+z=xyz$

$=4xyz$

$\therefore$ The given expression

$$=\frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}$$

10. (1) $\dfrac{x-a}{b+c+2a}+\dfrac{x-b}{c+a+2b}+\dfrac{x-c}{a+b+2c}+3=0$

or $\left(\dfrac{x-a}{b+c+2a}+1\right)+\left(\dfrac{x-b}{c+a+2b}+1\right)+\left(\dfrac{x-c}{a+b+2c}+1\right)=0.$

$\therefore \dfrac{x+(a+b+c)}{b+c+2a}+\dfrac{x+(a+b+c)}{c+a+2b}+\dfrac{x+(a+b+c)}{a+b+2c}=0$

$\therefore \{x+(a+b+c)\}\left(\dfrac{1}{b+c+2b}+\dfrac{1}{c+a+2b}+\dfrac{1}{a+b+2c}\right)=0$

$\therefore x+a+b+c=0$

$\therefore x=-(a+b+c)$

(2) $x+2y+3z=\ \ 20 \quad \dots (1)$

$2x+3y-5z=-\ 7 \quad \dots (2)$

$4x-5y+7z=\ \ 21 \quad \dots (3)$

Multiplying = "(1) by 2 and substracting = "(2) from it we get

$y+11z=47 \qquad .. \quad (4)$

Again multiplying = "(2) by 2 and subtracting = "(3) from it we get

$11y-17z=-35 \quad \dots \quad (5)$

Now multiplying = "(4) by 11 we get $11y+121z=517$

and $11y-17z=-35$

$\therefore 138z=552$

$\therefore z=4$

From (4) we get $y+44=47 \quad \therefore y=3$ and $x+6+12=20 \quad \therefore x=2$

11 Let x be the no of sheep of each kind

Then the money spent $=(6x+8x)$ Rs $=14x$ Rs

If he had expended $14x$ Rs equally in two kinds the no of sheep would have been $\dfrac{7x}{6}+\dfrac{7x}{8}$.

$$\therefore \frac{7x}{6}+\frac{7x}{8}=2x+3 \text{ or } 28x+21x=48x+72$$

$$\therefore x=72$$

13. $\dfrac{a}{d}=\dfrac{a}{b}\times\dfrac{b}{c}\times\dfrac{c}{d}=\dfrac{a}{b}\times\dfrac{a}{b}\times\dfrac{a}{b},=\dfrac{a^3}{b^3}$

Again $\dfrac{a^3}{b^3}=\dfrac{b^3}{c^3}=\dfrac{c^3}{d^3}=\dfrac{pa^3+qb^3+rc^3}{pb^3+qc^3+rd^3}$

$$\therefore \frac{a}{d}=\frac{pa^3+qb^3+rc^3}{pa^3+qc^3+rd^3}$$

otherwise, thus $\dfrac{a}{b}=\dfrac{b}{c}=\dfrac{c}{d}=k$ (say)

$a=bk$ and $c=dk$ $\quad\quad \therefore \dfrac{a}{d}=k^3$ &c

$b=ck \quad \therefore a=dk^3$

1898.—AFTERNOON.

1. Proposition 26, Book I
2 „ 42, Book I
3 „ 7, Book II.

The corresponding algebraical formula is

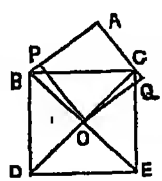
$$(a-b)^2=a^2-2ab+b^2$$

(Vide Page 133 Hall and Steven's Euclid).

4 Proportion 22, Book III.

5 See Page 203, Hall and Steven's Euclid.

6 Proposition 15, Book IV.



7 From O the point of intersection of the two diagonals draw OP and OQ perpendiculars to AB and AC produced Then shall OP=OQ

The diagonals of a square are equal and they bisect each other at right angles

$\therefore$ BO=OC

and also the $\angle$ BOC is a right angle

and in the quadrilateral APOQ, $\angle$ A is a rt angle, $\angle$s at P and Q are also right angles

$\therefore \angle$ POQ is a right angle

for the four angles are together equal to four right angles (I 32, Cor 1)

$\therefore \angle BOC=\angle POQ$

Taking away the common $\angle POC$

$\angle QOC=\angle BOP$

$\angle QOC=\angle BOP$

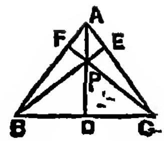
∴ In the two Δs BOP, COQ

BO=OC

$\angle$OPB=$\angle$OQC, being rt angles and $\angle$QOC=POB

∴ OP=OQ (I. 26)

8 2ΔABC=rect AD, BC (I 41, and def 1, Book II



also, 2ΔABC=rect BE, AC

and 2ΔABC= „ CF, AB

6 ΔABC=rect (AD BC+BE AC+CF AB)

Now AD BC = AP BC+PD BC (II 1)

∴ AP BC=AD BC−PD BC

=2ΔABC−2ΔBPC

Also, BP CA=2ΔABC−2ΔAPC

and CP AB=2ΔABC−2ΔAPB

∴ The sum of the rectangles given

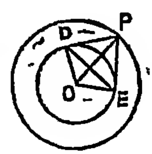
=6ΔABC−2(ΔBPC+ ΔAPC+ ΔAPB)

=6ΔABC−2ΔABC

=4ΔABC

9. See Hall and Steven's Euclid, page 232.

10 Let P be a pt. from which the tangents drawn to the ⊙ DEC include an angle of an equilateral triangle i.e ⅔rds of a right angle (I 32 and I 5, Cor.)



Find O the centre, Join PO, PE, EO and DO

Then PO is the bisector of the angle DPE (II. 1 Cor)

∴ $\angle$DPO is ⅓ of a right angle

and $\angle$PDO is a right angle (III 18)

∴ The ΔDOP is a right angled triangle of which one of the acute angles $\angle$DOP is double of $\angle$DPO

∴ The side OP is double of AO

∴ PO is equal to the diameter of the given circle

∴ The locus of P is a circle having O its centre and radius equal to the diameter of the given circle.

That every point of this circle satisfies the given condition may be very easily shown

1899.—MORNING.

Paper set by—BABU GOURI SANKAR DEY, M A.

1. Find the greatest number which will divide 1028, 1629 and 2130, leaving the remainders 3, 4 and 5 respectively.

2 (*a*) Simplify

$$\frac{\frac{3}{5}+\frac{7}{8}}{\frac{8}{9}+\frac{9}{10}} \text{ of } \frac{13s\ \ 5d}{9s\ \ 10d} - \frac{2}{3}\left(\frac{3}{7}+\frac{8}{9}\right) \text{ of } \frac{3 \text{ tons } 3 \text{ cwt.}}{4 \text{ tons } 3 \text{ cwt}}$$

(*b*) Prove that $\cdot 2\dot{3}\dot{4} = \frac{232}{990}$ without assuming the rule of converting a recurring decimal into a vulgar fraction.

3 Find by practice, or otherwise, the value of 7 tons 2 cwt. 2 qrs at Rs 3 2 as per maund assuming that 1 ton is equal to $27\frac{1}{4}$ maunds

4 Extract the square root of 51076, and of ·051076

5. A grocer mixed 20 maunds of one kind of rice at Rs. 4 a maund with a certain quantity of a second kind of rice at Rs. 3 8as. a maund, and selling the mixed rice at Rs 3 12as a maund, gained Rs 10 on the whole Find how many maunds of the second kind of rice he mixed, and the gain per cent on his outlay.

6 Find the discount on Rs 1218 due six months hence at 3 per cent per annum simple interest.

7 Divide $(a^2-bc)^3+8b^3c^3$ by a^2+bc

8. Resolve the following into simple factors —

(1) a^3-b^3,

(2) $a^2(b+c)+b^2(c+a)+c^2(a+b)$

9 (*a*) Find the G C M. of

$$x^4+x^3-12x+21 \text{ and } x^4-15x+14,$$

(*b*) Find the cube of $x-\frac{1}{x}$

10 Solve the following equations —

(1) $\frac{x}{2x-a}+\frac{x}{2x-b}=1$;

(2) $\begin{cases} x-2y+\ \ z=\ 0, \\ 5z-3x-4y=\ 0, \\ 7x+8y+9z=98. \end{cases}$

11. There is a number of two digits whose difference is 2, and if it be diminished by $\frac{1}{2}$ times the sum of the digits, the digits will be inverted find it

12 If $a : b = c : d$, prove that

$$pa^2+qc^2 : pb^2+qd^2 :: na^2-nc^2 : nb^2-nd^2.$$

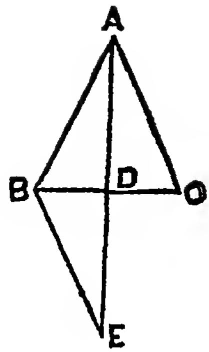
Multiply (1) by 3

Then, $3x+9y=30$ } ∴ subtracting,
From (2) $3x+2y=9$ } $7y=21$, ∴ $y=3$
and $x=10-3y=1$.

$\therefore \left.\begin{matrix} x=1 \\ y=3 \end{matrix}\right\}$

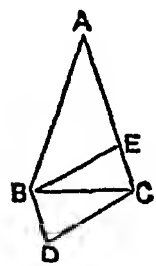
1861.—AFTERNOON.

1. Euclid I 47
2. See Question 3 of 1860.
3. Let BAC be the vertical angle of the △ABC if AD bisects the vertical angle as well as the base, then △ ABC is isosceles



Prod AD to E making DE equal to AD.
Join BE
∵ AD=DE, BD=DC
and ∠ADC=∠EDB (I 15)
∴ AC=BE and ∠CAD=∠BED (I. 4).
But ∠CAD=∠DAB (Hyp.)
∴ ∠DAB=∠DEB
∴ AB=BE=AC
Hence ABC is isosceles, (I. 6)

4. Euclid II. 10
5. Euclid II 5.
6. See Question 9 of I 1859.
7. Euclid III 31.
8. Euclid III 27.
9. Let ABC be the given angle.



Make the ∠ CBD=∠ABC (I 23)
Draw BE ⊥ BD, making BE=the given perpendicular
Through E draw CEA parallel to BD cutting BC in C and BA in A.
∵ ∠CBD=∠BCE (I. 29)
and ∠ CBD=∠ABC
∴ ∠ABC=∠BCE (Ax 1)
∴ AB=AC (I 6)
∴ △ ABC is isosceles and BE ⊥ AC (I 29)
∴ ABE is the reqd △.

1862.—MORNING.

Examiner,—J. G. MEDLEY, MAJOR, R E

1 What is the difference between $\dfrac{4\frac{3}{7}}{5\frac{1}{6}} - \dfrac{99}{310}$ and 06 ?

2 Reduce ·14 of a pie to the fraction of a Rupee, and find the value of 0875 of a pound sterling.

3. If the wages of 18 coolies for a month amount to 85 Rs.

1899 —AFTERNOON

Paper set by—BABU GOURI SANKER DEY, M A

All the questions are of equal value

1. If a side of a triangle be produced, then the exterior angle shall be equal to the sum of the two interior opposite angles also the three interior angles of a triangle are together equal to two right angles

2 Parallelograms on the same base, and between the same parallels, are equal in area

3 In an obtuse-angled triangle, if a perpendicular is drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side on which, when produced, the perpendicular falls, and the line intercepted without the triangle, between the perpendicular and the obtuse angle

4 Give Euclid's definition of a *tangent* to a circle

The straight line drawn from the centre of a circle to the point of contact of a tangent is perpendicular to the tangent

5 If two chords of a circle cut one another, the rectangle contained by the segments of one shall be equal to the rectangle contained by the segments of the other

6 Inscribe a circle in a given triangle

7 Define *parallel* straight lines and enunciate the axiom upon which Euclid's treatment of parallel straight lines depends

Show that the straight line bisecting the sides of a triangle is parallel to the base

8 A line of given length moves with its ends on two fixed straight lines at right angles to each other Show that the distance of its middle point from the intersection of the two fixed straight lines is equal to half its length Hence find the *locus* of its middle point

9 In any triangle the sum of the squares on two sides is equal to twice the square on half the base, together with twice the square on the straight line joining the vertex to the middle point of the base.

10 The straight line joining the centres of the inscribed and circumscribed circles of a triangle, subtends at each vertex an angle equal to half the difference of the angles at the other two vertices

SOLUTIONS

1899.—MORNING.

1. The question reduces to finding the greatest number which will divide $1028-3$ or 1025, $1629-4$ or 1625, and $2130-5$ or 2125 exactly

Hence the G. C M. of 1025, 1625 and 2125 is the required result.

```
1025) 1625 (1
      1025
      ----
       600) 1025 (1
             600
            ----
             425) 600 (1
                  425
                  ---
                  175) 425 (2
                       350
                       ---
                        75) 175 (2
                            150
                            ---
                             25) 75 (3
                                 75
                                 --

25) 2125 (85
    200
    ----
     125
     125
     ---
```

∴ 25 is the required number

2 (*a*) The given expression

$$=\frac{\dfrac{24+35}{40}}{\dfrac{80+81}{90}}\text{ of }\frac{161}{118}-\frac{2}{3}\text{ of }\frac{27+56}{63}\text{ of }\frac{63}{83}$$

$$=\frac{59}{40}\times\frac{90}{161}\times\frac{161}{118}-\left(\frac{2}{3}\times\frac{83}{63}\times\frac{63}{83}\right)$$

$$=\frac{9}{8}-\frac{2}{3}=\frac{9\times3}{8\times2}=\frac{27}{16}=1\frac{11}{16}$$

(*b*) Let s = 234

i e s = 2343434 . . . . .

∴ 10s = 2 343434 ....

and 1000 s = 234 3434 ....

∵ (1000 − 10) s = 234 − 2 = 232

∴ $s=\frac{232}{990}$.

3. Rs 3 2as per maund

and 1 ton = $27\frac{1}{4}$ mds

∴ Price of 1 ton = Rs 3 2as × 27 + $\frac{1}{4}$ of Rs 3 2as.

= Rs 84 6as + 12as 6 pies

= Rs 85 2as 6 pies

	Rs	A.	P.	
2 cwts $=\frac{1}{10}$ of 1 ton	85	2	6	= price of 1 ton
			7	
	596	1	6	= price of 7 tons
2 qrs. $=\frac{1}{4}$ of 2 cwts	8	8	3	= price of 2 cwts.
	2	2	$0\frac{3}{4}$	= price of 2 qrs.
	606	11	$9\frac{3}{4}$	= value of 7 tons 2 cwts 2 qrs.

4.

$$\begin{array}{r|l} & 51076 \,(\, 226 \text{ Ans} \\ & 4 \\ \hline 42) & 110 \\ & 84 \\ \hline 446) & 2676 \\ & 2676 \\ \hline \end{array}$$

The sq root of 051076 is therefore 226 Ans.

5. The 1st kind is at Rs 4 per maund,

and the 2nd „ at Rs 3 8as. per maund,

by mixing, 20 mds of the 1st kind with 20 mds of the second kind and selling the mixture at Rs 3 12as. per maund, there is neither gain nor loss.

∴ The gain of Rs 10 is due to his mixing more of the 2nd kind, and as the profit is Rs 3 12as — Rs 3 8as.

or 4as. per md.

∴ He must mix $\left(\frac{\text{Rs } 10}{4\text{as.}} = 40\right)$ mds more of the second kind

i e 60 mds. of the second kind must be mixed.

The cost price of the two kinds

= Rs $4 \times 20 +$ Rs 3 8as $\times 60$

= Rs 290.

∴ the profit of Rs. 10 is on Rs. 290.

∴ the gain per cent $= \frac{100 \times 10}{290}$

$= \frac{100}{29} = 3\frac{13}{29}$

6. The interest on Rs 100 for six months $= 1\frac{1}{2}$

∴ the discount on $101\frac{1}{2}$: is $1\frac{1}{2}$

$101\frac{1}{2}$: 1218 : . Rs $1\frac{1}{2}$: Reqd dis

∴ Discount = Rs. $\frac{3}{2} \times \frac{1218 \times 2}{203}$ = Rs. 18.

7 $(a^2-bc)^3+8b^3c^3=(a^2-bc)^3+(2bc)^3$

$=\{(a^2-bc)+2bc\}\{(a^2-bc)^2-2bc(a^2-bc)+(2bc)^2\}$

$=(a^2+bc)(a^4-2a^2bc+b^2c^2-2a^2bc+2b^2c^2+4b^2c^2)$

$=(a^2-bc)(a^4-4a^2bc+7b^2c^2)$

∴ the quotient is $a^4-4a^2bc+7b^2c^2$

8 (1) $a^3-b^3=a^3-a^2b+a^2b-ab^2+ab^2-b^3$

$=a^2(a-b)+ab(a-b)+b^2(a-b)$

$=(a^2+ab+b^2)(a-b)$

(2) The example should be

$a^2(b+c)+b^2(c+a)+c^2(a+b)+2abc$

or, $a^2(b+c)+b^2(c+a)+c^2(a+b)+3abc$

and then the result is

in the 1st case it is $(a+b)(b+c)(c+a)$

and in the 2nd case it is $(a+b+c)(bc+ca+ab)$

9 (a) $x^4-15x+14)\ x^4+x^2-12x+21\ (1$

$x^4 \qquad -15x+14$

$x^2+3x+7)\ x^4-15x+14\ (x^2-3x+2$

$x^4+3x^3+7x^2$

$-3x^3-7x^2-15x$

$-3x^3-9x^2-21x$

$2x^2+6x+14$

$2x^2+6x+14$

∴ x^2+3x+7 is the G. C. M.

(b) $\left(x-\frac{1}{x}\right)^3=x^3-3x^2\cdot\frac{1}{x}+3x\cdot\frac{1}{x^2}-\frac{1}{x^3}$

$=x^3-3x+\frac{3}{x}-\frac{1}{x^3}$

10. $\frac{x}{2x-a}+\frac{x}{2x-b}=1$

or $\frac{x}{2x-a}-\frac{1}{2}=\frac{1}{2}-\frac{x}{2x-b}$

$\frac{2x-2x+a}{2(2x-a)}=\frac{2x-b-2x}{2(2x-b)}$

or $\frac{a}{2x-a}=-\frac{b}{2x-b}$

∴ $2ax-ab=-2bx+ab$

$$\text{or } 2x(a+b)=2ab$$
$$\therefore x = \frac{ab}{a+b}.$$

(2) $\quad x-2y+z=0$

$\quad -3x-4y+5z=0 \qquad$ Rearranging eqn (2)

By the rule of cross multiplication

$$\text{we get, } \frac{x}{-10+4}=\frac{y}{-3-5}=\frac{z}{-4-6}$$

$$\text{or } \frac{x}{-6}=\frac{y}{-8}=\frac{z}{-10}$$

$$\text{i.e. } \frac{x}{3}=\frac{y}{4}=\frac{z}{5}=k \text{ (say)}$$

$$\therefore x=3k,\ y=4k,\ z=5k.$$

Now eqn (3) is $7x+8y+9z=98$

$$\therefore 7\times 3k+8\times 4k+9\times 5k=98$$
$$\text{or } k(21+32+45)=98$$
$$\therefore k=1$$
$$\therefore x=3,\ y=4,\ z=5$$

11. Let the digits be $x+2$ and x

Then the number is $10(x+2)+x$

By the question,

$$10(x+2)+x-\tfrac{3}{2}(x+2+x)=10x+(x+2)$$
$$\text{or } 9(x+2)-9x=\tfrac{3}{2}(2x+2)$$
$$\text{or } 18=3(x+1)$$
$$\therefore x+1=6$$
$$\therefore x=5$$

and the number is 75

12 $\quad \frac{a}{b}=\frac{c}{d}=k$ (say)

$$\therefore a=bk, \quad c=dk$$

$$\frac{pa^2+qc^2}{pb^2+qd^2}=\frac{pb^2k^2+qd^2k^2}{pb^2+qd^2}=k^2$$

Again, $\quad \dfrac{ma^2-nc^2}{mb^2-nd^2}=\dfrac{mb^2k^2-nd^2k^2}{mb^2-nd^2}=k^2$

$$\therefore pa^2+qc^2 : pb^2+qd^2 :: ma^2-nc^2 : mb^2-nd^2.$$

1899.—AFTERNOON.

1. Proposition 32, Book I.

2. Proposition 35, Book I

3 Proposition 12, Book II

4 A tangent to a circle is a straight line which meets the circumference but being produced, does not cut it
Proposition 18, Book III

5. Proposition 35, Book III

6. Proposition 4, Book IV.

7 Parallel straight lines are such as being in the same plane do not meet, however far they are produced in either direction.

Vide 12th axiom and deduction 2, page 96. (Hall and Steven's Euclid)

8 PQ is a position of the moving line, R is the middle point of PQ

Then because POQ is a right angle PQ is the diameter of the circle round POQ $\therefore$ R is its centre
and $\therefore$ OR = a radius of the circle
$= \frac{1}{2}$ PQ

$\therefore$ the locus of R is a circle with O as centre and radius equal to half the given length

9 Worked out in Hall and Steven's Euclid, page 147, deduction 24.

10 Bisect the angles ABC and ACB by BI and CI respectively
Then I is the in-centre
and let O be the circum-centre
then OB = OC = OA.

Difference between the angles ABC and ACB.
= Dif between the angles ABO and ACO
($\because \angle OBC = \angle OCB$)
= Dif between $\angle$s BAO and CAO
$\because \angle ABO = \angle BAO$.
and $\angle ACO = \angle CAO$
$\therefore \angle BAO - \angle CAO = (\angle BAI + \angle IAO) - (\angle CAI - \angle IAO)$
But $\angle BAI = \angle CAI$ ($\because$ AI bisects the angle at A)
$= 2 \angle IAO$
$\therefore \angle IAO = \frac{1}{2}$ the difference of the other two angles of the triangle, *i e.* the angles B and C.

1900.—MORNING.

Paper set by—Babu Haran Chandra Banerji, M.A., B L.

1 What do you understand by the Greatest Common Measure and the Least Common Multiple of two or more whole numbers ?

Nine bells begin to strike simultaneously, and strike at intervals 1, 2, 3, 4, 5, 6, 7, 8, 9 seconds respectively. After what interval of time will they next strike simultaneously ?

2. (*a*) Simplify

$$\frac{16\frac{1}{12}-3\frac{1}{3} \text{ of } 2\frac{1}{5}}{\frac{4}{7} \text{ of } 5\frac{1}{2}+3\frac{3}{4}} \times \frac{2\frac{5}{16} \text{ of } 4\frac{3}{4}+\frac{3}{8} \text{ of } 13\frac{3}{8}}{5\frac{3}{4}-4\frac{1}{2} \text{ of } \frac{5}{9}} - \frac{67}{117}+1\frac{1}{3}$$

(*b*) Reduce 0416 to its equivalent vulgar fraction in the lowest terms, and explain the reason for the process you employ,

3 Find the value of

$$(1\ 25)^3+2\ 25\times(1\ 25)^2+3\ 75\times(\ 75)^2-(\ 75)^3,$$

without reducing the decimals to vulgar fractions.

4. The length, the breadth, and the height of a room are 25 ft. 7 in, 20 ft. 5 in, and 14 ft respectively Its walls are papered at 3*s* 6*d* a sq yd, and its ceiling painted at 1*s* 2*d* a sq ft. Find the total cost

5 The subscriptions to a certain memorial fund amounted to Rs 976 9as, and each person subscribed as many annas as there were subscribers altogether Find the number of subscribers.

6. Explain clearly what do you mean by saying that the $3\frac{1}{2}$ per cent. Government Securities are at 101

A person invests Rs 19,700 in the $3\frac{1}{2}$ per cent. Government Securities at $98\frac{1}{2}$, and when they rise to $101\frac{1}{2}$, he sells out and invests the proceeds in $4\frac{1}{2}$ per cent Calcutta Municipal Deventures at $114\frac{3}{16}$. Find the change in his income.

7 Prove that $a^m \times a^n = a^{m+n}$, when m and n are positive integers

$$\text{Simplify } \left(\frac{x^m}{x^n}\right)^{m+n} + \left(\frac{x^n}{x^p}\right)^{n+p} \times \left(\frac{x^p}{x^m}\right)^{p+m}.$$

8 (*a*) Resolve into factors .—

(1) x^3+64, (2) x^4+64

(3) $a^2(b-c)+b^2(c-a)+c^2(a-b)$.

(*b*) If $x=a+b-2c$, $y=b+c-2a$, $z=c+a-2b$; find the value of $x^3+y^3+z^3-3xyz$

9. Solve the following equations —

(1) $\frac{2-x}{3}+\frac{3-x}{4}+\frac{4-x}{5}+\frac{5-x}{6}+\frac{3}{4}=0$

(2) $\left.\begin{aligned} x-2(3y-2z) &= 0 \\ 2y+(3x-3z) &= 0 \\ 5x+7y+9z &= 67 \end{aligned}\right\}$

10 A person bought a certain number of eggs, half of them at 2 a penny and half at 3 a penny He sold them again at the rate of 5 for $2d$ and lost a penny by the transaction What was the number of eggs ?

11 If $a : b = c : d$, prove that

$$\sqrt{(3a^2+4c^2)} : \sqrt[3]{(5a^3-6c^3)} = \sqrt{(3b^2+4d^2)} : \sqrt[3]{(5b^3-6d^3)}$$

1900.—AFTERNOON.

Paper set by—BABU HARAN CHANDRA BANERJI, M A, B L

1 If two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise their bases equal, then the angle which is contained by the two sides of the one shall be equal to the angle which is contained by the two sides, equal to them, of the other

Also enunciate the converse of the above Proposition

2 All the interior angles of any rectilineal figure, together with four right angles, are equal to twice as many right angles as the figure has sides

Find the magnitude of the interior angle of a regular polygon of twelve sides

3 If a straight line be divided into two equal and also into two unequal parts, the rectangle contained by the unequal parts, together with the square on the line between the points of section, is equal to the square on half the line

Divide a given straight line into two parts such that the rectangle contained by them shall be the greatest possible

4 Angles in the same segment of a circle are equal.

Also enunciate the converse of the above Propositon

5 Enunciate and prove Euc III 37

6 Describe a circle passing through three given points not in a straight line

When is it possible to describe a circle passing through four given points ?

7 Bisect a triangle by a straight line drawn from a given point in one of its sides

8 From the right angle C of a right-angled triangle ABC, CD is drawn perpendicular to the hypotenuse AB Prove that the square on CD is equal to the rectangle AD BD

9 Prove that, in an acute-angled triangle the point of intersection of the perpendiculars drawn from the vertices to the opposite sides is the centre of the inscribed circle of the triangle formed by joining of the feet of the perpendiculars

SOLUTIONS

1900.—MORNING.

1. The greatest Common Measure of two or more numbers is the greatest number which divides each of them exactly, i e. without a remainder

The Least Common Multiple of two or more numbers is the least number which is divisible by each of them exactly, i e. without a remainder

Int. of Time $=9\times8\times7\times5=2520$ seconds

$=42$ minutes.

2. (a) $$\frac{16\frac{1}{12}-\frac{7}{2}\times\frac{7}{3}}{\frac{3}{7}\times\frac{11}{2}+3\frac{3}{4}}\times\frac{\frac{37\times19}{16\times4}+\frac{3\times107}{8\times8}}{5\frac{3}{4}-\frac{9}{2}\times\frac{5}{9}}-\frac{67}{117}\times\frac{2}{3}$$

$$=\frac{16\frac{1}{12}-8\frac{1}{6}}{2\frac{5}{14}+3\frac{3}{4}}\times\frac{\frac{1024}{64}}{5\frac{3}{4}-2\frac{1}{2}}-\frac{134}{351}$$

$$=\frac{7\frac{11}{12}}{6\frac{3}{28}}\times\frac{16}{3\frac{1}{4}}-\frac{134}{351}$$

$$\frac{\overset{5}{\cancel{95}}}{\underset{3}{\cancel{12}}}\times\frac{28}{\underset{9}{\cancel{171}}}\times\frac{16\times4}{13}-\frac{134}{351}$$

$$\frac{2240-134}{351}=\frac{2106}{351}$$

$=6$ Ans

(b) $=\cdot041\dot{6}=\frac{416-41}{9000}=\frac{375}{9000}=\frac{1}{24}$

Let $x=\cdot041\dot{6}$

$=\cdot0416666$ ....ad infinitum

$1000x=41\cdot666$...... .. Do. (i)

$10000x=416\cdot666$ .... Do. (ii)

Subtracting (i) from (ii) we have

$9000x=416-41$

$\therefore x=\frac{416-41}{9000}$

Hence the rule applied above.

3 $(1\ 25)^3+2{\cdot}25(1\ 25)^2+3\ 75(\ 75)^2+(\ 75)^3$

$=(1\ 25)^3+3\times\ 75\times(1\ 25)^2+3\times1\ 25(\ 75)^2+(\ 75)^3$

$=(1\ 25+\ 75)^3=2^3=8$

4 Area of the walls

$=2\{25\text{ ft }7\text{ in }+20\text{ ft }5\text{ in}\}\times14\text{ ft}$

$=2\times46\times14$ sq feet

cost $=\dfrac{2\times46\times14\times7}{9\times2}s=£25\ 0s\ 10\frac{2}{3}d$

Area of ceiling

$=\frac{245}{12}\times\frac{301}{12}$ sq ft

cost $\frac{245}{12}\times\frac{307}{12}\times14d=\frac{526505}{72}d=£30\ 9s\ 4\frac{41}{72}d.$

Total cost $=£65\ 10s\ 3\frac{17}{72}d.$

5 Rs 976 9 ans
16

$\dot{1}5\dot{6}25$ (125 men.
1

22) 56
44

245) 1225
1225

6 It means that government pays interest at the rate of $3\frac{1}{2}$ per cent per annum on the nominal value of the bond, but if a man wants to invest money in these stocks he shall have to pay Rs 101 for every hundred of the such nominal value

$98\frac{1}{2}$ 19700 $3\frac{1}{2}$ x

$x=\dfrac{2\times19700\times7}{197\times2}$

Original income $=$ Rs 700

$98\frac{1}{2}$ 19700 $101\frac{1}{2}\cdot x$

$x=\dfrac{2\times19700\times203}{197\times2}$

Proceeds of sale $=$ Rs 20300

$114\frac{3}{16}$ 20300 $4\frac{1}{2}$ x

$x=\dfrac{\cancel{16}^{8}\times20300\times9}{\cancel{1827}_{203}\times2}$

when rice is 24 seers per Rupee—what ought the daily pay of a coolie to be in proportion when the price of rice is Rs 2-10-8 per maund ?

4 A and B run a race A has a start of 40 yards, and set off 5 minutes before B at the rate of 10 miles an hour How soon will B overtake him if his rate of running is 12 miles per hour ?

5 Extract the square root of $\frac{1}{1000}$ to 5 places of decimals.

6 Reduce to its simplest form $\frac{x+y}{x-y}+\frac{x-y}{x+y}-\frac{x^2+y^2}{x^2-y^2}$

7 Square $a^{\frac{1}{5}}-b^{\frac{1}{7}}+c^{\frac{1}{3}}$ and divide 1 by $(x+b)^2$ giving 3 terms of he quotient

8 Prove that if $a \quad b \quad c \quad d$ then $a \pm b \quad a \quad c \pm d \quad c$

9 Solve the following equations—

(i) $2x+11=7x-14$

(ii) $\sqrt{(x+9)}=7-\sqrt{x}$

(iii) $\frac{a-b}{x-c}=\frac{a+b}{x+2c}$

10 What fraction is that which, if 1 be added to the numerator becomes 1, and if 1 be added to the denominator becomes $\frac{1}{2}$?

1862.—AFTERNOON.

Examiner,—H Blochmann

1 Define a parallelogram and state what is meant by a line AB being cut externally in the point C.

2 If two angles of a triangle be equal to one another, the sides also which subtend equal angles shall be equal to one another.

3 All the interior angles of any rectilineal figure together with four right angles are equal to twice as many right angles as the figure has sides

4 Construct an isosceles triangle whose exterior vertical angle is 67 $\frac{1}{2}$ degrees

5 Prove (for the *obtuse-angled* triangle only) that the square on the side, subtending either of the two acute angles is less than the squares on the sides containing that angle by twice the rectangle contained by either of these sides and the straight line intercepted between the angle and the perpendicular let fall upon it from the opposite angle

6 In the side BC of a right angled triangle ABC, right angled at C, find a point D, such that the perpendicular DF drawn from D to a point F in the hypotenuse shall equal AF

7 If a straight line touches a circle and from the point of contact a straight line be drawn cutting the circle, the angles which the

Second income = Rs. 800.

He gains Rs 100 per annum

7. $a^m = a \times a \times a \ldots$ m factors

$a^n = a \times a \times a \ldots\ldots n$ factors.

$\therefore a^m \times a^n = (a \times a \times a \ldots m \text{ factors})$
$\times (a \times a \times a \ldots n \text{ factors})$

$= a \times a \times a \ldots (m+n)$ factors

$= a^{m+n}$

$$\left(\frac{x^m}{x^n}\right)^{m+n} \times \left(\frac{x^n}{x^p}\right)^{n+p} \times \left(\frac{x^p}{x^m}\right)^{p+m}$$

$= (x^{m-n})^{m+n} \times (x^{n-p})^{n+p} + (x^{p-m})^{p+m}$

$= x^{m^2-n^2} \times x^{n^2-p^2} \times x^{p^2-m^2}$

$= x^{m^2-n^2+n^2-p^2+p^2-m^2}$

$= x^0 = 1$

8. (a) (1) $x^3 + 64 = x^3 + (4)^3$

$= (x+4)(x^2 - 4x + 16)$

(2) $x^4 + 64 = x^4 + 16x^2 + 64 - 16x^2$

$= (x^2+8)^2 - (4x)^2$

$= (x^2 + 4x + 8)(x^2 - 4x + 8)$

(3) $a^2(b-c) + b^2(c-a) + c^2(a-b)$

$= a^2(b-c) - a(b^2-c^2) + bc(b-c)$

$= (b-c)\{a^2 - a(b+c) + bc\}$

$= (b-c)(a-b)(a-c)$

$= -(b-c)(c-a)(a-b)$

(b) $x^3 + y^3 + z^3 - 3xyz$

$= (x+y+z)(x^2+y^2+z^2-yz-zx-xy)$

but $(x+y+z) = 0$

$\therefore x^3 + y^3 + z^3 - 3xyz = 0.$

9. (1) $\frac{2-x}{3} + \frac{3-x}{4} + \frac{4-x}{5} + \frac{5-x}{6} + \frac{3}{4} = 0$

$40 - 20x + 45 - 15x + 48 - 12x + 50 - 10x + 45 = 0$

$57x = 228$

$x = 4$

(2) $$\left.\begin{aligned} x - 6y + 4z &= 0 \\ 3x + 2y - 3z &= 0 \\ 5x + 7y + 9z &= 67 \end{aligned}\right\}$$

From the 1st and 2nd we have

$$\frac{x}{18-8}=\frac{y}{12+3}=\frac{z}{2+18}$$

$$\frac{x}{10}=\frac{y}{15}=\frac{z}{20}$$

$$\frac{x}{2}=\frac{y}{3}=\frac{z}{4}=k \text{ suppose}$$

$$\therefore x=2k,\ y=3k,\ z=4k$$

Substituting k in the 3rd equation

$$k(10+21+36)=67$$

$$67k=67 \quad \therefore k=1$$

$$\therefore x=2,\ y=3,\ z=4$$

10 Let x be the no of eggs.

Then by the question

$$\frac{x}{2\times2}+\frac{x}{2\times3}=\frac{2x}{5}+1.$$

$$15x+10x=24x+60$$

$$x=60$$

11. $\frac{3a^2}{3b^4}=\frac{4c^2}{4d^2}$

$$\text{each}=\frac{3a^2+4c^2}{3b^2+4d^2}=\frac{a^2}{b^2}$$

$$\therefore \frac{\sqrt{3a^2+4c^2}}{\sqrt{3b^2+4d^2}}=\frac{a}{b} \text{ (i)}$$

Again $\frac{5a^3}{5b^3}=\frac{6c^3}{6d^3}$

$$\text{each}=\frac{5a^3-6c^3}{5b^3-6d^3}=\frac{a^3}{b^3}$$

$$\therefore \frac{\sqrt[3]{5a^3-6c^3}}{\sqrt[3]{5b^3-6d^3}}=\frac{a}{b} \text{ (ii)}$$

$$\therefore \frac{\sqrt{3a^2+4c^2}}{\sqrt{3b^2+4d^2}}=\frac{\sqrt[3]{5a^3-6}}{\sqrt[3]{5b^3-6}}$$

$$\therefore \frac{\sqrt{3a^2+4c^2}}{\sqrt[3]{5a^3-6c^3}}=\frac{\sqrt{3b^2+4}}{\sqrt[3]{5b^3-6}}$$

1900.—AFTERNOON

1. Prop. I. 18. Prop. I. 4 is the converse and others of that nature.

2. Cor. I. Prop. I. 32.

Angle$=\frac{24-4}{12}$ rt angles $=1\frac{2}{3}$ rt angles.

3 Prop II 5

Rect AD DB$+CD^2=AC^2=(\frac{1}{2}$ AA$)^2$ (II 5)

$\therefore$ The rectangle will be greatest when CD^2 $=0$, i e when C and D coincides or the line is bisected

4 Prop III 21

Equal angles standing on the same base and on the same side of it are in a segment of a circle of which the base is the chord

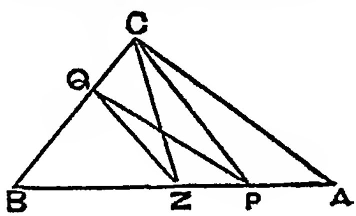
5 See Book

6 Join the three points so as to form a triangle and circumscribe a circle about the triangle thus formed (IV 5)

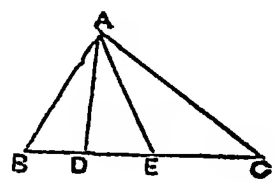
When the sum of any two of its opposite angles is together equal to two right angles (Converse to III. 22) or converse of III 21 and III 35

7 Let ABC be the Δ and P the given point in BA, join CP Through Z the middle point of BA, draw ZQ parallel to CP, meeting BC at Q join CZ and PQ Then PQ is the required line

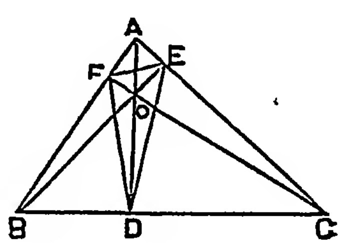
Δ CQZ$=\Delta$QZP (I 37)
to each add ΔQBZ
$\therefore$ the fig BPQ$=\Delta$CBZ
$=\frac{1}{2}\Delta$ABC (I 38)



8 Let E be the middle point of BC
BD DC$+DE^2=BE^2$ (II 5)
$=EA^2=AD^2+DE^2$ (I 47)
$\therefore$ BD CD$=AD^2$



9 $\angle$BFC$=\angle$BEC$=1$ rt angle
$\therefore$ A circle can be described round B, F, E, C
Similarly, a circle can be described round A, F, D, C
In the circle BFEC
$\angle$EFC$=\angle$EBC (II 21)
Similarly $\angle$CAD$=\angle$CFD
But $\angle$CAD$=\angle$EBC for the two Δs CAD and EBC have the angle at C common and the angles E and D rt. angles (I 32)



$\therefore <EFC = <CFD$

i e $<EFD$ is bisected by FC

Similarly it may be shown that $<BEF$ and $<FDE$ are bisected by BE and AD

Hence O the point of intersection of the lines is the centre of the inscribed circle in DFE.

1901.—MORNING.

Head Examiner,—BABU GOURISANKAR DE, M A

1. (*a*) Simplify

$$\frac{306}{323} - \frac{204}{221} \text{ of } \frac{22\frac{5}{11}}{32\frac{9}{11}} - 583 \times 142857,$$

expressing your answer as a decimal.

(*b*) Reduce £3 15*s* 4*d*. to the decimal of Rs 100 (£1 = Rs. 15)

2 (*a*) What is meant by an *aliquot part* of a number? Is $2\frac{3}{4}$ yds. an aliquot part of a mile?

(*b*) Find by Practice, or otherwise, the value of 25 tons 15 cwt. 3 qrs $17\frac{1}{2}$ lbs at £2 13*s* 4*d* per ton

3 If the fourpenny loaf weighs 3lbs 9 oz when wheat is at 9*s*. 4*d* per bushel, what ought the sixpenny loaf to weigh when wheat is at 11*s* 1*d* per bushel?

4. (*a*) Define *Interest* What do you understand by the expression *Rate per cent per annum*?

(*b*) At what rate per cent per annum simple interest will £200 amount to £236 13*s* 4*d* in four years 7 months?

5 Extract the square root of 7468·4164

6 A man invests one-third of his capital in the $3\frac{1}{2}$ per cent. Government Securities at $96\frac{1}{2}$, and the remaining two-thirds in the $4\frac{1}{2}$ per cent Calcutta Municipal Debentures at $105\frac{1}{2}$ If the difference of the two annual incomes be Rs 1997, find his capital

7. (*a*) Prove that $a^m \div a^n = a^{m-n}$, where m and n are positive integers, and m greater than n

(*b*) Divide $a5m + b5n$ by $a^m + b^n$.

8. (*a*) Resolve $a^4 + a^2b^2 + b^4$ into two factors.

(*b*) Find the G C M of

$$x^3 + 6x^2 + 11x + 6 \text{ and } x^4 + x^3 - 4x^2 - 4x.$$

9 Simplify

$$\frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} + \frac{(x-a)(x-b)}{(c-a)(c-b)}.$$

10. (*a*) Distinguish between an *Equation* and an *Identity*, and give an example of each

(*b*) Solve the following equations —

(1) $\frac{1}{3}(x-3)+\frac{1}{4}(x-8)+\frac{1}{6}(x-4)=2\frac{7}{15}$;

(2) $2x+3y+4z=38$,

$3x-2y+5z=26$,

$4x+6y-3z=21$.

11 A number consists of two digits, the digit in the units' place being four times that in the tens' place If the digits be inverted, the new number increased by 2 equals three times the old number Find the number

12. If $x : a = y : b = z : c$, prove that

$$\frac{x^3}{a^2}+\frac{y^3}{b^2}+\frac{z^3}{c^2}=\frac{(x+y+z)^3}{(a+b-c)^2}$$

1901 —AFTERNOON.

Paper set by—BABU GOURISANKAR DE, M.A

1. (*a*) Define parallel straight lines.

Prove that the opposite sides and angles of a parallelogram are equal to one another, and that the diagonal bisects the parallelogram, that is, divides it into two equal parts

(*b*) Prove that the diagonals of a parallelogram bisect each other.

2 If the square described on one of the sides of a triangle be equal to the squares described on the other two sides of it, the angle contained by these two sides is a rt. angle.

3 Divide a given straight line into two parts, so that the rectangle contained by the whole line and one of the parts may be equal to the square on the other part

4 (*a*) Prove that the opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles

(*b*) Enunciate and prove the converse of the above proposition

5 If from any point without a circle two straight lines be drawn, one of which cuts the circle, and the other touches it; the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, shall be equal to the square on the line which touches it

6. (*a*) Inscribe a circle in a given circle

(*b*) Describe a circle touching one of a given triangle and the other two sides produced

7. If two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise the angles opposite to one pair of equal sides equal, then the angles opposite to the other pair of equal sides are either equal or supplementary; and, in the former case, the triangles are equal in all respects.

8 The straight lines drawn from the angles of a triangle to the middle points of the opposite sides meet at the same point

9 Define the *locus* of a point

A is a fixed point and BC a fixed straight line of indefinite legth, AP is any straight line drawn through A to meet BC at P; and in AP, a point O is taken such that the rectangle AP, AQ is constant find the locus of Q

SOLUTIONS

1901.—MORNING.

1. (*a*) $\text{Ex} = \frac{306}{323} - \frac{204}{221} \text{ of } \frac{\frac{247}{11}}{\frac{361}{11}} - \frac{583-58}{900} \times \frac{142857}{999999}$

$$= \frac{17\times18}{17\times19} \div \frac{12\times\cancel{17}}{13\times\cancel{17}} \times \frac{247}{\cancel{11}} \times \frac{\cancel{11}}{361} - \frac{525}{900} \times \frac{142857}{7\times142857}$$

$= \frac{18}{19} \div \frac{12}{19} - \frac{7}{12} \times \frac{1}{7} = \frac{18}{19} \times \frac{19}{12} - \frac{1}{12} = \frac{18}{12} - \frac{1}{12} = \frac{17}{12} = 1\frac{5}{12} = 1.416$ Ans.

(*b*) £3. 15*s* 4*d* = £3 15⅓*s*. = £$3\frac{23}{30}$

Required decimal $= 3\frac{23}{30} \div \frac{100}{15} = \frac{113}{30} \times \frac{15}{100} = \frac{113}{200} = \frac{565}{1000} = .565$ Ans.

2 (*a*) An aliquot part of a number is such a part that it is contained an exact number of times in the number

1 mile = 1760 yds

$1760 \div 2\frac{3}{4} = 1760 \times \frac{4}{11} = 160 \times 4 = 640$

∴ $2\frac{3}{4}$ yds is an aliquot part of a mile.

(*b*)

	£	s	d	
	2	13	4	value of 1 ton
			5	
10 cwt = ½ os 1 ton	13	6	8	value of 5 tons
			5	
	66	13	4	value of 25 tons
	1	6	8	,, ,, 10 cwt
5 cwt ½ of 10 cwt		13	4	,, ,, 5 cwt
2 qrs = $\frac{1}{10}$ of 5 cwt		1	4	,, ,, 2 qrs
1 qr = ½ of 2 qrs			8	,, ,, 1 qr
14 lbs = ½ of 1 qr			4	,, ,, 14 lbs
3½ lbs = ¼ of 14 lbs			2	,, ,, 3½ lbs
	£68	15	10	value of 25 tons 15 cwt 3 qrs 17½ lbs

3. 4 : 6
$11\frac{1}{12}$: $9\frac{1}{3}$ :: $3\frac{9}{16}$ lbs. : x lbs

$$x = \frac{6\times28\times57\times12}{4\times133\times3\times16} \text{ lbs} = \tfrac{9}{2} \text{ lbs.} = 4\tfrac{1}{2} \text{ lbs} \quad \text{Ans.}$$

4. (*a*) Interest is the excess of money paid for the use of money borrowed Rate per cent per annum is a number for every hundred for the use of a year

(*b*) £236 13*s* 4*d*

£200 0 0

£ 36 13*s* 4*d* Interest of £200 for $4\frac{7}{12}$ years

£$18\frac{1}{3}$ Interest of £100 for $4\frac{7}{12}$ years

$18\frac{1}{3} - 4\frac{7}{12} = \frac{55}{3} \times \frac{12}{55} =$ £4 Interest of 100 for 1 year

∴ Rate per cent. per annum is 4. Ans

5.

$$
\begin{array}{r|l|l}
 & 74\dot{6}8\,4164 & 86\,42 \quad \text{Ans} \\
 & 64 & \\
\hline
166 & 1068 & \\
 & 996 & \\
\hline
1724 & 7241 & \\
 & 6896 & \\
\hline
17282 & 34564 & \\
 & 34564 & \\
\hline
\end{array}
$$

6 Suppose the capital to be Rs. 3

$96\frac{1}{2}$ 1 $3\frac{1}{2}$ G S. Income

$$\text{G S. Income} = \frac{7\times2}{193\times2} = \frac{7}{193}$$

$105\frac{1}{2}$ 2 $4\frac{1}{2}$ C M Income

$$\text{C M. Income} = \frac{2\times2\times9}{211\times2} = \frac{18}{211}$$

$$\text{Difference} = \frac{18}{211} - \frac{7}{193} = \frac{3474-1477}{211\times193} = \frac{1997}{211\times193}$$

$$\frac{1997}{211\times193} : 1997 : \quad 3 : \text{capital}$$

$$\text{Capital} = \text{Rs.}\ \frac{1997\times3\times211\times193}{1997} = \text{Rs}\ 122169\ \text{Ans.}$$

7. (*a*) $a^m = a\times a\times a\times a\times\ldots$. m factors

$a^n = a\times a\times a\times$ . ..... n factors

∴ $a^m - a^n = a\times a\times a\times$ ....$(m-n)$ factors

$= a^{m-n}$

(b)
$$
\begin{array}{l}
a^m+b^n\,\big)\; a^{5m}+b^{5n} \;\big(\, a^{4m}-a^{3m}b^n+a^{2m}b^{2n}-a^mb^{3n}+b^{4n} \quad \text{Ans.} \\
\qquad\qquad\;\; \underline{a^{5m}+a^{4m}b^n} \\
\qquad\qquad\;\; -a^{4m}b^n+b^{5n} \\
\qquad\qquad\;\; \underline{-a^{4m}b^n-a^{3m}b^{2n}} \\
\qquad\qquad\;\; a^{3m}b^{2n}+b^{5n} \\
\qquad\qquad\;\; \underline{a^{3m}b^{2n}+a^{2m}b^{3n}} \\
\qquad\qquad\;\; -a^{2m}b^{3n}+b^{5n} \\
\qquad\qquad\;\; \underline{-a^{2m}b^{3n}-a^mb^{4n}} \\
\qquad\qquad\;\; a^mb^{4n}+b^{5n} \\
\qquad\qquad\;\; \underline{a^mb^{4n}+b^{5n}}
\end{array}
$$

8. (a) $a^4+a^2b^2+b^4=a^4+2a^2b^2+b^4-a^2b^2=(a^2+b^2)^2-(ab)^2$
$=(a^2+b^2+ab)(a^2+b^2-ab)$

(b)
$$
\begin{array}{l}
x^3+6x^2+11x+6\,\big)\; x^4+x^3-4x^2-4x \;\big(\, x \\
\qquad\qquad\qquad\qquad \underline{x^4+6x^3+11x^2+6x} \\
\qquad\qquad\qquad -5x \,\big|\, \underline{-5x^3-15x^2-10x} \\
\qquad\qquad\qquad\qquad x^2+3x+2\,\big)\; x^3+6x^2+11x+6 \;\big(\, x+3 \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\;\; \underline{x^3+3x^2+2x} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\;\; 3x^2+9x+6 \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\;\; \underline{3x^2+9x+6}
\end{array}
$$

$\therefore\ x^2+3x+2$ is the G. C. M.

9. $\text{Ex.}=\dfrac{x^2-x(b+c)+bc}{(a-b)(a-c)}-\dfrac{x^2-x(c+a)+ac}{(b-c)(a-b)}+\dfrac{x^2-x(a+b)+ab}{(a-c)(b-c)}$

L. C. M. of Denrs $=(a-b)(a-c)(b-c)$

Numerator $=(b-c)\{x^2-x(b+c)+bc\}$
$-(a-c)\{x^2-x(a+b)+ac\}$
$+(a-b)\{x^2-x(a+b)+ab\}$
$=x^2(b-c+c-a+a-b)-x(b^2-c^2-a^2+c^2+a^2-b^2)$
$+bc(b-c)-ac(a-c)+ab(a-b)$
$=bc(b-c)-a^2c+ac^2+a^2b-ab^2$
$=bc(b-c)+a^2(b-c)-a(b^2-c^2)$
$=(b-c)\{bc+a^2-a(b+c)\}$
$=(b-c)\{bc+a^2-ab-ac\}$
$=(b-c)\{a(a-c)-b(a-c)\}$
$=(b-c)(a-c)(a-b)$

$\text{Ex.}=\dfrac{(a-b)(a-c)(b-c)}{(a-b)(a-c)(b-c)}=1.$ Ans.

10 (a) In an identity the two expressions are equal for all the values of the unknown quantities while in an equation the two expressions are equal only for a particular value or values of the unknown quantites, as $(x-2)(x-3)=x^2-5x+6$, here the two expressions are equal for all the values of x while in $(x-2)(x-3)=x^2-4x+8$ the two sides are equal only when $x=-2$ and for no other value of x.

(b) (1) $\frac{1}{3}(x-3)+\frac{1}{4}(x-8)+\frac{1}{5}(x-4)=2\frac{7}{15}$. Multiplying by 60

$20(x-3)+15(x-8)+12(x-4)=37\times 4$

or $20x-60+15x-120+12x-48=148$

or $47x=148+60+120+48$

or $47x=376$ $\quad\therefore x=\frac{376}{47}=8$ Ans

(2) $6x+9y+12z=114$ $\quad(1)\times 3$

$6x-4y+10z=\ 52$ $\quad(2)\times 2$

$\therefore 13y+2z=\ 62$ $\quad(4)$

$4x+6y+8z=\ 76$ $\quad(1)\times 2$

$4x+6y-3z=\ 21$ $\quad(3)$

$11z=\ 55$ $\quad\therefore z=5$

$13y=62-2z$ $\quad(4)$

or $13y=62-10=52$ $\quad\therefore y=4$

$2x+3y+4z=38$

or $2x+12+20=38$

or $2x=38-32=\ 6$

$\therefore x=3$

$\left.\begin{array}{l}x=3\\y=4\\z=5\end{array}\right\}$ Ans

11. Let x be the 1st digit, *i.e.*, in the ten's place then $4x$ is the 2nd digit

$\therefore$ the number is $10x+4x$.

Then by the question

$10\times 4x+x+2=3(10x+4x)$

or $40x+x+2=42x$ $\quad\therefore x=2$

$\therefore$ the number is 28

12 $\quad\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=k$ (suppose)

Then $x=ak$

$y=bk$

$z=ck$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=\frac{a^2k^2}{a^2}+\frac{b^2k^2}{b^2}+\frac{c^2k^2}{c^2}=k^2(a+b+c)$$

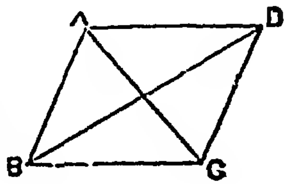
$$\text{Again } \frac{(x+y+z)^2}{(a+b+c)^2}=\frac{(ak+bk+ck)^2}{(a+b+c)^2}=\frac{k^2(a+b+c)^2}{(a+b+c)^2}=k^2(a+b+c)$$

$$\therefore \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=\frac{(x+y+z)^2}{(a+b+c)^2}$$

1901.—AFTERNOON.

1 (*a*) Euclid, Bk I 35 def
Euclid Bk I 34

(*b*) Let ABCD be the parallelogram and AC and DB the diagonals They will bisect one another



Proof—In the triangles AOB and DOC

$\because \left\{\begin{array}{l}\angle OAB=\angle OCD\\ \angle OBA=\angle ODC\end{array}\right\}$ (I 29)

and AB=DC (I 34)

$\therefore \left.\begin{array}{l}AO=OC\\ BO=OD\end{array}\right\}$ (I 26)

2 Euclid, Bk I. 18. 3 Euclid, Bk. II 11

4. (*a*) Bk III 11. (*b*) See Hall and Stevens' Euclid.

5 Bk III 36

6 (*a*) Bk IV. 4

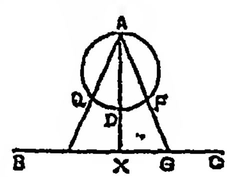
(*b*) See Hall and Stevens' Bk. IV. Escribed circle after the 4th prop.

7. See Hall and Stevens' Bk. I page 100.

8 See Hall and Stevens' Bk. I. page 111.

9 The locus of a point is a line, lines or part of a line in which every point and no other point besides, satisfies a given condition.

10 From A draw AX perp on BC Divide AX in the point D so that AX AD=AP AQ On AD as diameter describe a circle. This circle is the locus required



Proof —From A draw any line AFG join DF $\because$ $\angle$DFA is a rt angle $\therefore$ $\angle$DFG is a rt angle and $\angle$DXG is a rt angle $\therefore$ a circle will pass through D, F, G, X $\therefore$ AF AG=AD AX= AP.AQ $\therefore$ The circle is the locus

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1902.—MORNING.

Head Examiner—Babu Gouri Sankar Dey, M A.

1 (*a*) How can you ascertain whether a given vulgar fraction can be reduced to a terminating or a recurring decimal, without actually converting it into a decimal? What kind of decimal will the fraction $\frac{161}{17920}$ produce?

straight line makes with the line touching the circle shall be equal to the angles which are in the alternate segments.

8. The area of a rhombus is equal to half the rectangle contained by the diagonals

9 To inscribe an equilateral and equiangular quindecagon in a circle.

10 Given a chord AB of a circle and a point C in it. Find in the circumference a point D such that the line DC shall bisect the vertical angle of the triangle ABD.

SOLUTIONS.

1862.—MORNING.

1. $\dfrac{4\frac{3}{4}}{5\frac{1}{6}}-\dfrac{99}{310}=\dfrac{19}{4}\times\dfrac{6}{31}-\dfrac{99}{310}=\dfrac{57}{62}-\dfrac{99}{310}=\dfrac{285-99}{310}=\dfrac{186}{310}=\frac{3}{5}=\cdot 6.$

$\therefore\ \cdot 6-\cdot 06=\cdot 54.$

2 $\cdot 14$ of a pie $=\frac{14}{100}$p $=\frac{7}{50}$ pie Re $1=16\times 12=192$ pies

$\therefore$ the fraction required $=\dfrac{7}{50\times 192}=\dfrac{7}{9600}$

(a) £$\cdot 0875\times 20=1\cdot 7500s=1s+\cdot 75\times 12d=1s\ 9d$

3 24 seers : 40 seers :: Re 1 : x $\therefore\ x=$ Rs. $\frac{40}{24}=$ Rs $\frac{5}{3}$

$$\left.\begin{array}{l}18 \text{ co} : 1 \text{ co} \\ 30 \text{ days} : 1 \text{ day} \\ \text{Rs } \frac{5}{3} : \text{Rs } 2\frac{2}{3}\end{array}\right\} :: \text{Rs } 85 : x$$

$\therefore\ x=$ Rs $\dfrac{8\times 85\times 3}{18\times 30\times 3\times 5}=$ Rs $\dfrac{2\times 4\times 17\times 5\times 3}{9\times 2\times 2\times 15\times 3\times 5}$

$=$ Rs $\frac{34}{135}=4\frac{4}{135}$ as

4. $60' : 5' :: 10$ mi $: x$ $x=\dfrac{5\times 10}{60}$ mi. $=\frac{5}{6}$ mi.

40 yds $=\frac{40}{1760}$ of a mile $=\frac{1}{44}$ mile

$\therefore\ \frac{5}{6}+\frac{1}{44}=\frac{113}{132}$ mi is to be gained by B

And B gains $(12-10)$ mi in every hour

$\therefore$ time required $=\dfrac{113\times 60}{132\times 2}$ min $=25\frac{15}{22}$ min

5. $\sqrt{(\frac{1}{1000})}=\sqrt{\cdot 001}=\sqrt{\cdot 0010000000}=\cdot 03162$

6 Ans $=\dfrac{(x+y)^2+(x-y)^2-(x^2+y^2)}{x^2-y^2}$

$=\dfrac{x^2+y^2+2xy+x^2+y^2-2xy-x^2-y^2}{x^2-y^2}=\dfrac{x^2+y^2}{x^2-y^2}$

(b) Simplify

$$1-\cfrac{2}{3+\cfrac{4}{5-\cfrac{6}{7+\frac{8}{9}}}} \quad -2\cdot08\dot{3} \text{ of } \frac{2 \text{ cwt } 2 \text{ qrs } 21 \text{ lbs}}{10 \text{ cwt } 2 \text{ qrs } 11 \text{ lb}}.$$

and reduce the result to the decimal of 1 1.

2. The area of a rectangular field whose breadth is 500 yds. is 100 acres. Find the cost of cultivating it at Rs 3 2as 8 p. per 100 sq yds, and also the cost of fencing it round at Rs 2 8as. per yard

3 If 12 men and 15 boys can do a piece of work in 30 days, working $7\frac{1}{2}$ hours a day, how many boys must assist 21 men to do a piece of work twice as great in 25 days, working 9 hours a day? (3 men are equivalent to 5 boys)

4 Extract the square roots of $5\frac{1}{16}$ and and 76 195441

5 (a) Define *Discount*

(b) Find the discount on £700 due 3 years 4 months hence at 5 per cent per annum simple interest

6 Which is the better investment, the $3\frac{1}{2}$ per cent Government Securities at $95\frac{5}{8}$ or the 4 per cent Calcutta Municipal Debentures at $101\frac{1}{4}$? What will be the difference in annual income by investing Rs 22127 in each of them?

7 (a) Divide

$$x^2(y+z)+y^2(z+x)+z^2(x+y)+3xyz \text{ by } yz+zx+xy$$

(b) If $s=a+b+c$, prove that

$$(as+bc)(bs+ca)(cs+ab)=(b+c)^2(c+a)^2(a+b)^2$$

8 (a) Prove that $(x^p)^q=x^{pq}$, where p and q are positive integers.

(b) Simplify $\left(\dfrac{x^p}{x^q}\right)^{p+q} - \left(\dfrac{x^{p+q}}{x^{p-q}}\right)^{\frac{p^2}{q}}$.

9 Find the L C M. of

$$x^3-x^2-14x+24,\ x^3-2x^2-5x+6, \text{ anb } x^2-4x+3$$

10 (a) Prove that in a simple equation there cannot be more than one value of the unknown quantity

(b) Solve the following equations —

(1) $$\frac{bc(ax-1)}{b+c}+\frac{ca(bx-1)}{c+a}+\frac{ab(cx-1)}{a+b}=a+b+c,$$

(2) $12x+34y=8\frac{1}{15}$, $34x+12y=8\frac{4}{5}$

11 A sum of money was divided equally among a certain number of persons, had there been six more, each would have received a shilling less, and had there been four fewer, each would

have received a shilling more than he did find the sum of money and the number of men.

12 (*a*) If $a \quad b \quad b \quad c \quad c \quad d$, prove that $a \quad d \quad a^3 \cdot b^3$;

(*b*) If $x \quad ax+by+cz=y \quad bx+cy+az=z \quad cx+ay+bz$,

show that each of these ratios $=\dfrac{1}{a+b+c}$, supposing $x+y+z$ is not $=0$.

1902 —AFTERNOON

Head Examiner —BABU GOURI SANKER DEY, M A

1. (*a*) If a straight line fall on two parallel straight lines, it makes the alternate angles equal to one another, and the exterior angle equal to the interior and opposite angle on the same side; and also the two interior angles on the same side together equal to two right angles

(*b*) Write out the axioms which are used in the proof of this proposition

2 Enunciate and prove Euc I 47

3 Describe a square that shall be equal to a given rectilineal figure

4 (*a*) Draw a straight line from a given point, either without or in the circumference, which shall touch a given circle

(*b*) The two tangents which can be drawn to a circle from an external point are equal

5 (*a*) If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle will be equal to the angles which are in the alternate segments of the circle

(*b*) Enunciate and prove the converse of this proposition.

6 (*a*) Describe an isosceles triangle, having each of the angles at the base double of the third angle

(*b*) Divide a right angle into five equal parts

7 In a straight line of indefinite length, find a point such that the sum of its distances from two given points, on the same side of the given line, shall be a minimum

8 Draw a common tangent to two given circles which are external to each other and do not intersect

9 Define the *locus* of a point

From a fixed point O a straight line OPQ is drawn to cut a fixed circle in P and Q If R and S be the middle points of OP and OQ, prove that R and S always lie on another fixed circle, and find its centre and radius

SOLUTIONS

1902.—MORNING.

1 (*a*) When the denominator of the vulgar fraction is 2 or 5 or any power of 2 and 5 or product of them then it can be reduced

to a terminating decimal In all other cases it is reduced to a recurring decimal.

$$17920 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times5\times7 \text{ and } 161 = 7\times23$$

$\therefore$ Fraction $= \dfrac{23}{2^9\times5}$ $\therefore$ The fraction can be reduced to a terminating decimal

$$(b)\ \text{Ex.} = 1 - \cfrac{2}{3+\cfrac{4}{5-\cfrac{6}{7\frac{1}{9}}}} \div \frac{2083-208}{900} \text{ of } \frac{10\frac{3}{4}\text{ qrs}}{42\frac{1}{28}\text{ qrs}}$$

$$= 1 - \cfrac{2}{3+\cfrac{4}{5-\frac{54}{71}}} - \frac{1875}{900} \text{ of } \frac{43}{4} \times \frac{28}{1187}$$

$$= 1 - \cfrac{2}{3+\cfrac{4}{\frac{355-54}{71}}} - \frac{75\times25}{36\times25} \times \frac{43\times7}{1187}$$

$$= 1 - \frac{2}{3+\frac{284}{301}} - \frac{25}{12} \times \frac{43\times7}{1187}$$

$$= 1 - \cfrac{2}{\frac{903+284}{301}} \div \frac{25}{12} \times \frac{43\times7}{1187}$$

$$= 1 - \frac{602}{1187} \div \frac{7525}{12\times1187}$$

$$= \frac{1187-602}{1187} \div \frac{7525}{12\times1187}$$

$$= \frac{585}{1187} \times \frac{12\times1187}{7525} = \frac{117\times12}{1505} = \frac{1404}{1505} \text{ Ans.}$$

$$\frac{1404}{1505} + \frac{11}{19} = \frac{1404}{1505} \times \frac{19}{11} = \frac{808}{1111} = .\dot{8}\dot{4}808. \ldots \text{Ans}$$

2\. Cost of cultivating $= \dfrac{100\times4840}{100} \times$ Rs $3\frac{1}{6}$ = Rs. $\dfrac{4840\times19}{6}$

$$= \text{Rs.} \frac{2420\times19}{3} = \text{Rs.} \frac{45980}{3} = \text{Rs } 15326 \text{ 10as. 8p Ans.}$$

Length $= \dfrac{100\times4840}{500} = 968$ yds

2 length + 2 breadth = 968 × 2 + 500 × 2 = 1936 + 1000 = 2936 yds.

Cost of fencing = 2936 × Rs $2\frac{1}{2}$ = Rs $\dfrac{2936\times5}{2}$ = Rs. 1468 × 5

= Rs 7340 Ans.

3 $\quad$ 3 men $=$ 5 boys

$\therefore$ 12 men $=$ 20 boys

21 men $=$ 35 boys

12 men $+$ 15 boys $=$ 20 $+$ 15 boys $=$ 35 boys

$35 \times 30 \times 7\frac{1}{2} \quad x \times 25 \times 9 \quad 1 \quad 2$

$$x = \frac{35 \times 30 \times 15 \times 2}{2 \times 25 \times 9} = 70$$

$70 - 35 = 35$ boys $\quad$ Ans

4. $\sqrt{5\frac{1}{16}} = \sqrt{\frac{81}{16}} = \frac{9}{4} = 2\frac{1}{4}$ $\quad$ Ans

	76 195441	8 729 Ans
	64	
167	1219	
	1169	
1742	5054	
	3484	
17449	157041	
	157041	

5 (a) Discount is the sum of money deducted for the payment of a sum of money before it is due

$5 \times 3\frac{1}{3} = \frac{5 \times 10}{3} = \frac{50}{3}$

$100 + \frac{50}{3} \quad 700 \;. \quad \frac{50}{3}$ $\quad$ Discount

Discount $= £\ \frac{700 \times 50 \times 3}{350 \times 3} = £100$ Ans

6. $\quad 95\frac{3}{8} \quad 101\frac{1}{2} \quad 3\frac{1}{2} \quad x$

$$x = \frac{203 \times 7 \times 8}{763 \times 2 \times 2} = \frac{406}{109} = 3\frac{79}{109}$$

$\therefore\ 4 - 3\frac{79}{109} = \frac{30}{109}$

$\therefore$ 2nd is the better investment

$101\frac{1}{2} \quad 22127 \quad \frac{30}{109}$ $\quad$ Difference

Diff $=$ Rs $\frac{22127 \times 30 \times 2}{203 \times 109} =$ Rs 60 $\quad$ Ans.

7. (a) $xy + xz + yz)\ x^2y + x^2z + y^2z + xy^2 + xz^2 + yz^2 + 3xyz\ (x + y + z$ $\quad$ Ans.

$\qquad x^2y + x^2z \qquad\qquad + xyz$

$\qquad xy^2 + y^2z + xz^2 + yz^2 + 2xyz$

$\qquad xy^2 + y^2z \qquad\qquad + xyz$

$\qquad xyz + xz^2 + yz^2$

$\qquad xyz + xz^2 + yz^2$

(b) $(as+bc)(bs+ca)(cs+ab)$

$=\{a(a+b+c)+bc\}\{b(a+b+c)+ca\}\{c(a+b+c)+ab\}$

$=\{a(a+b)+c(a+b)\}\{b(a+b)+c(a+b)\}\{c(b+c)+a(b+c)\}$

$=(a+b)(a+c)(a+b)(b+c)(b+c)(b+a)$

$=(a+b)^2(c+a)^2(b+c)^2$

8 (a) $(x^p)^q=x^q\times x^p\times x^p\times x^p\times\ldots\ldots\ldots q$ factors

$=(x\times x\times x..\ p$ factors$)\times(x\times x\times x\ldots\ p$ factors$)$

$\times(x\times x\times x\times\ldots\ldots p$ factors$)\times\ldots\ldots\ q$ factors

$=x\times x\times x\times x\times\quad pq$ factors

$=x^{pq}$

(b) Ex. $=(x^{p-q})^{p+q}+\{x^{p+q-(p-q)}\}^{\frac{p^2}{q}}$

$=x^{p^2-q^2}+x^{2q})^{\frac{p^2}{q}}=x^{p^2-q^2}+x^{2p^2}=x^{p^2-q^2-2p^2}=x^{-p^2-q^2}$

$=\dfrac{1}{x^{p^2+q^2}}$ Ans.

9. $x^3-x^2-14x+24=x^3-2x^2+x^2-2x-12x+24$

$=x^2(x-2)+x(x-2)-12(x-2)=(x-2)(x^2+x-12)$

$=(x-2)\{x^2+4x-3x-12\}=(x-2)\{x(x+4)-3(x+4)\}$

$=(x-2)(x+4)(x-3)$

$x^3-2x^2-5x+6=x^3-3x^2+x^2-3x-2x+6$

$=x^2(x-3)+x(x-3)-2(x-3)$

$=(x-3)(x^2+x-2)=(x-3)\{x^2+2x-x-2\}$

$=(x-3)\{x(x+2)-1(x+2)\}=(x-3)(x+2)(x-1)$

$x^2-4x+3=x^2-3x-x+3=x(x-3)-1(x-3)=(x-1)(x-3)$

$\therefore$ L. C. M $=(x-2)(x+4)(x-3)(x+2)(x-1)$

$=(x^2-4)(x+4)(x^2-4x+3)$. Ans

10. (a) If possible let a and β be the two roots of the equation $ax+b=0$ Then substituting these values of x

$a\alpha+b=0$

$a\beta+b=0$

substracting $a(\alpha-\beta)=0$

But a is not 0 $\therefore\ \alpha-\beta=0\quad\therefore\ \alpha=\beta$

$\therefore$ a simple equation cannot have two different roots

(b) (1) or $\dfrac{bc(ax-1)}{b+c}-a+\dfrac{ca(bx-1)}{c+a}-b+\dfrac{ab(cx-1)}{a+b}-c=0$

$$\text{or } \frac{bcax-bc-ab-ac}{b+c}+\frac{cabx-ca-bc-ab}{c+a}$$
$$+\frac{abcx-ab-ac-bc}{a+b}=0$$

$\therefore bcax-bc-ab-ac=0$

or $abcx=ab+ac+bc$

$\therefore x=\dfrac{ab+ac+bc}{abc}$ Ans

(2) $12x+34y=8\frac{1}{15}$
$34x+12y=8\frac{4}{5}$

adding and substracting these two equations

$46x+46y=16\frac{13}{15}$ or $x+y=\frac{11}{30}$

and $22x-22y=\frac{11}{15}$ or $x-y=\frac{1}{30}$

$$\left.\begin{array}{lll} \therefore 2x=\frac{12}{30} & \therefore x=\frac{6}{30} \text{ or } & x=\frac{1}{5} \\ 2y=\frac{10}{30} & y=\frac{5}{30} & y=\frac{1}{6} \end{array}\right\} \text{Ans}$$

11 Let x be the no of men

ys the sum. Then by the question

$\dfrac{y}{x+6}=\dfrac{y}{x}-1$ or $xy=xy+6y-x^2-6x$

$\dfrac{y}{x-4}=\dfrac{y}{x}+1$ or $xy=xy-4y+x^2-4x$

or $x^2+6x-6y=0$
$x^2-4x-4y=0$

$10x-2y=0$ $\therefore y=5x$

$\therefore x^2+6x-6y=0$ or $x^2+6x-30x=0$ or $x+6-30=0$

$\therefore x=24$

$y=120s=£6$ $\therefore$ 24 men and £6

12. (a) $\dfrac{a}{b}=\dfrac{b}{c}=\dfrac{c}{d}$ $\therefore \dfrac{a}{b}\times\dfrac{b}{c}\times\dfrac{c}{d}=\dfrac{a}{b}\times\dfrac{a}{b}\times\dfrac{a}{b}$ or $\dfrac{a}{d}=\dfrac{a^3}{b^3}$.

(b) $\dfrac{x}{ax+by+cz}=\dfrac{y}{bx+cy+az}=\dfrac{z}{cx+ay+bz}$

each of them $=\dfrac{x+y+z}{ax+by+cz+bx+cy+az+cx+ay+bz}$

$$=\frac{x+y+z}{a(x+y+z)+b(x+y+z)+c(x+y+z)}$$

$$=\frac{x+y+z}{(a+b+c)(x+y+z)}=\frac{1}{a+b+c}$$

$\therefore x+y+z$ is not $=0$

1902.—AFTERNOON

1. (*a*) Euclid, I. 29. (*b*) 4th and 12th aximos.

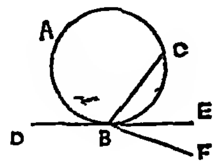
2 See Euclid. 3. Euclid, II 14.

4 (*a*) Euclid, III 17. (*b*) Euclid, III. 17, Cor

5 (*a*) Euclid, (III 32)

(*b*) If a straight line meet a circle and from the point of meeting a straight line be drawn cutting the circle and if the angles which this straight line makes with the line meeting the circle be equal to the angles in the alternate segments of the circle then the straight line meeting the circle shall be a tangent to the circle

Let the straight line DE meet the circle ABC at the point B and let BC be drawn cutting the circle and the angle CBE equal to the angle in the segment BAC



Then DE shall be a tangent to the circle ABC For if not let BF be a tangent at the point B

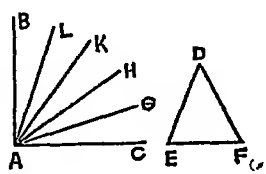
Then $\angle$FBC=angle in the alternate segment BAC (III 32)
But $\angle$EBC= (Hyp)
$\because$ $\angle$FBC=$\angle$EBC the less equal to the greater which is impossible
$\therefore$ BF is not tangent Similarly it can be proved that no other straight line but DE is the tangent $\therefore$ DE is the tangent

6. (*a*) Euclid, IV 10

(*b*) Let BAC be a rt angle it is required to divide it into 5 equal parts

Describe a triangle DEF having each of the angles at E and F double of the angle D (IV. 10)

At the point A in BA make the angle BAG equal to $\angle$DEF.



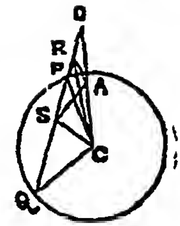
Bisect $\angle$BAG by AK (I 9)
and bisect the angles BAK and KAG by AL and AH

Then the angle BAC is divided into five equal parts
$\because$ $\angle$DEF and $\angle$DFE are each of them double of $\angle$EDF
and $\because$ $\angle$DEF+$\angle$DFE+$\angle$EDF are together=2 rt angles
$\therefore$ DEF is $\frac{2}{5}$ of 2 rt angles, *i e*, $\frac{4}{5}$ of rt angles
$\therefore$ $\angle$BAG=$\frac{4}{5}$ of a rt angle
$\therefore$ Each of the angles BAL LAK, KAH, HAG, and GAC is one-fifth of a right angle
$\therefore$ $\angle$BAC is divided into five equal parts

7. See Hall and Steavens' Euc Bk. III. Maximum and minimum, p 261

8. See Hall and Stevens' Euc Bk III. common tangent, p 266

9. The Locus of a point is a line, lines or part of a line in which every point and no other points besides satisfies a given condition



Find C the centre of the circle and join OC

Bisect OC at A and join AR and CP

∵ R and A are the middle points of OP and OC

∴ RA is parallel to and half of PC.

Similarly SA is half of QC

But PC and QC are the radii of the given circle and the point A is fixed therefore the locus of R and S is a circle of which the centre is A and radius is half the radius of the given circle.

1903 —MORNING.

Head Examiner—Babu Gouri Sankar Dey, M A.

1 (*a*) Simplify

$$\frac{67\times 67\times 67-001}{\cdot67\times 67+\cdot067+01}+\frac{57}{1+\frac{1}{3\frac{1}{14}}}$$

(*b*) What decimal of a mile is a yard?

2 (*a*) What is meant by the aliquot part of a number? Is an acre an aliquot part of a square mile?

(*b*) Find by Practice or otherwise the price of 25 tons 12 cwt. 3 qrs 17½ lb at 6*l* 13*s*. 4*d*. per ton.

3 Three taps *A*, *B*, and *C* can fill a cistern in 5, 6 and 7½ minutes respectively They are all turned on at once; but after one minute, *A* is turned off How much longer will *B* and *C* take to fill the cistern?

4 (*a*) Define the square root of a number.

(*b*) Extract the square root of $10\frac{1}{36}$; and of $\frac{22}{7}$ to four places of decimals

5 A man buys wine at 5*s* a gallon; he mixes it with water and by selling the mixture at 4*s* a gallon gains 12½ per cent. on his outlay How much water did each gallon of the mixture contain?

6. (*a*) Define *Present Worth.*

(*b*) A tradesman marks his goods with two prices, one for ready money and the other for 3 months' credit, allowing interest at 4½ per cent per annum If the credit price be marked at Rs. 50 9as, what ought to be the cash price?

7. (*a*) Prove that $(ac+bd)^2-(ad+bc)^2=(a^2-b^2)(c^2-d^2)$.

(*b*) Divide $8a^3-b^3-27c^3-18abc$ by $2a-b-3c$.

8. Simplify

$$\frac{(x+1)^2}{(x-y)(x-z)}+\frac{(y+1)^2}{(y-z)(y-x)}+\frac{(z+1)^2}{(z-x)(z-y)}.$$

9 (*a*) Prove that $a^m \times a^n = a^{m+n}$, where m and n are positive integers.

(*b*) Simplify

$$\left(\frac{x^m}{x^n}\right)^m \times \left(\frac{x^n}{x^m}\right)^n \div \{(x^m)^n \times (x^n)^m\} \times \{(x^n)^m \times (x^m)^n\}$$

10 Solve the following equations —

(1) $$\frac{x}{x+a-b}+\frac{x}{x+b-c}=2,$$

(2) $$\left.\begin{aligned} \frac{x}{2}+\frac{y}{3}+\frac{z}{4}&=23 \\ \frac{x}{3}+\frac{y}{4}+\frac{z}{2}&=28, \\ \frac{x}{4}+\frac{y}{2}+\frac{z}{3}&=27 \end{aligned}\right\}$$

11 A general wishing to draw up his regiment in the form of a hollow square found that he had 50 men over when it was 4 deep, but that he wanted 50 men to complete it when it was 5 deep, the number of men in the front being the same in both cases Find the number of men in the regiment

12 If $x \div (b+c)=y \div (c+a)=z \div (a+b)$, prove that

$$a \div (y+z-x)=b \div (z+x-y)=c \div (x+y-z).$$

1903.—AFTERNOON.

Head Examiner—Babu Gourisankar De, M A

1 (*a*) Enunciate and prove Euc I 4

(*b*) Write out the axioms which are used in the proof of this proposition.

2 Describe a parallelogram that shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

3 If a straight line is divided into any two parts, the sum of the squares on the whole line and on one of the parts is equal to twice the rectangle contained by the whole and that part, together with the square on the other part

Write down the corresponding algebraical formula

4 (*a*) The angle at the centre of a circle is double of an angle at the circumference, standing on the same arc

(*b*) Prove immediately from this proposition that the angle in a semicircle is a right angle.

5 If two chords of a circle cut one another, the rectangle contained by the segments of one shall be equal to the rectangle contained by the segments of the other

State and prove the converse of this.

6 Circumscribe a circle about a given triangle

AB, AC are two straight lines given in position, BC is a straight line of given length, D, E are the middle points of AB, AC, DF, EF are drawn at right angles to AB, AC respectively. Prove that the locus of F is an arc of a circle of which A is the centre

7. In any triangle, the angle contained by the bisector of the vertical angle and the perpendicular from the vertex to the base is equal to half the difference of the angles at the base.

8 The sum of the squares on the sides of any triangle is equal to three times the sum of the squares of the distances of the angular points from the point of intersection of the medians

SOLUTIONS

1903.—MORNING

1 (a) $\text{Ex} = \dfrac{\cdot300763 - \cdot001}{4489 + 067 + 06} + \dfrac{\frac{57}{100}}{1 + \dfrac{1}{\frac{43}{14}}} = \dfrac{299763}{5259} + \dfrac{\frac{57}{100}}{1 + \frac{14}{43}}$

$= \frac{299763}{1000000} \times \frac{10000}{5259} + \frac{57}{100} \times \frac{43}{57}$

$= \frac{57}{100} + \frac{43}{100} = \frac{100}{100} = 1$ Ans.

(b) 1 mile = 1760 yds

$$
\begin{array}{r|l}
1760 \overline{)\,1\,0000\,(} & \cdot0005681 \text{ Ans} \\
8800 & \\
\hline
12000 & \\
10560 & \\
\hline
14400 & \\
14080 & \\
\hline
3200 & \\
1760 & \\
\hline
1440 &
\end{array}
$$

2 (a) See Ques 1901, 2 (a)

1 sq. mile = 1760 × 1760 sq. yds

1 acre = 4840 sq yds

$\dfrac{1760 \times 1760}{4840} = \dfrac{11 \times 16 \times 10 \times 11 \times 4 \times 4 \times 10}{10 \times 11 \times 11 \times 4} = 64$

∴ an acre is an aliquot part of a square mile

7. $(a^{\frac{1}{5}} - b^{\frac{1}{7}} + c^{\frac{1}{3}})^2 = \{(a^{\frac{1}{5}} - b^{\frac{1}{7}}) + c^{\frac{1}{3}}\}^2 = (a^{\frac{1}{5}} - b^{\frac{1}{7}})^2$
$+ (c^{\frac{1}{3}})^2 + 2c^{\frac{1}{3}}(a^{\frac{1}{5}} - b^{\frac{1}{7}})$

$= a^{\frac{2}{5}} + b^{\frac{2}{7}} - 2a^{\frac{1}{5}}b^{\frac{1}{7}} + c^{\frac{2}{3}} + 2a^{\frac{1}{5}}c^{\frac{1}{3}} - 2c^{\frac{1}{3}}b^{\frac{1}{7}}$

(a) $a^2 + 2ab + b^2$) 1 ($a^{-2} - 2a^{-3}b + 3a^{-4}b^2$ &c. $=$ Quot
since $a^0 = 1$ $\quad 1 + 2a^{-1}b + a^{-2}b^2$

$-2a^{-1}b - a^{-2}b^2$
$-2a^{-1}b - 4a^{-2}b^2 - 2a^{-3}b^3$

$3a^{-2}b^2 + 2a^{-3}b^3$

8 Since $\dfrac{a}{b} = \dfrac{c}{d}$ $\quad\therefore \dfrac{a}{b} \pm 1 = \dfrac{c}{d} \pm 1$

$\therefore \dfrac{a \pm b}{b} = \dfrac{c \pm d}{d}$ $\quad\therefore \dfrac{a \pm b}{b} - \dfrac{a}{b} = \dfrac{c \pm d}{d} + \dfrac{c}{d}$

or $\dfrac{a \pm b}{a} = \dfrac{c \pm d}{c}$. $\quad\therefore a \pm b \quad a \quad c \pm d \quad c$

9 (1) $2x + 11 = 7x - 14$ $\quad\therefore 7x - 2x = 11 + 14$
$\therefore 5x = 25$ $\quad\therefore x = 5$

(2) $\sqrt{(x+9)} = 1 + \sqrt{x}$ $\quad x + 9 = 1 + x + 2\sqrt{x}$
$\therefore 2\sqrt{x} = 8$ $\quad\therefore \sqrt{x} = 4$ $\quad\therefore x = 16$

(3) $\dfrac{a-b}{x-c} = \dfrac{a+b}{x+2c}$ $\quad\therefore (a-b)(x+2c) = (a+b)(x-c)$
or $ax + 2ac - bx - 2bc = ax - ac + bx - bc$
$\therefore 2bx = -bc + 3ac$ $\quad\therefore x = \dfrac{c(3a-b)}{2b}$

10 Let x = numerator and y = denominator

Then $\dfrac{x+1}{y} = 1$ (1) $\quad$ (1) From (1) $x + 1 = y$ $\quad\therefore x = y - 1$

and $\dfrac{x}{y+1} = \frac{1}{2}$ (2) $\quad$ (2) $x = \frac{1}{2}(y+1)$

$\therefore y - 1 = \frac{1}{2}y + \frac{1}{2}$ $\quad\therefore y = 3$
and $x = 3 - 1 = 2$ $\quad\therefore$ the fraction $= \frac{2}{3}$

1862.—AFTERNOON.

1 Euc I Def 37
(a) The straight line AB is said to be cut externally in the point C when C is taken in AB produced

2 Euclid I 6

(b)

	£	s.	d	
	6	13	4	price of 1 ton
			5	
	33	6	8	price of 5 tons
			5	
	166	13	4	price of 25 tons
10 cwt = $\frac{1}{2}$ of 1 ton	3	6	8	,, ,, 10 cwt
2 cwt. = $\frac{1}{5}$ of 10 cwt	0	13	4	,, ,, 2 cwt
2 qrs. = $\frac{1}{4}$ of 2 cwt	0	3	4	,, ,, 2 qrs
1 qr. = $\frac{1}{2}$ of 2 qrs	0	1	8	,, ,, 1 qr.
14 lbs. = $\frac{1}{2}$ of 1 qr.	0	0	10	,, ,, 14 lbs
$3\frac{1}{2}$ lbs = $\frac{1}{4}$ of 14 lbs.	0	0	$2\frac{1}{2}$	,, ,, $3\frac{1}{2}$ lbs
	£170	19	$4\frac{1}{2}$	,, ,, 25 tons 12 cwt 3 qrs $17\frac{1}{2}$ lbs.

3. In one minute A can fill $\frac{1}{5}$ of the cistern
.. .. B . . ... $\frac{1}{6}$ ....
... . C ... $\frac{2}{15}$ . ...
... . . . A + B & C ... $\frac{1}{5}+\frac{1}{6}+\frac{2}{15}=\frac{6+5+4}{30}=\frac{15}{30}$
$=\frac{1}{2}$ of the cistern
. ...B & C . ...$\frac{1}{6}+\frac{2}{15}=\frac{5+4}{30}=\frac{9}{30}=\frac{3}{10}$ of the cistern
$1-\frac{1}{2}=\frac{1}{2}$ of the cistern to be filled by B and C
∴ Time required $=\frac{1}{2}\div\frac{3}{10}=\frac{1}{2}\times\frac{10}{3}=\frac{5}{3}=1\frac{2}{3}$ minutes Ans

4. (a) Square root of a number is such a number that it being multiplied by itself gives the number.

$\sqrt{10\frac{1}{36}}=\sqrt{\frac{361}{36}}=\frac{19}{6}=3\frac{1}{6}$.

$\frac{22}{7}=3{\cdot}142857\dot{1}4...$.

```
      | 3 14285714 , 1 7728...Ans.
      | 1
      |-----------
   27 | 214
      | 189
      |-----------
  347 |  2528
      |  2429
      |-----------
 3542 |    9957
      |    7084
      |-----------
35448 |    287314
      |    283584
```

5 $100 : 5 :: 112\frac{1}{2} : x$

$$x = \frac{5 \times 225}{100 \times 2} = \frac{45}{8}$$

$\frac{45}{8} \div 4 = \frac{45}{32}$ gallons mixture

$\frac{45}{32} - 1 = \frac{13}{32}$ gallon of water to be mixed with one gallon of wine

$\frac{45}{32} : 1 :: \frac{13}{32} : x$

$$x = \frac{13 \times 32}{45 \times 32} = \frac{13}{45} \text{ gallon. Ans}$$

6 (a) Present worth of a sum of money is such a sum which with its interest is equal to the given sum.

(b) $4\frac{1}{2} \times \frac{3}{12} = \frac{9}{2} \times \frac{3}{12} = \frac{9}{8}$

$100 + \frac{9}{8} : 50\frac{9}{16} :: 100 :$ cash price

$$\text{cash price} = \text{Rs} \frac{809 \times 8 \times 100}{809 \times 16} = \text{Rs } 50.$$

7. (a) $(ac+bd)^2 - (ad+bc)^2$
$= a^2c^2 + b^2d^2 + 2abcd - a^2d^2 - b^2c^2 - 2abcd$
$= a^2c^2 - a^2d^2 + b^2d^2 - b^2c^2$
$= a^2(c^2 - d^2) - b^2(c^2 - d^2) = (a^2 - b^2)(c^2 - d^2)$

(b)
$$
\begin{array}{l|l|l}
2a - b - 3c & 8a^3 - 18abc - b^3 - 27c^3 & 4a^2 + 2ab + 6ac + b^2 - 3bc + 9c^2 \text{ Ans.} \\
 & \underline{8a^3 - 4a^2b - 12a^2c} & \\
 & 4a^2b + 12a^2c - 18abc & \\
 & \underline{4a^2b - 2ab^2 - 6abc} & \\
 & 12a^2c + 2ab^2 - 12abc & \\
 & \underline{12a^2c \quad - 6abc - 18ac^2} & \\
 & 2ab^2 - 6abc + 18ac^2 - b^3 - 27c^3 & \\
 & \underline{2ab^2 \quad - b^3 - 3b^2c} & \\
 & -6abc + 18ac^2 + 3b^2c - 27c^3 & \\
 & \underline{-6abc \quad + 3b^2c + 9bc^2} & \\
 & 18ac^2 - 9bc^2 - 27c^3 & \\
 & \underline{18ac^2 - 9bc^2 - 27c^3} & \\
\end{array}
$$

8 $$\text{Ex.} = \frac{x^2+2x+1}{(x-y)(x-z)} - \frac{y^2+2y+1}{(y-z)(x-y)} + \frac{z^2+2z+1}{(x-z)(y-z)}$$

L. C. M. of the Denrs $= (x-y)(x-z)(y-z)$

Numer $= (y-z)\{x^2+2x+1\} - (x-z)\{y^2+2y+1\} + (x-y)\{z^2+2z+1\}$
$= x^2(y-z) - y^2(x-z) + z^2(x-y)$
$= x^2(y-z) - xy^2 + y^2z + xz^2 - yz^2$
$= x^2(y-z) - x(y^2 - z^2) + yz(y-z)$
$= (y-z)\{x^2 - xy - xz + yz\}$
$= (y-z)\{x(x-y) - z(x-y)\}$
$= (y-z)(x-z)(x-y)$

$\therefore$ Ex $=\frac{(y-z)(x-z)(x-y)}{(x-y)(x-z)(y-z)}=1.$ Ans

9 (a) $a^m=a\times a\times a\times a\times\ldots\ldots m$ factors

$a^n=a\times a\times a\times a\times$ n factors

$\therefore$ $a^m\times a^n=a\times a\times a\times a\times\ldots$ $(m+n)$ factors

$\therefore$ $a^m\times a^n=a^{m+n}$

(b) Ex. $=(x^{m-n})^m\times(x^{n-m})^n\div\{x^{m^2}\times x^{n^2}\}\times\{x^{nm}\times x^{mn}\}$

$=x^{m^2-nm}\times x^{n^2-nm}\div x^{m^2+n^2}\times x^{2nm}$

$=x^{m^2-2nm+n^2-m^2-n^2+2nm}$

$=x^0=1$ Ans.

10. (1) or $\frac{x}{x+a-b}-1=1-\frac{x}{x+b-c}$ or $\frac{b-a}{x+a-b}=\frac{b-c}{x+b+c}$

or $x(b-a)+b^2+bc-ab-ac=x(b-c)+ab-b^2-ac-bc$

or $x(c-a)=-2b^2+2bc+2ab-2ac$

$\therefore$ $x=\frac{-2(b^2-bc-ab+ac)}{-(a-c)}=\frac{2(b-a)(b-c)}{a-c}$ Ans.

(2) $x+\frac{2y}{3}+\frac{z}{2}=46$ (1) $\times$ 2

$x+\frac{3y}{4}+\frac{3z}{2}=84$ (2) $\times$ 3

$\frac{y}{12}+z=38$ (4)

again $x+2y+\frac{4z}{3}=108$ (3) $\times$ 4

$x+\frac{2y}{3}+\frac{z}{2}=46$ (1) $\times$ 2

$\frac{4y}{3}+\frac{5z}{6}=62$ (5)

$\frac{4y}{3}+16z=608$ (4) $\times$ 16

$\frac{4y}{3}+\frac{5z}{6}=62$ (5)

$\frac{91}{6}z=546$ $\therefore$ $z=546\times\frac{6}{91}=36$

$\frac{y}{12}+z=38$ (4)

or $\frac{y}{12}=38-36=2$ $\therefore y=24$

$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=23$ (1)

or $\frac{x}{2}+8+9=23$

or $\frac{x}{2}=6$ $\therefore x=12$

$$\therefore \left.\begin{array}{r} x=12 \\ y=24 \\ z=36 \end{array}\right\} \text{ Ans.}$$

11. Let x be the no of men in front of the squares

$\therefore$ no. of men in the 1st hollow square $x^2-(x-8)^2$ i e $16x-64$

.... 2nd . ... $x^2-(x-10)$ i.e. $20x-100$

Then by the question

$16x-64+50=20x-100-50$

or $4x=-64+50+100+50$ or $4x=136$ $\therefore x=34$

$\therefore$ no of men in the regiment $=16x-64+50=16\times 34-14$

$=544-14=530$

12 $\frac{x}{b+c}=\frac{y}{c+a}=\frac{z}{a+b}$

$\therefore$ each of them $=\frac{y+z-x}{(c+a)+(a+b)-(b+c)}=\frac{y+z-x}{2a}$

each of them $=\frac{(z+x-y)}{(c+a)+(a+b)-(c+a)}=\frac{z+x-y}{2a}$

each of them $=\frac{x+y-z}{(b+c)+(c+a)-(a+b)}=\frac{x+y-z}{2c}$

$\therefore \frac{y+z-x}{2a}=\frac{z+x-y}{2b}=\frac{x+y-z}{2c}$

or $\frac{y+z-x}{a}=\frac{z+x-y}{b}=\frac{x+y-z}{c}$

or $\frac{a}{y+z-x}=\frac{b}{z+x-y}=\frac{c}{x+y-z}$

or $a\ (y+z-x)=b\ (z+x-y)=c\ (x+y-z)$.

1903.—AFTERNOON.

1. (a) See Euclid (b) 8th and 10th axioms.
2. Euc. Bk I 42
3 Euc. Bk. II. 7. $(a-b)^2=a^2+b^2-2ab$

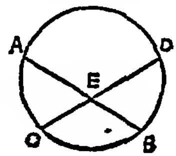
4 (*a*) Euc Bk III 20

(*b*) Because the angle at the centre is double of the angle at the circumference

$\therefore$ The angle at the circumference, *i e*, the angle in the semicircle is half of two rt angles, *i e*, a rt angle

5 III 35

If two straight lines intersect and if the rectangle contained by the segments of one be equal to the rectangle contained by the segments of the other, then the extremities of these lines are concyclic



Let AB and CD intersect at E

and let AE EB$=$CE ED

Then A, C, B, D, shall be concyclic

Describe a circle passing through A, C, and D

It will pass through B For if not let it cut AB at F

Then CE ED$=$AE EF (III 35)

But CE ED$=$AE EB (Hyp)

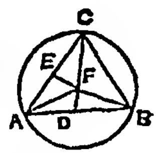
$\therefore$ AE EF$=$AE EB (Ax)

$\therefore$ EF$=$EB the part equal to the whole which is impossible

$\therefore$ the circle will pass through B

$\therefore$ A, C, B and D are concyclic.

6 Euc Bk IV. 5



Describe a circle passing through B, A, and C

Then because BC is constant , the angle BAC is constant $\because$ The segment BAC is constant $\therefore$ the other segment is constant

$\therefore$ The circle BAC is constant

and because F is the centre of this circle (IV 5)

$\therefore$ FA is the radius of this circle $\therefore$ FA is constant

$\therefore$ Locus of F is an arc of a circle of which A is the centre and AF the radius

7 Let AF be the bisector of the angle BAC and AD the perpendicular from A on BC

Then shall $\angle$ DAE be equal to half the difference of the angles ABC and ACB

Proof —$\because$ $\angle$ABD$+\angle$ BAD$=$1 rt. angle (I 32.)

and $\angle$ACD$+\angle$CDA$=$1 rt angle (I 32)

$\therefore$ $\angle$ABD$+\angle$BAD$=\angle$ACD$+\angle$CAD to each add the angle DAE

$\therefore$ $\angle$ABD$+\angle$BAD$+\angle$DAE$=\angle$ACD$+\angle$CAE$+\angle$DAE$+$DAE

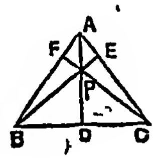
But $\angle$BAD$+\angle$CAE$=\angle$BAE$=\angle$EAC (Hyp)

$\therefore$ $\angle$ABD$=\angle$ACD$+2\angle$DAE

$\therefore$ $2\angle$DAE$=\angle$ABD$-\angle$ACD

$\therefore$ $\angle$DAE is half the difference of $\angle$ABD and $\angle$ACD

8 Let ABC be a triangle and AD, BE and CF be the medians intersecting at G



It is a well known theorem that AG$=$2GD also the square on a straight line is 4 time the square on half the line

It is also a well known theorem that the sum of the squares on two sides of a triangle is equal tot wice the

square on half the third side together with twice the square on the median which bisects the third side

Now $AG^2+GB^2=2AF^2+2FG^2$ $\therefore 2AG^2+2GB^2=4AF^2+4EG^2$

$\therefore 2GA^2+2GB^2=AB^2+GC^2$

Similarly $2GB^2+2GC^2=BC^2+GA^2$

$2GC^2+2GA^2=AC^2+GB^2$

Adding $4GA^2+4GB^2+4GC^2=AB^2+BC^2+AC^2+GA^2+GB^2+CB.$

$\therefore 3GA^2+3GB^2+3GC^2=AB^2+BC^2+AC^2.$ Q.E.D.

1904.—MORNING.

Head Examiner,—BABU GOURI SANKAR DEY, M A

1 Define the G C M and the L C M of two or more numbers

Find the greatest number of six digits which is exactly divisible by 27, 45, 60, 72 and 96

2 Write down the local value of each of the figures in the number 0·0203

Simplify

$$\frac{(\cdot 0')^2+\cdot 0')^2+(\cdot 03)^2}{(\cdot 001+\cdot 002+\cdot 003)^2}-\cdot 0208\dot{3}-\frac{2l\ 3s}{25l\ 16}\text{ of }\frac{1}{2}.$$

3 A can do a piece of work in 25 days, B in 50 days, and C in 24 days They three work together for 2 days, and then A and B leave; but C continues, and after $8_{\frac{?}{}}$ days is rejoined by A, who brings D along with him, and these three finish the remainder of the work in 3 days. In what time would D alone have done the whole work?

4 The area of a square cricket field is 9 ac. 3 ro 8 16 po.; find the length of a side.

5 Define *Discount*

The difference between the interest and the discount on a certain sum for 3 years 4 months at 5 per cent per annum is 16*l* 13*s* 4*d* Find the sum.

6 A person invests a certain sum in the 3½ per cent Government Securities when they are at 97¼; had he waited till they had fallen to 97⅛, he would have had Rs 400 more of Government Securities How much money did he invest, ⅛ per cent. being charged as brokerage in both cases?

7. Simplify

(1) $(2x+3y)^3-3(2x+3y)^2(2x-3y)$
$$+3(2x+3y)(2x-3y)^2-(2x-3y)^3;$$

(2) $\left(\frac{x^l}{x^m}\right)^{l^2+lm+m^2}\times\left(\frac{x^m}{x^n}\right)^{m^2+mn+n^2}+\left(\frac{x^n}{x^l}\right)^{n^2+nl+l^2}.$

8. Reduce $\dfrac{3x^5-5x^3+2}{2x^5-5x^4+3}$ to its lowest terms.

9. Show that

$$(x-1)(x-2)(x-3)(x-4)+1$$

is a perfect square.

10. (a) Distinguish between an *equation* and an *identity*, and give an example of each.

(b) Solve the following equations:—

$$(1)\quad \frac{x(a+b)+c}{x+d}+\frac{x(a-b)+d}{x+c}=2a,$$

$$(2)\quad \begin{aligned} &x+y+z=0,\\ &ax+by+cz=0,\\ &a^2x+b^2y+c^2z+(b-c)(c-a)(a-b)=0. \end{aligned}$$

11. At 7-40 a.m. an ordinary train starts from P, and reaches Q at 11-40 a.m.; an express train which starts from Q at 9 a.m. arrives at P at 11-40 a.m.; if both trains travel at a uniform speed without stopping, find the time when they meet.

12. If $a \quad b \quad x \quad y$, then $a^2+b^2 \quad \dfrac{a^3}{a+b} \quad x^2+y^2 \quad \dfrac{x^3}{x+y}$.

1904.—AFTERNOON

Head Examiner,—Babu Gauri Sankar Dey, M.A.

1. If, at a point in a straight line, two other straight lines on opposite sides of it make the adjacent angles together equal to two right angles, then these two straight lines shall be in one and the same straight line.

2. In a right angled triangle the square described on the hypotenuse is equal to the sum of the squares described on the other two sides.

If from any point P within a triangle ABC, perpendiculars PX, PY, PZ are dropped on the sides BC, CA, AB respectively, then

$$BX^2+CY^2+AZ^2=XC^2+YA^2+ZB^2$$

3. (a) If a straight line is divided equally and also unequally, the sum of the squares on the two unequal parts is twice the sum of the squares on half the line and on the line between the points of section.

(b) Divide a given straight line into two parts such that the sum of the squares on the two parts may be the least possible.

4. (a) The straight line drawn from the centre of a circle to the point of contact of a tangent is perpendicular to the tangent.

(b) In two concentric circles, any chord of the outer circle which touches the inner is bisected at the point of contact.

5. Similar segments of circles on equal chords are equal to one another

6 Inscribe a regular pentagon in a given circle

7. Construct a triangle, having given the base, one of the angles at the base, and the difference of the remaining sides

8 The sum of the squares on the sides of a parallelogram is equal to the sum of the squares on the diagonals.

9 AB is a fixed diameter of a circle, and AP any chord through A; PQ is drawn parallel to AB, and of a given length prove that the locus of Q is a circle, and find its centre and radius

SOLUTIONS

1904 —MORNING.

1 The G C M of two or more numbers is the greatest number which divides each of them exactly, *i e*, without any remainder

The L C M of two or more numbers is the least number which is exactly divisible by each of the numbers.

First, we are to find the L C M of the given numbers

$$\begin{array}{r|l} 2 & 27,\ 45,\ 60,\ 72,\ 96 \\ \hline 2 & 27,\ 45,\ 30,\ 36,\ 48 \\ \hline 2 & 27,\ 45,\ 15,\ 18,\ 24 \\ \hline 3 & 27,\ 45,\ 15,\ 9,\ 12 \\ \hline 3 & 9,\ 15,\ 5,\ 3,\ 4 \\ \hline 5 & 3,\ 5,\ 5,\ 1,\ 4 \\ \hline & 3,\ 1,\ 1,\ 1,\ 4 \end{array}$$

L C M $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 3 \times 4 = 4320$

Now, the greatest number of six digits is 999999

Divide this number by the L C M. found

$$\begin{array}{r} 4320)\ 999999\ (231 \\ 8640 \\ \hline 13599 \\ 12960 \\ \hline 6399 \\ 4320 \\ \hline 2079 \end{array}$$

$\therefore$ The greatest number reqd is $999999 - 2079 = 997920$ Ans.

2 In ·010203, the local value of 1 is $\frac{1}{10^2}$ or $\frac{1}{100}$

" " 2 is $\frac{2}{10^4}$ or $\frac{2}{10000}$

" " 3 is $\frac{3}{10^6}$ or $\frac{3}{1000000}$

(a) $$\frac{(\cdot01)^3+(\cdot02)^3+(\cdot03)^3}{(\cdot001+\cdot002+\cdot003)^2}=\frac{\cdot000001+\cdot000008+\cdot000027}{\cdot006\times\cdot006}$$

$$=\frac{\cdot000036}{\cdot000036}=1$$

$$\cdot02083+\frac{£2\ 3s}{£25\ 16s}\text{ of }\tfrac{1}{2}=\frac{2083-208}{90000}\times\frac{516s}{43s}\times 2$$

$$=\frac{\overset{25}{\cancel{1875}}}{\underset{\underset{\underset{\underset{2}{\cancel{4}}}{\cancel{300}}}{\cancel{7500}}}{\cancel{90000}}}\times\frac{\overset{12}{\cancel{516}}}{\cancel{43}}\times\cancel{2}=\frac{1}{2}$$

∴ The whole expression $=1-\frac{1}{2}=\frac{1}{2}$ or ·5

3 From the question, it is evident that the work is done by A, working $2+3$ or 5 days, B working 2 days, C, working $2+8\frac{3}{5}+3$, or $13\frac{3}{5}$ days and B working for 3 days only

Now, A can do the whole in 25 days

∴ in 5 days he does $\frac{5}{25}$ths or $\frac{1}{5}$th of the work

B can do the whole in 20 days

∴ B does $\frac{2}{20}$ or $\frac{1}{10}$th in 2 days

C can do the whole in 24 days

∴ C does $\frac{13\frac{3}{5}}{24}$ or $\frac{68}{120}$ in $13\frac{3}{5}$ days

∴ The portion of the work done by D in 3 days

$$=1-\left(\tfrac{1}{5}+\tfrac{1}{10}+\tfrac{68}{120}\right)=1-\frac{24+12+68}{120}=1-\tfrac{104}{120}=\tfrac{16}{120}.$$

∴ D can do $\frac{1}{3}$ of $\frac{16}{120}$ of the work in 1 day

∴ He would alone have done the work

in $\left(\frac{3\times120}{16}\text{ or }\frac{45}{2}\right)$ days i.e. $22\frac{1}{2}$ days.

4 Area $=$ 9 ac 3 ro 8.16 po

$= 39$ ro 8.16 po

$= (39 \times 40 + 8.16)$ sq po

$=$ 1568.16 sq po

$\therefore$ Length of a side $= \sqrt{(1568.16 \text{ sq po})}$

$= 39.6$ po

$$\begin{array}{r|l} 15\dot{6}8.16 & 39.6 \quad \text{Ans} \\ 9 & \\ \hline 69 \quad 668 & \\ 621 & \\ \hline 786 \quad 4716 & \\ 4716 & \\ \hline \end{array}$$

5 Discount is the abatement made in consideration of payment of a debt before it is due; or, in other words, it is the Interest of the Present worth.

The interest on £100 for 3 years 4 months at 5 per cent

$= £5 \times 3\frac{1}{3} = £16\frac{2}{3}$

$\therefore$ The discount on $£116\frac{2}{3}$ due 3 years 4 months hence $= £16\frac{2}{3}$

$\therefore$ The discount on $£100 = £\dfrac{16\frac{2}{3} \times 100}{116\frac{2}{3}} = £\dfrac{50 \times 100}{350} = £\frac{100}{7}$

$\therefore$ The difference between the Interest and Discount

$= £(\frac{50}{3} - \frac{100}{7}) = £\frac{50}{21}$

£16 13s 4d $= £16\frac{2}{3}$

$£\frac{50}{21}$: $£16\frac{2}{3}$ :: £100 : Reqd sum (x)

$\therefore x = £\dfrac{100 \times 50}{3} \times \frac{21}{50} = £700$

6 The total cost was $97\frac{3}{8} + \frac{1}{8} = 97\frac{1}{2}$ with the fall in price, the total cost would be $97\frac{1}{8} + \frac{1}{8} = 97\frac{1}{4}$

Suppose he had Rs $97\frac{1}{2} \times 97\frac{1}{4}$ to invest

Then, when the cost was $97\frac{1}{2}$ for Rs 100 stock, he could purchase Rs $97\frac{1}{4} \times 100$ Rs 9725 Govt. security and at the latter cost, he would have Rs $97\frac{1}{2} \times 100 =$ Rs 9750 Govt securities

$\therefore$ The difference is Rs 25

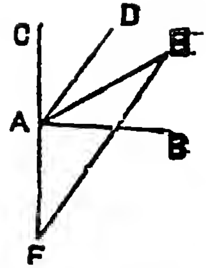
But the difference given is Rs 400

$\therefore$ He had Rs $97\frac{1}{2} \times 97\frac{1}{4} \times \frac{400}{25}$

or Rs $\dfrac{195 \times 389}{8} \times \frac{400}{25} =$ Rs $195 \times 389 \times 2$ invested

$=$ Rs 151710

3. Euclid I. 1st cor Prop 32.

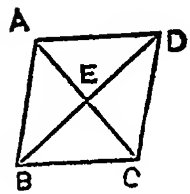


4. Let AC be at rt ∠s to AB
Bisect the ∠BAC by AD and the ∠DAB by AE
In CA prod take AF=AE.
Join FE Then AEF is the reqd △
∵ ∠CAD=45° and the ∠DAE=22½°,
∴ ∠CAE=67½°, and △ AEF is isosceles

5 Euclid II 13, 2nd case

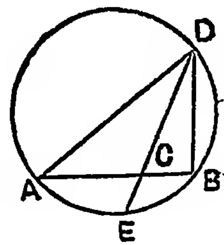
6. Bisect the ∠ BAC by AD meeting BC in D. From D draw DF perpendicular to BC, meeting AB in F.
∵ ∠ADF=∠DAC (I 29)=∠DAF (Cons.)
∴ AF=DF (I 6).

7. Euclid III. 23



8 Let ABCD be a rhombus The diagonals AC and BD bisect each other, and cut at rt ∠s in E (See Question 2 of 1st 1859)
∵ △ABD=½ BD AE (I 41)
and △BDC=½ BD CE
∴ △ABD+△BDC=½ BD (AE+CE)
or rhom ABCD=½ BD AC.

9 Euclid IV 16.



10 Bisect the arc AB in E
Join EC and prod. it to meet the circum in D
Join AD and DB.
∴ arc AE=arc BE (Cons)
∴ ∠ADE=BDE (III 26)
Therefore D is the required point so that the ∠ADB is bisected by CD.

1863.—MORNING.

Examiner,—H Scott Smith, B A

1 Find the value in vulgar and decimal fractions of —

$$\frac{15\frac{2}{3}+6-\frac{1}{4}}{7\frac{4}{9}\times\frac{3}{12}}$$

2. Find the fractional value of —
(2 37979 + 4 22)—(3 041— 937)

3 The weight of five casks of coffee being 31 cwt 3 qrs 13 lbs. calculate the price at 90 shillings per cwt

4 If a man can perform a journey of 170 miles in 4¼ days of 11 hours each, how many days of 8¾ hours will he perform a journey of 470 miles?

7. (1) Put $2x+3y=a$ and $2x-3y=b$.

Then the expression stands as $a^3-3a^2b+3ab^2-b^3$

$$=(a-b)^3=\{(2x+3y)-(2x-3y)\}^3$$
$$=(6y)^3=216y^3.$$

(2) $\left(\dfrac{x^l}{x^m}\right)^{l^2+lm+m^2}=\left(x^{l-m}\right)^{l^2+lm+m^2}=x^{l^3-m^3}$

Similarly, the 2nd $=x^{m^3-n^3}$ and the 3rd $=x^{n^3-l^3}$

$\therefore$ Product $=x^{l^3-m^3+m^3-n^3+n^3-l^3}$

$=x^0=1$

8 First, we are to find the H. C. F. of

$3x^5-5x^3+2$ and $2x^5-5x^2+3$.

$$3x^5-5x^3+2$$
$$2$$
$$2x^5-5x^2+3)\ 6x^5-10x^3+4\ (3$$
$$6x^5-15x^2+9$$
$$-10x^3+15x^2-5=-5(2x^3-3x^2+1)$$

Neglecting the factor -5, which is not common in the given expressions

$$2x^3-3x^2+1)\ 2x^5-5x^2+3\ (x^2$$
$$2x^5-3x^4+x^2$$
$$3x^4-6x^2+3$$
$$2$$
$$6x^4-12x^2+6\ (3x$$
$$6x^4-9x^3+3x$$
$$9x^3-12x^2-3x+6$$
$$2$$
$$18x^3-24x^2-6x+12\ (9$$
$$18x^3-27x^2\qquad+9$$
$$3x^2-6x+3=3(x^2-2x+1)$$
$$x^2-2x+1)\ 2x^3-3x^2+1\ (2x+1$$
$$2x^3-4x^2+2x$$
$$x^2-2x+1$$
$$x^2-2x+1$$

$\therefore$ x^2-2x+1 is the H. C. F.

Now, divide both the expressions by the H C F

$$
\begin{array}{r|l|l}
x^2-2x+1 & 3x^5-5x^3+2 & 3x^3+6x^2+4x+2 \\
 & \underline{3x^5-6x^4+3x^3} & \\
 & 6x^4-8x^3+2 & \\
 & \underline{6x^4-12x^3+6x^2} & \\
 & 4x^3-6x^2+2 & \\
 & \underline{4x^3-8x^2+4x} & \\
 & 2x^2-4x+2 & \\
 & \underline{2x^2-4x+2} & \\
\end{array}
$$

$$
\begin{array}{r|l|l}
x^2-2x+1 & 2x^5-5x^2+3 & 2x^3+4x^2+6x+3 \\
 & \underline{2x^5-4x^4+2x^3} & \\
 & 4x^4-2x^3-5x^2+3 & \\
 & \underline{4x^4-8x^3+4x^2} & \\
 & 6x^3-9x^2+3 & \\
 & \underline{6x^3-12x^2+6x} & \\
 & 3x^2-6x+3 & \\
 & \underline{3x^2-6x+3} & \\
\end{array}
$$

∴ The expression reduced to its lowest terms

$$=\frac{3x^3+6x^2+4x+2}{2x^3+4x^2+6x+3}.\quad \text{Ans}$$

9
$$
\begin{aligned}
&(x-1)(x-2)(x-3)(x-4)+1 \\
&=(x-1)(x-4)(x-2)(x-3)+1 \\
&=(x^2-5x+4)(x^2-5x+6)+1 \\
&=y(y+2)+1 \qquad \text{putting } x^2-5x+4=y \\
&=y^2+2y+1 \\
&=(y+1)^2\text{, a perfect square}
\end{aligned}
$$

The sq root is $y+1$ or $x^2-5x+4+1$, i e x^2-5x+5

10 (a) An equality which holds good for all the values of the letters involved in them is an identical equation, or more simply, an identity, as $(x+a)(x-a)=x^2-a^2$.

But an equality which holds good for mere particular value or values of one or more letters involved in them is an equation of condition, or more simply, an equation, as $x+2=6$.

(b) 1) $\dfrac{x(a+b)+c}{x+d}+\dfrac{x(a-b)+d}{x+c}=2a$

$$\text{or,}\quad \frac{x(a+b)+c}{x+d}-a+\frac{x(a-b)+d}{x+c}-a=0$$

$$\text{or,}\quad \frac{ax+bx+c-ax-ad}{x+d}+\frac{ax-bx+d-ax-ac}{x+c}=0$$

$$\therefore\ (bx+c-ad)(x+c)-(bx+ac-d)(x+d)=0$$

$$\text{i.e.}\ bx^2+(c-ad)x+bcx+c(c-ad)-bx^2-(ac-d)x-bdx-d(ac-d)=0$$

$$x\{-a(c+d)+(c+d)+b(c-d)\}=-(c^2+d^2-2acd)$$

$$\therefore\ x=\frac{2acd-c^2-d^2}{(c+d)(1-a)+b(c-d)}$$

(2) $ax+by+cz=0$
$x+y+z=0$

By the method of cross-multiplication, we get

$$\frac{x}{b-c}=\frac{y}{c-a}=\frac{z}{a-b}=\lambda\ \text{(suppose)}$$

$\therefore\ x=\lambda(b-c),\ y=\lambda(c-a)$ and $z=\lambda(a-b)$

Substituting these in the 3rd equation we get

$$\lambda\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}+(b-c)(c-a)(a-b)=0$$

$$\therefore\ \lambda=\frac{-(b-c)(c-a)(a-b)}{a^2(b-c)+b^2(c-a)+c^2(a-b)}=1$$

$\therefore\ x=b-c,\ y=c-a$ and $z=a-b$

P——————————————Q

11 Let a miles be the distance from P to Q Then the ordinary train runs with the velocity of $\frac{a}{4}$ miles per hour

$\because$ It takes $11.40-7.40=4$ hrs in running the whole distance.

The express train runs the whole distance in $2\frac{2}{3}$ hrs

$\therefore$ Its rate is $\left(\frac{a}{2\frac{2}{3}}=\frac{3a}{8}\right)$ miles per hour

Let x be the number of hours after 9 A M when they met, then the 1st train runs over $(1\frac{1}{3}+x)\times\frac{a}{4}$ miles when they meet and the 2nd $x\times\frac{3a}{8}$ miles

we get the equation

$$(\tfrac{4}{3}+x)\frac{a}{4}+\frac{3a}{8}x=a$$

or $\frac{1}{3}+\frac{x}{4}+\frac{3x}{8}=1$ i e $\frac{5}{8}x=\frac{2}{3}$

$\therefore x=\frac{16}{15}$ hrs or 1 hr 4 min

$\therefore$ The trains meet at 10 4' A M

12 $\frac{a}{b}=\frac{x}{y}=k$ (say)

$\therefore a=bk, \quad x=yk$

Then, $a^2+b^2 \div \frac{a^3}{a+b}=\frac{(a^2+b^2)(a+b)}{a^3}=\frac{(b^2k^2+b^2)(bk+b)}{b^3k^3}$

$$=\frac{b^3(k^2+1)(k+1)}{b^3k^3}=\frac{(k^2+1)(k+1)}{k^3}$$

Again, $x^2+y^2 \div \frac{x^3}{x+y}=\frac{(x^2+y^2)(x+y)}{x^3}=\frac{(y^2k^2+y^2)(yk+y)}{y^3k^3}$

$$=\frac{(k^2+1)(k+1)y^3}{y^3k^3}=\frac{(k^2+1)(k+1)}{k^3}$$

$\therefore a^2+b^2 \div \frac{a^3}{a+b}=x^2+y^2 \div \frac{x^3}{x+y}$

We can do this example in another way, thus —

$\frac{a}{b}=\frac{x}{y} \quad \therefore \frac{a+b}{b}=\frac{x+y}{y}$ (By componendo) (1)

$\therefore \frac{a^2+b^2}{b^2}=\frac{x^2+y^2}{y^2}$ (2)

Now multiplying (1) by (2) we get,

$$\frac{(a^2+b^2)(a+b)}{b^3}=\frac{(x^2+y^2)(x+y)}{y^3}$$

But $\frac{a^3}{b^3}=\frac{x^3}{y^3} \quad \therefore \frac{b^3}{a^3}=\frac{y^3}{x^3}$

Also, $\frac{a^3}{b^3}=\frac{x^3}{y^3} \quad \therefore \frac{(a^2+b^2)(a+b)}{a^3}=\frac{(x^2+y^2)(x+y)}{x^3}$

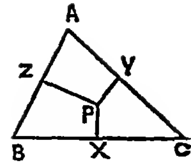
$\therefore a^2+b^2 \div \frac{a^3}{a+b}=x^2+y^2 \div \frac{x^3}{x+y}$

1904—AFTERNOON.

1 Proposition 14, Book I.

2. Proposition 47, Book I.

(a) Join AP BP & CP.



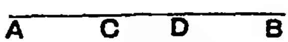
Then $\left.\begin{array}{l} BP^2=BX^2+PX^2 \\ CP^2=CY^2+PY^2 \\ \text{and } AP^2=AZ^2+PZ^2 \end{array}\right\}$ (I 47)

$\therefore BX^2+CY^2+AZ^2+(PX^2+PY^2+PZ^2)$
$=AP^2+BP^2+CP^2$

Again, $CP^2=CX^2+PX^2$
$AP^2=YA^2+PY^2$
$BP^2=ZB^2+PZ^2$

$\therefore AP^2+BP^2+CP^2=XC^2+YA^2+ZB^2+(PX^2+PY^2+PZ^2)$
$\therefore$ Taking away the common $PX^2+PY^2+PZ^2$
we get $BX^2+CY^2+AZ^2=XC^2+YA^2+ZB^2$

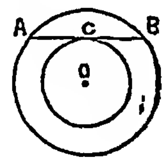
3 (a) Proposition 9, Book II



C is the mid-point of AB and D any other pt

Then $AD^2+BD^2=2(AC^2+CD^2)$
$\therefore AD^2+BD^2$ is the least possible when CD vanishes, *i e*, when D coincides with C
$\therefore$ When the straight line is divided into two equal parts, the sum of the squares of the two parts is the least possible.

4 (a) Proposition 18, Book III.



(b) O is the common centre and AB, a chord of the outer circle is a tangent of the inner one, touching it at C

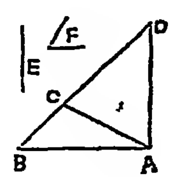
Join OC

Now, OC is at right angles to AB (III 18)
$\therefore$ AB is bisected at C (III 3)

5 Proposition 24 Book III.

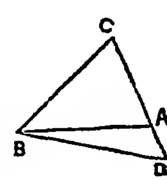
6 Proposition 11, Book III.

7. Let AB be the given base, E the difference of the remaining sides and F, one of the angles at the base



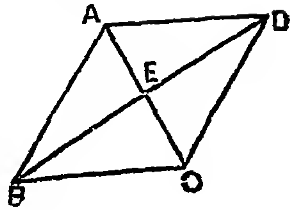
At B in AB, make the angle $ABC=\angle F$ (I 23) and make $BC=E$ (I 3)
Produce BC to D and at A in CA make the $\angle CAD=\angle ACD$
Then ABD is the required triangle.
$AD=CD$ (I. 6)
$\therefore BD-AD=BC=E$ and $\angle ABC=\angle F$

Q Fig



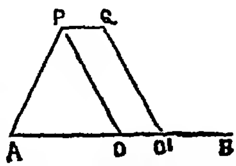
If the given angle is adjacent to the less side then at A make the angle BAD equal to the supplement of the given angle F and make $AD=E$
Join BD
$\angle ABD \angle \angle ADB$
$\because AD \angle AB$ (the difference of any two sides of a triangle being less than the 3rd sides)

Make the $\angle DBC = \angle ADB$
Produce DA to meet BC at C
Then ABC is the required triangle
$\because$ $BC = CD$ $\therefore BC - AC = AD$ and $\angle CAB = \angle F$
$\therefore$ [$\angle CAB + \angle BAD = 2$ rt angles (I 13)
and $\angle BAD + \angle F = 2$ rt angles (constr)]



8 BD and AC bisect each other at E
Also, $AB^2 + BC^2 = 2(AE^2 + BE^2)$ by the well known theorem proved in Hall and Stevens' Euclid, page 161 (Ed 1902)

$\therefore$ The sum of the squares on the sides which is equal to $2(AB^2 + BC^2) = 4(AE^2 + BE^2) = BD^2 + AC^2$ (II 4 Cor)



9 Bisect AB at O Then O is the centre of the given circle
Take $OO' = PQ$
Join OP and O'Q
Then $O'Q =$ and $\parallel$ to OP (I, 33)
$\therefore$ The locus of Q is a circle with O' as centre and $O'Q = OP$, the radius of the given circle as radius

So the centre of the locus of Q is on AB and at the distance o the given length PQ from the centre of the given circle and its radius is equal to the radius of the given circle

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1905 —MORNING

Head Examiner,—GOURI SANKAR DEY, M A.

1 When is one number said to be a *measure* of another? What is a *Prime Number*?

A man bought two heaps of mangoes, one for Rs 10 5 as, and the other for Rs 18 0 as 9 p If the price of each mango be the same, and not less than two and not more than three annas, find the total number of mangoes he bought

2 (1) What is the meaning of $\frac{2}{3}$ and of $\frac{1}{2}$ of $\frac{3}{5}$

(2) Simplify

$$(5\tfrac{5}{8} - 1\tfrac{5}{12}) \text{ of } \left(\frac{\frac{5}{6}}{3\frac{1}{3}} + \tfrac{7}{8} \text{ of } \tfrac{4}{5}\right) \div \tfrac{5}{7} \text{ of } \frac{3 \text{ tons } 3 \text{ cwt}}{9 \text{ cwt}}.$$

3 Extract the square root of 19 951 and of $\frac{5}{8}$ correct to three places of decimals

4 Find the cost of paving a pathway 6 feet wide, round and immediately outside a flower garden, 21 yards long and 10 yards broad, at $5\frac{1}{7}$ pies per square yard

5 Find the price of 35 maunds $13\frac{3}{4}$ seers of rice at Rs. 3 2 as. per maund

If it is sold at the rate of Rs 3 $3\frac{1}{2}$ as per maund, what is the profit per cent ?

6 I pay Rs 4590 to a Bank for a Bill of Exchange payable in London The rate of exchange is 1s 4d for the rupee and the Bank charges me 2 per cent on the amount payable in England. How much will my agent in London receive ?

7. (1) Given $x+y=5$ and $xy=7$, find the value of

$$x^3+y^3+4(x-y)^2.$$

(2) If $x^2+y^2=1$, prove that

$$(3x-4x^3)^2+(4y^3-3y)^2=1$$

8 Divide

$$a^3(b-c)+b^3(c-a)+c^3(a-b) \text{ by } a+b+c$$

9 Simplify —

$$\frac{b-c}{a^2-(b-c)^2}+\frac{c-a}{b^2-(c-a)^2}+\frac{a-b}{c^2-(a-b)^2}$$

10 Solve

(1) $$\frac{x-bc}{b+c}+\frac{x-ca}{c+a}+\frac{x-ab}{a+b}=a+b+c,$$

(2) $$\left.\begin{aligned} x-y+z&=2,\\ 4x+6y+5z&=81,\\ 5x-11y+13z&=22\end{aligned}\right\}$$

11 A company of men is formed into a hollow square, 10 deep If the company be increased by 1600 men the whole number may also be formed into a hollow squre 10 deep, so that the front in the latter formation shall contain twice the number of men contained in the front of the former Find the original number of men.

12 (1) If $a : b :: c : d$, prove that

$$\frac{a+b}{a-b}=\frac{c+d}{c-d}$$

(2) If $(a+b+c)x=(b+c-a)y,$

$$=(c+a-b)z,$$

$$=(a+b-c)w,$$

show that $$\frac{1}{y}+\frac{1}{z}+\frac{1}{w}=\frac{1}{x}.$$

1905.—AFTERNOON.

Head Examiner,—GOURI SANKAR DEY, M A

1. If two triangles have two angles of the one equal to two angles of the other, each to each, and a side of one equal to a side of the other, these sides being adjacent to the equal angles in each, prove that the triangles are equal in all respects

2 Prove that parallelograms on the same base, and between the same parallels, are equal in area

3 If a straight line is divided into any two parts, prove that the sum of the squares on the whole line and on one of the parts is equal to twice the rectangle contained by the whole and that part, together with the square on the other part

If a straight line AB is divided internally at H so that the rectangle AB, BH is equal to the square on AH, prove that the sum of the squares on AB, BH is three times the square on AH

4 If from a point within a circle more than two equal straight lines can be drawn to the circumference, prove that that point is the centre of the circle

Prove that a circle has only one centre

5. If a straight line touch a circle, and from the point of contact a chord be drawn, prove that the angles which this chord makes with the tangent are equal to the angles in the alternate segments of the circle

6 Circumscribe a circle about a given regular pentagon.

7 Divide a given straight line into two parts such that the difference of the squares on the two parts may be equal to the square on a given line

8 P, Q, R are the middle points of the sides of a triangle, and X is the foot of the perpendicular let fall from one vertex on the opposite side show that the four points P, Q, R, X are concyclic

9 A straight line PQ moves in such a manner that the sum of the perpendiculars AP and BQ on it from two fixed points A and B is constant Prove that the straight line always touches a fixed circle, and find the centre and radius of that circle

SOLUTIONS.

1905 —MORNING.

1. A number is said to be a measure of another, when it divides the latter exactly A prime number is one which is not divisible by any other number except itself and unity.

Rs 10 5as = 165 12 pies = 1980 pies

Rs 18 0a 9 pie = (288 × 12 9 pies = 3465 pies

Find the H. C F of 1980 and 3465

$$\begin{array}{r|l|l} 1980 & 3465 & 1 \\ & 1980 & \\ \hline 1485 & 1980 & 1 \\ & 1485 & \\ \hline 495 & 1485 & 3 \\ & 1485 & \end{array}$$

Now, $495 = 3 \times 3 \times 5 \times 11$

∴ The price of each mango can be only 33 pies (the no 33 lying between 24 and 36)

$$\therefore \text{The total number of mangoes} = \frac{3465 + 1980}{33} = 105 + 60 = 165.$$

The Examiner evidently means that the price of each mango is an exact number of pies

Without this instruction in the question, we might get the price of each mango $24\frac{3}{4}$ pies $27\frac{1}{2}$, 33, $35\frac{5}{13}$, $30\frac{5}{13}$ $28\frac{3}{17}$ or $26\frac{1}{19}$ pies, each of which would give a separate answer But most probably these latter are not meant

2 (1) $\frac{2}{3}$rds means unity is divided into three equal parts of which two are taken

$\frac{1}{2}$ of $\frac{2}{3}$ means that we are to divide $\frac{2}{3}$ regarded as a whole) into 2 equal parts and take only one of these parts

$$(2)\ 5\frac{5}{6} - \frac{5}{42} = 5 + \frac{35 - 5}{42} = 5\frac{30}{42} = 5\frac{5}{7}$$

$$\frac{5\frac{5}{6}}{3\frac{1}{3}} \div \frac{7}{8} \text{ of } \frac{4}{3} = \frac{\overset{5}{\cancel{35}} \times 3}{\underset{2}{\cancel{6}} \times \underset{2}{\cancel{10}}} \times \frac{\overset{4}{\cancel{6}} \times 3}{\cancel{7} \times \cancel{4}} = \frac{3}{2}$$

$$\frac{5}{7} \text{ of } \frac{3 \text{ tons } 3 \text{ cwt}}{9 \text{ cwt}} = \frac{5}{7} \text{ of } \frac{63}{9} = 5$$

∴ The expression stands thus —

$$5\frac{5}{7} \text{ of } \frac{3}{2} \div 5 = \frac{40}{7} \cdot \frac{3}{2} \times \frac{1}{5} = \frac{12}{7} = 1\frac{5}{7}$$

3.

```
     19 0.1000 ( 4 4 6 ... .
     16
    ------
  84 ) 305
       336
      ------
  886 ) 5910
        5316
       ------
  8926 ) 5 400
         53556
        ------
         5844
```

$\frac{5}{8} = .625$ ∴ $\sqrt{\frac{5}{8}} = \sqrt{.625} = .790\ldots \ldots$.

$$\begin{array}{r|l} & 625000\,(\,790 \\ & 49 \\ 149 & 1350 \\ & 1341 \\ 158 & 900 \end{array}$$

4. 6ft = 2 yds

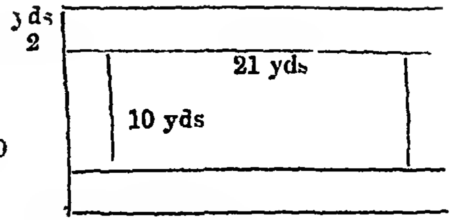
The area of the path may all round

$=\{(21+4)2+10\times 2\}\times 2$ sq yd

$=140$ sq yds

$\therefore$ cost $=5\frac{4}{7}\times 140$ pies $=39\times 20$ pies

$=$ Rs 4 1 anna



5. 10 srs $=\frac{1}{4}$ of 1 md.

$2\frac{1}{2}$ srs $=\frac{1}{4}$ of 10 srs

$1\frac{1}{4}$ srs. $=\frac{1}{2}$ of $2\frac{1}{2}$ srs

Rs 3	2 as		— price of 1 md
		5	
15	10	0	,, 5 mds
		7	
109	6	0	35 mds
0	12	6	10 srs
0	3	$1\frac{1}{2}$	$2\frac{1}{2}$ srs
0	1	$6\frac{3}{4}$	$1\frac{1}{4}$ srs
110	7	$2\frac{1}{4}$	price of 35 mds $13\frac{3}{4}$ srs

Rs 3 2 as is the cost price and Rs 3 $3\frac{1}{2}$ as is the selling price.

$\therefore$ $1\frac{1}{2}$ as. is the profit on Rs 3 2 as or 50 as

$\therefore$ 3 as is the . 100 as

$\therefore$ The profit is 3 per cent

6. 1s 4d = 1 rupee or, £ $\frac{1}{15}$ = 1 rupee

$\therefore$ £ 1 = Rs 15 £ 100 = Rs 1500

and the bank charges 2 per cent. or £ 2 for £ 100 payable and

£ 2 = Rs = 30

$\therefore$ For every Rs 1530, my agent receives £ 100

$\therefore$ For Rs 45900, on £ $\dfrac{45900\times 100}{1530}$

= £ 3000.

7. (1) $x^3+y^3+4(x-y)^2$

$=(x+y)^3-3xy(x+y)+4\{(x+y)^2-4xy\}$

$=5^3-3\times 7\times 5+4\{5^2-4\times 7\}$

$=125-105+4(25-28)$

$=20-12=8$

5 Extract the square root of 964 226704

6 What sum of money will produce £43 interest in $3\frac{1}{4}$ years at $2\frac{1}{2}$ per cent simple interest ?

7 Prove that —

$$\frac{x+y}{y}-\frac{x}{x+y}-\frac{x^3-x^2y}{x^2y-y^3}=1$$

8 Divide a^3+x^3 by $a+x$

Multiply $x^{\frac{1}{4}}y+y^{\frac{1}{3}}$ by $x^{\frac{1}{3}}-y^{\frac{1}{3}}$

9 Solve the following equations —

(i) $\frac{2x}{5}+\frac{x-2}{3}=2x-7$

(ii) $\sqrt{3x}-4=\sqrt{3x+4}$

(iii) $\begin{cases} 2x-\frac{y-3}{5}=4 \\ 3y+\frac{x-2}{3}=9 \end{cases}$

10 A post is a fourth of its length in the mud, a third of its length in the water, and 10 feet above the water, what is its length ?

1863—AFTERNOON.

Examiner,—Rev W. Sampson

1 (*a*) Define accurately *parallelogram*, *rectangle*, *square*.

(*b*) Every rectangle is a parallelogram Is it true that every parallelogram is a rectangle ? Give reasons for your answer

(*c*) Two triangles that have three sides of the one equal to three sides of the other each to each, are equal in every respect Two triangles that stand on the same base and between the same parallels are likewise equal Is there any difference between the equality of the triangles in these two cases ? If so, what ?

(*d*) In the first book of Euclid, what properties are shown to belong to triangles ?

2 Given two equal and parallel straight lines AB and DC ; prove that AC and BD bisect each other. Under what circumstances will AC equal DB ?

3 The angles at the base of an isosceles triangle are equal to each other Give Euclid's proof of this proposition How might it be proved if you were permitted to bisect an angle ?

4 Three straight lines meet in a point Draw another line cutting them so that the segment of it intercepted between the first and second shall be equal to that intercepted between the second and third.

(2) $(3x-4x^3)^2+(4y^3-3y)^2$

$=9x^2-24x^4+16x^6+16y^6+9y^2-24y^4$

$=9(x^2+y^2)-24(x^4+y^4)+16(x^6+y^6)$

$=9\times 1-24\{(x^2+y^2)^2-2x^2y^2\}+16\{(x^2+y^2)^3-3x^2y^2(x^2+y^2)\}$

$=9-24(1-2x^2y^2)+16(1-3x^2y^2)$

$=9-24+48x^2y^2+16-48x^2y^2=25-24=1$

8 $a^3(b-c)+b^3(c-a)+c^3(a-b)$

$=a^3(b-c)-a(b^3-c^3)+bc(b^2-c^2)$

$=(b-c)\{a^3-a(b^2+bc+c^2)+bc(b+c)\}$

$=(b-c)\{a(a^2-b^2)-bc(a-b)-c^2(a-b)\}$

$=(b-c)(a-b)\{a(a+b)-bc-c^2\}$

$=(b-c)(a-b)\{(a^2-c^2)+b(a-c)\}$

$=(b-c)(a-b)(a-c)(a+c+b)$

∴ The quotient

$=(a-b)(a-c)(b-c)$

9 $\dfrac{b-c}{a^2-(b-c)^2}+\dfrac{c-a}{b^2-(c-a)^2}+\dfrac{a-b}{c^2-(a-b)^2}$

$=\dfrac{b-c}{(a+b-c)(a-b+c)}+\dfrac{c-a}{(b+c-a)(b-c+a)}+\dfrac{a-b}{(c+a-b)(c-a+}$

$=\dfrac{(b-c)(b+c-a)+(c-a)(c+a-b)+(a-b)(a+b-c)}{(a+b-c)(a+c-b)(b+c-a)}$

Numerator $=b^2-c^2-ab+ac+c^2-a^2-bc+ab+a^2-b^2-ac+bc.$

$=0$ ∴ Answer $=0$

(1) $\dfrac{x-bc}{b+c}+\dfrac{x-ca}{c+a}+\dfrac{x-ab}{a+b}=a+b+c$

or, $\left(\dfrac{x-bc}{b+c}-a\right)+\left(\dfrac{x-ca}{c+a}-a\right)+\left(\dfrac{x-ab}{a+b}-c\right)=0$

i e. $\dfrac{x-bc-ab-ac}{b+c}+\dfrac{x-ca-bc-ab}{c+a}+\dfrac{x-ab-ac-bc}{a+b}=0$

∴ $(x-ab-ac-bc)\left(\dfrac{1}{b+c}+\dfrac{1}{c+a}+\dfrac{1}{a+b}\right)=0$

∴ $x-ab-ac-bc=0$

∴ $x=ab+ac+bc$

(2) $x-y+z=2$

∴ $4x-4y+4z=8$ (1)

and $4x+6y+5z=31$ (2)

∴ $10y+z=23$, subtracting (1) from (2)

Again, $5x-5y+5z=10$

$$5x-11y+13z=22$$

$$\therefore\ 6y-8z=-12$$

and $80y+8z=184$

$$\therefore\ 86y=172$$

$$\therefore\ y=2$$

$$10y+z=23$$

$$\therefore\ 20+z=23$$

$$z=3.$$

Now, $x-y+z=2$

or, $x-2+3=2$

$$\therefore\ x=1$$

$\therefore\ x=1,\ y=2,\ z=3$

11 Let x be the number of men in the front of the former square

$\therefore$ No of men $=x^2-(x-20)^2$

$\therefore$ The no of men in front of the 2nd square $=2x$

$\therefore$ No of men in the 2nd square

$$=(2x)^2-(2x-20)^2$$

By the question,

$$x^2-(x-20)^2+1600=(2x)^2-(2x-20)^2$$

or, $20(2x-20)+1600=20(4x-20)$

$$\therefore\ 2x-20+80=4x-20$$

or, $2x=80$

$$\therefore\ x=40$$

$\therefore$ No of men reqd $=40^2-20^2$

$$=20\times60=1200$$

12 (1) $\dfrac{a}{b}=\dfrac{c}{d}$

adding 1 to both sides, we get $\dfrac{a+b}{b}=\dfrac{c+d}{d}$ (1)

Again, subtracting 1 from each side

we get $\dfrac{a-b}{b}=\dfrac{c-d}{d}$ (2)

Dividing (1) by (2) we have

$$\frac{a+b}{a-b}=\frac{c+d}{c-d}$$

(2) $(a-b+c)x=(b+c-a)y=(c+a-b)z=(a+b-c)w$

or $\dfrac{a+b+c}{\frac{1}{x}}=\dfrac{b+c-a}{\frac{1}{y}}=\dfrac{c+a-b}{\frac{1}{z}}=\dfrac{a+b-c}{\frac{1}{w}}$

$$=\frac{(b+c-a)+(c+a-b)+(a+b-c)}{\frac{1}{y}+\frac{1}{z}+\frac{1}{w}}=\frac{a+b+c}{\frac{1}{y}+\frac{1}{z}+\frac{1}{w}}$$

$$\therefore \frac{1}{x}=\frac{1}{y}+\frac{1}{z}+\frac{1}{w}$$

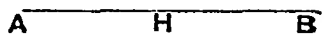
1905.—AFTERNOON.

1 Case 1, proposition 26, Book I

2 Proposition 35, Book I

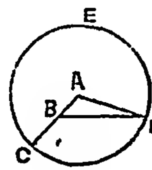
3. Proposition 7, Book II

(a) $AB^2+BH^2=2AB\ BH+AH^2$ (II 7)
But $2\ AB\ BH=2AH^2$ (Hyp)
$\therefore AB^2+BH^2=2AH^2+AH^2=3AH^2$



4 Proposition 9, Book III

(a) If possible let the circle CDE have two centres A and B



Join AB and produce it to C
Join AD and BD
Then AC=AD (Def of a circle)
also BC=BD ,,

$\therefore AD=AB+BC=AB+BD$

i e, one side of a triangle is equal to the sum of the other two sides, which is impossible by I 20

5 Proposition 32, Book III

6 Proposition 14, Book IV

7 Let AB be the given straight line and C be the given line the square on which is to be equal to the difference of the squares on the two parts into which AB is to be divided

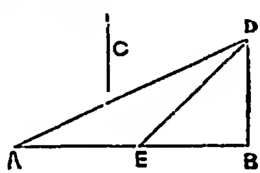
At B draw BD at rt angles to AB and make BD=C

Join AD

A B is clearly greater than C or BD

$\therefore$ angle at D > angle at A

In AD make the angle $ADE=\angle DAE$ (I 23)

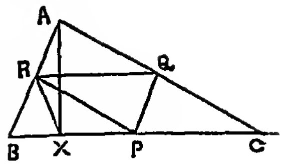


Then E shall be the reqd. point of division

AE=ED (I 6)

$\therefore AE^2-EB^2=ED^2-EB^2=DB^2$ (I 47)

i e., the difference of the squares on the two parts is equal to the sq. on DB or C.



8 P, Q, R are the mid points of BC, CA and AB respectively.

The four points P, Q, R and X are concyclic if the angles RQP and RXP are together equal to two right angles (converse of prop 22, Book III.)

Now, R+Q being the mid points of AB and AC

RQ is ‖ to BC

and QP is similarly ‖ to AB

∴ ∠RQP=∠RBP (I 34)

and AXB is a rt angled triangle in which the middle point of the hypotenuse is joined to the vertex at which the right angle is

∴ RX=½ of the hypotenuse=RB

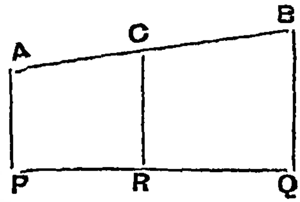
∠RBX=∠RXB

∴ ∠RQP+∠RXP=∠RBP+∠RXP

=∠RXB+∠RXP

=2 rt angles (I. 13)

∴ the four points are concyclic



9 Bisect AB at C (I 10) From C draw CR at right angles to PQ

Then CR=½ (AP+BQ)

This may be proved very easily by drawing AN ‖ to PQ N being on BQ,

and as AP+BQ is constant=C (say)

∴ CR=½C

∴ PQ is a line moving so that its distance from a fixed point C is always the same

∴ PQ touches a circle of which the centre is C (the mid point of AB) and radius equal to half the given constant

1906—MORNING.

Head Examiner,—Dr Cullis

1 (1) When is one number said to be a multiple of another? How can you ascertain by inspection whether a given number is a multiple of 3?

(2) What is the greatest number consisting of five digits which can be added to 8321 so that the sum may be exactly divisible by 15, 20, 24, 27, 32, and 36?

2 (1) What is the meaning of $\frac{3}{4}$ of $\frac{1}{5}$? Give an illustration

(2) Simplify

(1) $12\times\left(\frac{2}{29}-\frac{1}{24}-\frac{1}{38}-\frac{1}{174}-\frac{1}{232}\right)+\frac{337}{381}-\frac{2\frac{5}{8}}{2\frac{5}{7}}\text{ of }\frac{11s\ 4d}{12s\ 3d}$;

(2) $\frac{1\ 59\times15\ 9-\ 41\times4\ 1}{15\ 9-4\ 1}$

3 The cost of matting a room 16 ft. broad and 12 ft high at 3 as per sq yd is Rs 7 9 as 4 p What will be the cost of paper-

ing its walls at the same rate, allowing for six doors, each 6 ft by 3 ft ?

4 Extract the square root of $0\dot{2}\dot{7}$, and of $\frac{1}{3}$ correct to four places of decimals

5 A book sent from England costs me (including R. 1 2as postage) Rs 12 1a But my bookseller allows me a discount of 2*d.* in the shilling on the published price What is the published price in English money, the rate of exchange being 1s 4d for the rupee ?

6. Define *Present Worth*

A man bought a horse for 30 guineas and sold him immediately for £36 1s payable at the end of 6 months If interest be reckoned at 6 per cent per annum, find his gain per cent upon the transaction.

7 Resolve into factors

x^4+4a^4 and $(a+b+c)(bc+ca+ab)-abc$.

8 (1) Prove that $(a^m)^n=a^{mn}$, when m and n are positive integers

(2) If $a^x=b$, $b^y=c$, $c^z=a$, prove that $xyz=1$

9. (1) If $a^2+b^2+c^2=1$, $x^2+y^2+z^2=1$, prove that $(bz-cy)^2+(cx-az)^2+(ay-bx)^2+(ax+by+cz)^2=1$

(2) If $bc+ca+ab=0$, prove that

$$\frac{1}{a^2-bc}+\frac{1}{b^2-ca}+\frac{1}{c^2-ab}=0$$

10. (1) Distinguish between an *equation* and an *identity* Give an example of each

(2) Solve —

(1) $$\frac{x-a^3}{b^2-bc+c^2}+\frac{x-b^3}{c^2-ca+a^2}+\frac{x-c^3}{a^2-ab+b^2}=2(a+b+c)$$

(2) $$\left.\begin{array}{l} x+y+z=0, \\ (b+c)x+(c+a)y+(a+b)z=0, \\ bcx+cay+abz=1 \end{array}\right\}$$

11. A man walks from A to B and back in a certain time at the rate of $3\frac{1}{2}$ miles an hour But if he had walked from A to B at the rate of 3 miles an hour, and back from B to A at the rate of 4 miles an hour, he would have taken 5 minutes longer Find the distance between A and B

12 (1) If $a : b=c : d=e : f$, prove that each ratio

$$=\frac{a+c+e}{b+d+f}$$

(2) If $\frac{py+qz}{b+c}=\frac{pz+qx}{c+a}=\frac{px+qy}{a+b}$, prove that

$$(x+y+z)\{(b+c)x+(c+a)y+(a+b)z\} = 2(a+b+c)(yz+zx+xy).$$

1906—AFTERNOON.

Head Examiner,—Dr Cullis.

1 If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of every triangle are together equal to two right angles

The bisectors of the exterior angles of any quadrilateral form a quadrilateral whose opposite angles are supplementary

2 Enunciate and prove Euc I 47

3 If a straight line be divided into two equal parts and also into two unequal parts, the rectangle contained by the unequal parts, together with the square on the line between the points of section, is equal to the square on half the line

Enunciate the Proposition corresponding to the algebraical formula $(a-b)^2+2ab=a^2+b^2$

4 The diameter is the greatest straight line in a circle; and of all others, that which is nearer to the centre is always greater than one more remote, and the greater is nearer to the centre than the less

Through a given point within a circle draw the least possible chord

5 In a circle the angle in a semicircle is a right angle, but the angle in a segment greater than a semicircle is less than a right angle, and the angle in a segment less than a semicircle is greater than a right angle.

AB is a fixed diameter of a given circle, and PQ a variable chord of length equal to the radius AP and BQ are produced to meet in R, prove that the locus of R consists of arcs of two equal circles

6 Inscribe an equilateral and equiangular hexagon in a given circle

The side of a regular hexagon inscribed in a circle is equal to the radius of the circle

7 Given the three middle points of the sides of a triangle, construct the triangle.

8 A straight rod slips between two straight rulers placed at right angles to one another, in what position is the area of the triangle formed by the rulers and the rod greatest?

SOLUTIONS

1906.—MORNING.

1 (1) If a number can be divided by another number without a remainder then the first number is a multiple of the second. If the sum of the digits of a number be divided by 3 exactly then the number is a multiple of 3

(2) L. C. M of 15, 20, 24, 27, 32, and 36 is

$32 \times 27 \times 5 = 27 \times 160 = 4320.$

```
4320) 99999 (23          4320
      8640                 24
      -----            ------
      13599            103680
      12960              8321
      -----            ------
        639             95359
                         4320
                       ------
                        99679   Ans
```

2. (1) A thing is divided into two equal parts and one part is again divided into 4 parts and 3 such parts are taken. Take an orange, divide it into two equal parts and then divide one of these two parts into 4 equal parts and take 3 such parts.

$$(1) = 12 \times \left(\frac{48-29-12-4-3}{29 \times 8 \times 3}\right) + \frac{17 \times 19}{19 \times 19} \div [illegible] \text{ of } [illegible] \text{ of } [illegible]$$

$$= \frac{12 \times 0}{29 \times 8 \times 3} + [illegible] \times [illegible] \times [illegible] \times \frac{7 \times 21}{8 \times 17} = 0 + 1 = 1 \quad \text{Ans}$$

$$(2) = \frac{\frac{1}{10} \text{ of } \{15\cdot9 \times 15\cdot9 - 4\cdot1 \times 4\cdot1\}}{15\cdot9 - 4\cdot1} = \frac{1}{10} \text{ of } \{15\cdot9 + 4\cdot1\} = \frac{1}{10} \text{ of } 20$$

$$= 2 \quad \text{Ans}$$

(3) area of the floor $= \frac{121\frac{1}{3}a}{3a}$ sq yds $= \frac{364}{9}$ sq yd $= 364$ sq ft

length $= 364 \div 16 = 22\frac{3}{4}$ ft

area of 4 walls $= 2 \times 12 \times (22\frac{3}{4} + 16) = 24 \times 38\frac{3}{4} = 6 \times 155 = 930$ sq ft

area of 6 doors $= 6 \times 6 \times 3 = 108$ sq ft

area to be papered $= 930 - 108 = 822$ sq ft

cost $=$ Rs. $\frac{822}{9} \times \frac{3}{16} = \text{Rs} \frac{3 \times 274) \times 3}{9 \times 16} = \text{Rs } 17 \ 2a$ Ans

4. $\sqrt{(\cdot027)} = \sqrt{(\frac{[illegible]}{1000})} = \sqrt{(\frac{25}{900})} = \sqrt{\frac{1}{36}} = \frac{1}{6} = \cdot1\dot{6}$ Ans

$\frac{4}{5} = \cdot6$

```
       | 60000000 | ·7745 or ·7746 nearly Ans
       | 49
147    | 1100
       | 1029
1544   |   7100
       |   6176
15485  |   92400
       |   77425
```

5 Rs 12 – 1
Rs 1 – 2

Rs 10 – 15

Rs 10 – 15 = 175a = 175d

published price = $\frac{175 \times 12}{10} = 35 \times 6d = 17s\ 6d$ Ans.

6 Present worth of a given sum due at the end of a certain time is a sum which with its interest for that time amount to the given sum

103 36$\frac{1}{20}$ 100 p w

p w = £$\frac{721 \times 100}{103 \times 20}$ = £35

gain £35 – £31 10s = £3 10s

31$\frac{1}{2}$ 100 3$\frac{1}{2}$ x

$x = \frac{2 \times 100 \times 7}{63 \times 2} = \frac{100}{9} = 11\frac{1}{9}$ p c Ans

7 $x^4 + 4a^4 = x^4 + 4a^2x^2 + 4a^4 - 4a^2x^2 = (x^2 + 2a^2)^2 - (2ax)^2$

$= (x^2 + 2a^2 + 2ax)(x^2 + 2a^2 - 2ax)$ Ans

$(a + b + c)(bc + ca + ab) = (a + b)(bc + ca + ab) + bc^2 + c^2a + abc$
$- abc$

$= (a + b)(bc + ca + ab) + c^2(a + b) = (a + b)(bc + ca + ab + c^2)$

$(a + b)\{c(b + c) + a(b + c)\} = (a + b)(b + c)(c + a)$ Ans

8 (1) $(a^m)^n = a^m \times a^m \times a^m \times a^m \times$ &c (n factors)

$= a^{m + m + m + m + \&c}$ ... (n factors)

$= a^{mn}$

(2) $ax = a^x$ ∴ $a^{xy} = by = c$ ∴ $a^{xyz} = c^z = a$ ∴ $a^{xyz} = a^1$ ∴ $xyz =$

9 (1) $Ex = b^2z^2 + c^2y^2 - 2bcyz + c^2x^2 + a^2z^2 - 2acxz + a^2y^2 + b^2x^2 -$
$-abxy$

$+ a^2x^2 + b^2y^2 + c^2z^2 + 2abxy + 2acxz + 2bcyz$

$= a^2x^2 + a^2y^2 + a^2z^2 + b^2x^2 + b^2y^2 + b^2z^2 + c^2x^2 + c^2y^2 + c^2z^2$

$= a^2(x^2 + y^2 + z^2) + b^2(x^2 + y^2 + z^2) + c^2(x^2 + y^2 + z^2)$

$(x^2 + y^2 + z^2)(a^2 + b^2 + c^2) = 1 \times 1 = 1$

(2) $bc+ca+ab=0$

$\therefore\ ca+ab=-bc$

$bc+ab=-ca$

$bc+ca=-ab$

$$\text{Ex}=\frac{1}{a^2+ac+ab}+\frac{1}{b^2+bc+ab}+\frac{1}{c^2+bc+ca}$$

$$=\frac{1}{a(a+b+c)}+\frac{1}{b(a+b+c)}+\frac{1}{c(a+b+c)}$$

$$=\frac{bc+ac+ab}{abc(a+b+c)}=\frac{0}{abc(a+b+c)}=0$$

10. (1) When two algebraical expressions are equal for all values of the unknown quantity it is an *identity*, whereas when two algebraical expressions are equal for a particular value or values of the unknown quantity it is an *equation*

as $(x-3)(x-4)=x^2-7x+12$ is an identity

$(x-c)\ (x-4)=x^2-5x+6$ is true only when $x=4$ is an equation.

(2) (1) or $\left\{\frac{x-a^3}{b^2-bc+c^2}-(b+c)\right\}+\left\{\frac{x-b^3}{c^2-ca+a^2}-(c+a)\right\}$

$$+\left\{\frac{x-c^3}{a^2-ab+b^2}-(a+b)\right\}=0$$

or $\frac{x-a^3-b^3-c^3}{b^2-bc+c^2}+\frac{x-a^3-b^3-c^3}{c^2-ca+a^2}+\frac{x-c^3-a^3-b^3}{a^2-ab+b^2}=0$

$\therefore\ x-a^3-b^3-c^3=0$

$\therefore\ x=a^3+b^3+c^3$ Ans

(2) $x+y+z=0$

$(b+c)x+(c+a)y+(a+b)z=0$

$$\text{or}\ \frac{x}{\begin{matrix}1 & 1\\ c+a & a+b\end{matrix}}=\frac{y}{\begin{matrix}1 & 1\\ a+b & b+c\end{matrix}}=\frac{z}{\begin{matrix}1 & 1\\ b+c & c+a\end{matrix}}$$

$$\text{or}\ \frac{x}{b-c}=\frac{y}{c-a}=\frac{z}{a-b}=k \text{ suppose}$$

or $x=(b-c)k$

$y=(c-a)k$

$z=(a-b)k$

substituting these values in the 3rd equation $k\{bc(b-c)+ca(c-a)+ab(a-b)\}=1$

or $-k\{(b-c)(c-a)(a-b)=1$

or $k=-\dfrac{1}{(b-c)(c-a)(a-b)}$

$$\left.\begin{array}{l} \therefore x=-\dfrac{1}{(c-a)(a-b)} \text{ or } \dfrac{1}{(a-b)(a-c)} \\ y=-\dfrac{1}{(b-c(a-b)} \text{ or } \dfrac{1}{(b-a)(b-c)} \\ z=-\dfrac{1}{(b-c)(c-a)} \text{ or } \dfrac{1}{(c-b)(c-a)} \end{array}\right\} \text{Ans}$$

11. Let x miles be the distance from A to B Then by the question

$$\frac{2x}{3\frac{1}{2}}=\frac{x}{3}+\frac{x}{4}-\frac{1}{12}$$

or $\dfrac{4x}{7}=\dfrac{x}{3}+\dfrac{x}{4}-\dfrac{1}{12}$

or $48x=28x+21x-7$

or $x=7$ miles Ans

12 (1) Let $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{e}{f}=k$ then $a=bk$, $c=dk$, $e=fk$,

$a+c+e=k\ (b+d+f)$ or $\dfrac{a+c+e}{b+d+f}=k=$ each ratio

(2) each ratio$=\dfrac{\text{sum of all the numerators}}{\text{sum of all the denominators}}$

$\therefore$ each ratio$=\dfrac{(p+q)(x+y+z)}{2\ (a+b+c)}$

again Multiplying numr and denor of 1st by x 2nd by y and 3rd by z

we get $\dfrac{pxy+qxz}{(b+c)x}=\dfrac{pyz+qxy}{(c+a)y}=\dfrac{pxz+qyz}{(a+b)}$

each$=\dfrac{(p+q)(xy+xz+yz)}{(b+c)x+(c+a)y+(a+b)z}$

$\therefore \dfrac{(p+q)(x+y+z)}{2(a+b+c)}=\dfrac{(p+q)(xy+xz+yz)}{(b+c)x+(c+a)y+(a+b)z}$

or $(x+y+z)\{(b+c)x+(c+a)y+(a+b)z\}$

$=2(a+b+c)(xy+xz+yz)$

5 If a straight line be divided into any two parts, the square of the whole line is equal to the squares on the two parts together with twice the rectangle contained by the parts

To what algebraical formula is this equivalent?

6 Describe a square that shall be equal to a given triangle

7. (*a*) Equal chords in a circle are equally distant from the centre.

(*b*) How do you measure the distance of a straight line from a point?

(*c*) What is the locus of the middle points of equal straight lines in a circle?

8 The opposite angles of any quadrilateral figure inscribed in a circle are equal to two right angles

9. A tangent is drawn parallel to a chord Show that the intercepted arc is bisected at the point of contact

10. Inscribe in a circle an equilateral and equiangular pentagon. Is it necessary to say equilateral and equiangular?

SOLUTION.

1863.—MORNING.

1. $\dfrac{15\frac{2}{3}+6-\frac{3}{4}}{7\frac{4}{9}\times\frac{1}{4}}=\dfrac{15\frac{2}{3}+5\frac{1}{4}}{\frac{67}{9}\times\frac{1}{4}}=\dfrac{20\frac{11}{12}}{\frac{67}{36}}=\dfrac{251}{12}\times\dfrac{36}{67}$

$=\frac{753}{67}=11\frac{16}{67}=11.2388\ldots$

2. Ex $=6.59979\div2.104=\frac{659979}{100000}\times\frac{1000}{2104}=\frac{659979}{210400}=3\frac{28779}{210400}$.

3. 31 cwt 3 qrs. 13lbs. $=31\frac{97}{112}$ cwt.

$\therefore$ the price $=31\frac{97}{112}\times90s.=£\frac{3569}{112}\times\frac{9}{2}=£143\ 7s\ 11\frac{5}{14}d$

4. $\left.\begin{matrix}170\text{ mi.} & 470\text{ mi.}\\ 8\frac{3}{4}\text{ hrs} & 11\text{ hrs}\end{matrix}\right\}\quad 4\frac{1}{4}\text{ days}\quad x$

$$\therefore x=\frac{470\times11\times17\times4}{170\times35\times4}\text{ days}=14\tfrac{27}{35}\text{ days}$$

5. $\sqrt{(964.22\dot{6}704)}=31.052$

6. The interest on £100 for $3\frac{1}{4}$ yrs @ $2\frac{1}{2}$ p. c $=£\frac{65}{8}$;

$£\frac{65}{8}\quad £43\quad £100\quad x$

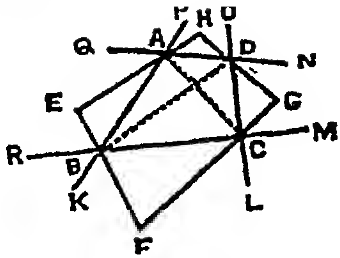
$$\therefore x=£\,\frac{43\times100\times8}{65}=£529\ 4s.\ 7\tfrac{5}{13}d.$$

7 Since $\dfrac{x^3-x^2y}{x^2y-y^3}=\dfrac{x^2(x-y)}{y(x+y)(x-y)}=\dfrac{x^2}{y(x+y)}$

$$\therefore \text{fraction}=\frac{x+y}{y}-\frac{x}{x+y}-\frac{x^2}{y(x+y)}=\frac{(x+y)^2-xy-x^2}{y(x+y)}$$

$$=\frac{x^2+2xy+y^2-xy-x^2}{y(x+y)}=\frac{y(x+y)}{y(x+y)}=1$$

1906.—AFTERNOON.



1 Book I Prop 32

Let ABCD be a quadrilateral and let its exterior angles be bisected by EF, FG, GH and HE to form the quadrilateral EFGH then its opposite angles shall be supplementary, i e the angle E and G shall be supplementary as well as the angles F and G shall be supplementary. Join AC and BD

Proof —$\left.\begin{array}{l}\angle CBK=\angle CAB+\angle BCA\\ \angle CDA=\angle DAC+\angle DCA\end{array}\right\}$ I—32

$\therefore \angle CBK+\angle OAD=\angle DAB+\angle DCB$

Similarly $\angle BCD+\angle BAD=\angle ABC+\angle ADC$

$\therefore \angle CBK+\angle BAD+\angle BCD+\angle BAD=\angle DAB+\angle DCB+\angle ABC+\angle ADC=4$ rt angles inference I—32

$\therefore$ Their halves (CBF$+\angle$OAD$+$(BCF$+\angle$HAD$=2$ rt. angles

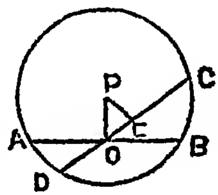
But these 4 angles$+\angle H+\angle F=4$ rt angles I 32

$\therefore \angle H+\angle F=2$ rt angles

2 Book I Prop 47

3 Book II Prop 5 $(a-b^2)^2+2ab=a^2+b^2$ is the II 7.

4 Book III Prop. 15



Let ABC be a circle and P the given point Find O the centre of the circle IV 1 join OP and draw the chord AB at rt angles to OP AB shall be the least chord through P Through P draw CD any other chord and draw OR perpendicular to CD

Then in the triangle OPE the angle ORP is a rt. angle

$\therefore \angle$OPE is less than a rt angle

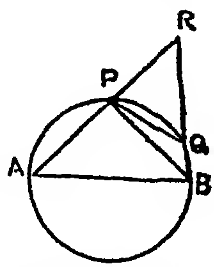
$\therefore \angle$OEP is greater than $\angle$OPE

$\therefore$ OP is greater them OE

$\therefore$ AB is less then CD II 15

Similarly it can be proved that AB is less than any other chord

$\therefore$ AB is the least chord through OP. Q.E D



4 Book III Prop 31

Join PB $\angle$PBQ is half the angle at the centre subtended by PQ But the angle at the centre subtended by PQ is $\frac{2}{3}$ rt angle $\therefore$ PQ is equal to the radius and $\therefore$ equal to the side of an inscribed hexagon

$\therefore \angle PBQ=\frac{1}{3}$ rt angle

The $\angle$APB is a rt angle III 31

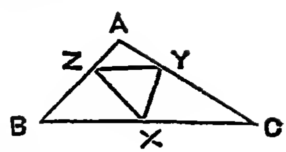
and $\angle APB=\angle PQB+\angle PBR$

$\therefore \angle PRB=\frac{2}{3}$rd rt angle (constant)

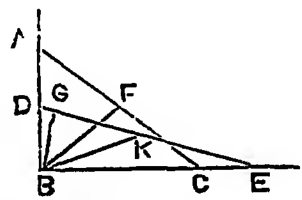
Now AB is fixed and the angle ARB is constant $\therefore$ The locus is a arc of which the base is AB and vertical angle$=\frac{2}{3}$ rt angle similarl another arc on the other side may be found with the same condition.

$\therefore$ The locus of R consists of arcs of two equal circles.

6 Book IV. Prop. 15 cor to IV. 15.



7. Let X, Y, Z, be the given points.
Join XY, XZ, ZX,
Through X, Y, Z draw BC, CA, AB respectively parallel to YZ, ZX, XY intersecting two and two at A, B, C
ABC shall be the triangle required.
By construction the figure BXYZ is a parallelogram
∴ ZB=YX
Again the figure AZXY is a parallelogram
∴ AZ=YX
∴ AZ=ZB ∴ Z is the middle point of AB.
Similarly it can be proved that X and Y are the middle points of BC, AC
∴ ABC is the triangle required. Q. E. F.



8. Let AB and BC be two str rulers placed at rt angles to one another Let AC be the rod. The area will be greatest when AC makes equal angles with the rulers Take any other poitions of the rod as DE from B draw BF and BG perpendiculars to AC and DE respectively
∴ ABC is an isosceles triangle ∴ F is the middle point of AC
Again ABC is a rt angled triangle ∴ BF is half of AC
Let K be the middle point of DE
The angle BGK is a rt angle ∴ > ∠BKG
∴ BG is less than BK
But BK is half of DE the rod
∴ BG is less than half the rod ∴ less than BF
Now area of ABC $=\frac{1}{2}$ AC BF
area of DBE $=\frac{1}{2}$ DE BG
But AC=DE and BF is greater than BG
∴ area of ABC > area of DBE
Similarly it can be proved that the area of ABC is greater than the area of any other triangle formed by any other position of the rod Q F D

8 $\dfrac{a^3+x^3}{a+x}=a^2-ax+x^2$

(a) $x^{\frac{1}{4}}y+y^{\frac{2}{3}}$

$x^{\frac{1}{3}}-y^{\frac{1}{3}}$

$x^{\frac{7}{12}}y+x^{\frac{1}{4}}y^{\frac{2}{3}}-x^{\frac{1}{4}}y^{\frac{4}{3}}-y$

9 (1) $\dfrac{2x}{5}+\dfrac{x-2}{3}=2x-7$

$\therefore 6x+5x-10=30x-105. \quad \therefore 19x=95 \quad \therefore x=5.$

(2) $\sqrt{(3x)}-4=\sqrt{(3x+4)}$ or $3x+16-8\sqrt{(3x)}=3x+4$ (sq)

$\therefore 12=8\sqrt{3x}$ or $3=2\sqrt{(3x)}$ or $9=12x \ \therefore x=\frac{3}{4}$

(3) $2x-\dfrac{y-3}{5}=4$ (1) From (1) $10x-y=17$

$3y+\dfrac{x-2}{3}=9$ (2) ,, (2) $9y+x=29$

or $90y+10x=290$

By subtraction $91y=273 \quad \therefore y=3$

And $10x=17+y=20 \quad \therefore x=2$

10 Let x be the length of the post in *ft*

Then $\dfrac{x}{4}+\dfrac{x}{3}+10=x$ or $3x+4x+120=12x$ or $5x=120$

$\therefore x=24$ *ft*

1863.—AFTERNOON.

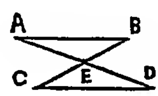
1 (a) Euc I Def 38, Def. 32

(b) Euc II Def 1 No, for a rectangle is a right angled parallelogram

(c) In the first case the triangles are equal in every respect in sides, angles and area, in the second case the triangles are equal in area only

(d) The following are the propositions, 4, 5, 6, 8, 16, 17, 18, 19, 20, 21, 24, 25, 26 32, 37, 38, 39, 40, 47, 48

2 Let AC and BD intersect each other in E



$\because \angle ABE=\angle EDC$ } (I 29)
and $\angle BAE=\angle EDC$ }
and AB=DC, (Hyp)
$\therefore$ AE=EC and EB=DE (I 26)
AC=BD, when ABCD is a rectangle

3. Euc I 5 In the Fig of Euc I. 5, draw AD bisecting the $\angle$BAC and meeting BC in D

$\because$ AB=AC and AD common (Hyp)
also $\angle$BAD=ACD. (Cons)
$\therefore \angle$ABD=$\angle$ACD. (I 4)

4 Let BA CA and DA meet in A.

From C any point in AC draw CF ‖ BA, and CE ‖ DA.

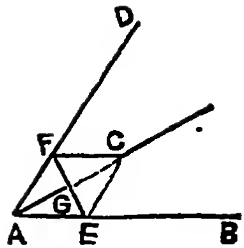
Join FE cutting AC in G

Then FE is the reqd line

∵ FAEC is a ▱

and diagonals of a parallelogram bisect one another

∴ FG=GE



5 Euclid II 4 $(a+b)^2=a^2+b^2+2ab$.

6 Euclid II 14

7 (*a*) Euclid III 14 A

(*b*) The distance of a str line from a point is the perpendicular upon the str line drawn from that point

(*c*) All str lines drawn from the centre to the middle points of the chords are perpendiculars of those lines (III 3) ∴ they are equal (III 14). ∴ The locus is a circle whose centre is the centre of the given circle and radius is any of these perpendiculars

8 Euclid III 22

9 Let AB be the chord and CPD a tangent ‖ AB

Let E be the centre Join AP, BP, and EP

Let EP cut AB in F

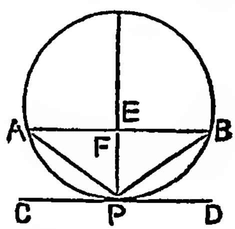
Then ∠EPD is rt ∠ (III 18)

∴ ∠PFB is also a right angle ; (I 29)

hence AF=FB also FP com (III 3)

∴ AP=BP (I 4)

Hence arc AP=arc BP (II 26)



10 Euclid IV 11 No for every equil fig inscribed in a circle is equiangular

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1864.—MORNING

Examiner,—J SUTCLIFFE, M A.

1. How many paving stones, measuring 14 in by 12 in are required to pave a verandah 70 ft long and 9 ft broad ?

2 Add together $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{11}$, $\frac{13}{33}$ and $\frac{3}{77}$

And simplify $\dfrac{2\frac{1}{4}}{2\frac{1}{3}}+\dfrac{2\frac{1}{2}+5\frac{1}{5}}{3\frac{1}{4}+9\frac{1}{2}}+\frac{1}{2}+\frac{3}{8}$ of $\frac{5}{26}$

3 Find the value of 17 cwt 3 qrs 22lbs, at £4 6*s* 7½*d*. per cwt

4 Add together 0125 of a pound, 0625 of a shilling, and ·5 of a penny, and reduce 11*s* 9¼*d* to the decimal of a pound.

5 Extract the square root of 000196, and divide the result by 140

6 A company guarantees to pay 5 per cent on shares of 1000 rupees each, another guarantees to pay $4\frac{3}{8}$ per cent on shares of 75 rupees each, the price of the former is 1245 rupees, and of the latter 85 rupees Compare the rates of interest which the shares return to purchasers

7. Add together $x^2-(x-y+z)(x+y-z)$,
$y^2-(y-x+z)(y+x-z)$, and $z^2-(z-x+y)(z+x-y)$

8 Multiply $x+y+z-\sqrt{xy}-\sqrt{yz}-\sqrt{xz}$ by $\sqrt{x}+\sqrt{y}+\sqrt{z}$,
and divide $x^8+a^4x^4+a^8$ by x^2-ax+a^2.

9. Simplify the expression—

$$\frac{1}{2}\cdot\frac{1}{x-1}+\frac{x-5}{x^2-7x+10}+\frac{1}{2}\;\frac{x-6}{x^2-9x+18}.$$

10 Solve the equations—

(a) $\frac{x-1}{3}-\frac{x-9}{3}+\frac{3x-2(x-2)}{7}=4\frac{1}{2}$

(a) $\begin{cases} 5x+11y=146 \\ 11x+5y=110 \end{cases}$

1864.—AFTERNOON

Examiners,— Rev. K S Macdonald, M A
Rev W Johnson, B A

1. What are the two definitions generally given for a straight line? What advantage has the one over the other?

2. Any two sides of a triangle are together greater than the third side

(a) Prove this by the preceding proposition of Euclid

(b) And also show how it may be regarded as a corollary to one of your definitions.

3 Show that every four sided figure whose opposite sides are equal is a parallelogram

4 If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles and the three interior angles of every triangle are together equal to two right angles

5 Enunciate and prove the two corollaries to this proposition as to the value of the interior and exterior angles of any rectilineal figure.

6 If a straight line be divided into two equal and also into two unequal parts, the squares of the two unequal parts are together double of the square of half the line, and of the square of the line between the points of section

7 The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles

8 In a right-angled triangle the line joining the right angle and the point of bisection of the hypotenuse is equal to half the hypotenuse.

9 To describe a regular pentagon about a given circle.

SOLUTIONS.

1864.—MORNING.

1. The area of verandah $=(70\times 9)$ sq. ft
And the area of each stone $=(14\times 12)$ sq. in.

$\therefore$ the required number of stones $=\dfrac{70\times 9\times 144}{14\times 12}=540$

2. $\frac{1}{3}+\frac{1}{7}+\frac{1}{11}+\frac{13}{33}+\frac{3}{77}=\dfrac{77+33+21+91+9}{231}=\dfrac{231}{231}=1.$

Ex. $=\frac{9}{4}\times\frac{3}{8}+\dfrac{7\frac{7}{10}}{12\frac{5}{6}}+\frac{1}{2}+\dfrac{3\times 3}{8\times 20}$

$=\frac{27}{32}+\frac{77}{10}\times\frac{6}{77}+\frac{1}{2}+\frac{9}{160}=\frac{27}{32}+\frac{3}{5}+\frac{1}{2}+\frac{9}{160}$

$=\dfrac{135+96+80+9}{160}=\frac{320}{160}=2$

3

	£	s	d.	
2 qrs $=\frac{1}{2}$ of 1 cwt	4	6	$7\frac{1}{2}$	= value of 1 cwt
			17	
	73	12	$7\frac{1}{2}$	= value of 17 cwt
1 qr. $=\frac{1}{2}$ of 2 qrs.	2	3	$3\frac{3}{4}$	= value of 2 qrs
14 lbs $=\frac{1}{2}$ of 1 qr.	1	1	$7\frac{7}{8}$	= value of 1 qr
7 lbs. $=\frac{1}{2}$ of 14 lbs.		10	$9\frac{15}{16}$	= value of 14 lbs.
1 lb $=\frac{1}{7}$ of 7lbs		5	$4\frac{31}{32}$	= value of 7 lbs
			$9\frac{9}{32}$	= value of 1 lb

£77 14 $7\frac{5}{16}d$ = value of 17 cwt. 3 qrs. 22lbs.

4. 0125 of £1 $=3d.$; 0625 of $1s.=\frac{3}{4}d$ · ·5 of $1d.=\frac{1}{2}d.$

$\therefore$ the value required $=3d+\frac{3}{4}d+\frac{1}{2}d=4\frac{1}{4}d.$

11s $9\frac{1}{4}d.$

12|11s. 9·25d. or 11s $9\frac{1}{4}d=141\frac{1}{4}d=565$q.

20|11 77083 £1 $=240d=960$q.

$5885416\dot{6}$ $\therefore \frac{565}{960}=5885416\dot{6}$

5 $\sqrt{(\dot{0}00\dot{1}9\dot{6})}=014$

and $014-140=\text{·}0001$

6. The int. on a share of Rs 1000 at 5 p c = Rs 50, and the value of the share = Rs 1245.

$\therefore$ the rate of interest = Rs. $\dfrac{50\times 100}{1245}$ = Rs $\frac{1000}{249}$.

And the int on a share of Rs. 75 at $4\frac{5}{6}$ p. c = Rs. $\frac{29}{6}\times\frac{75}{100}$ = Rs. $\frac{29}{8}$, and the value of the share = Rs 85

$\therefore$ the rate of interest = Rs. $\dfrac{29\times 100}{8\times 85}$ = Rs. $\frac{145}{34}$.

$\therefore$ Rs $\frac{1000}{249}$ Rs $\frac{145}{34}$. 6800 x $\therefore x=7221$.

7. $x^2-(x-y+z)(x+y-z)=x^2-\{x-(y-z)\}\{x+(y-z)\}$

$=x^2-\{x^2-(y-z)^2\}=y^2+z^2-2yz$

Similarly $y^2-(y-x+z)(y+x-z)=x^2+z^2-2xz$

and $z^2-(z-x+y)(z+x-y)=x^2+y^2-2xy$

$\therefore$ sum $=2\,(x^2+y^2+z^2-xy-xz-yz)$

8 $$\begin{array}{l} x+y+z-x^{\frac{1}{2}}y^{\frac{1}{2}}-y^{\frac{1}{2}}z^{\frac{1}{2}}-x^{\frac{1}{2}}z^{\frac{1}{2}} \\ \underline{x^{\frac{1}{2}}+y^{\frac{1}{2}}+z^{\frac{1}{2}}} \\ x^{\frac{3}{2}}+x^{\frac{1}{2}}y+x^{\frac{1}{2}}z-xy^{\frac{1}{2}}-x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}}-xz^{\frac{1}{2}} \\ xy^{\frac{1}{2}}+y^{\frac{3}{2}}+y^{\frac{1}{2}}z-x^{\frac{1}{2}}y-yz^{\frac{1}{2}}-x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} \\ \underline{xz^{\frac{1}{2}}+yz^{\frac{1}{2}}+z^{\frac{3}{2}}-x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}}-y^{\frac{1}{2}}z-x^{\frac{1}{2}}z} \\ \quad x^{\frac{3}{2}}+y^{\frac{3}{2}}+z^{\frac{3}{2}}-3x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}} \end{array}$$

(a) $$x^2-ax+a^2\,\big)\,x^8+a^4x^4+a^8\,\big(\,x^6+ax^5-a^3x^3+a^5x+a^6=\text{Quot.}$$

$$\begin{array}{l} \underline{x^8-ax^7+a^2x^6} \\ ax^7-a^2x^6+a^4x^4 \\ \underline{ax^7-a^2x^6+a^3x^5} \\ -a^3x^5+a^4x^4+a^8 \\ \underline{-a^3x^5+a^4x^4-a^5x^3} \\ a^5x^3 \qquad\qquad +a^8 \\ \underline{a^5x^3-a^6x^2+a^7x} \\ a^6x^2-a^7x+a^8 \\ \underline{a^6x^2-a^7x+a^8} \end{array}$$

Otherwise thus —

Divd. $=(x^4-a^2x^2+a^4)(x^4+a^2x^2+a^4)$

$=(x^4-a^2x^2+a^4)(x^2+ax+a^2)(x^2-ax+a^2)$.

$\therefore$ Quot $=(x^4-a^2x^2+a^4)(x^2+ax+a^2)$

9. Since $x^2-7x+10=(x-2)(x-5)$

and $x^2-9x+18=(x-3)(x-6)$

$\therefore$ fraction $=\dfrac{1}{2(x-1)}-\dfrac{1}{x-2}+\dfrac{1}{2(x-3)}$

$=\dfrac{(x-2)(x-3)-2(x-1)(x-3)+(x-1)(x-2)}{2(x-1)(x-2)(x-3)}$

Numr. $=x^2-5x+6-2(x^2-4x+3)+x^2-3x+2=2$

$\therefore$ Ans. $=\dfrac{1}{(x-1)(x-2)(x-3)}$

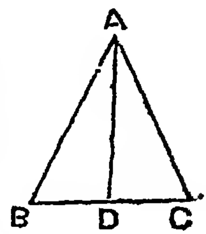
10. (a) $\dfrac{x-1}{3}-\dfrac{x-9}{2}+\dfrac{3x-2(x-2)}{7}=4\frac{1}{3}$.

or $14(x-1)-21(x-9)+6(x+4)=189$
$\therefore 14x-14-21x+189+6x+24=189$
$\therefore 20x-21x=\ -24+14$ $\qquad\therefore x=10$
(*b*) $5x+11y=146$ .... (1)
$11x+5y=110$ .. ..(2)
By Addt $16x+16y=256$ or $x+y=16$.
By Subtr $6x-6y=-36$ or $x-y=-6$
By Addt $2x=10$, $\therefore x=5$
By Subtr $2y=22$ $\therefore y=11$.

1864.—AFTERNOON.

Euclid I. Def. 4 and the shortest distance between any two points is another definition of a straight line. The 2nd is not properly a definition but a property of the straight line

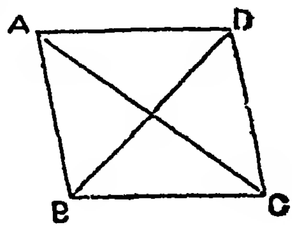
2 Euclid I 20



(*a*) In the Fig of Euc I. 20, bisect the $\angle$BAC by AD, meeting BC in D.
$\because$ $\angle$ADC > $\angle$BAD (I. 16) and $\angle$BAD = $\angle$CAD (Cons.)
$\therefore$ $\angle$ADC > $\angle$CAD $\quad\therefore$ AC > CD (I 18)
Likewise AB > DB.
$\therefore$ AB + AC > CD + DB or BC.

(*b*) A straight line is the shortest distance between any two points and the two sides of the triangle may be taken as a curved line. Therefore any two sides of a triangle are together greater than the third side.

3 Let ABCD be a four sided figure.
Join BD
$\because$ AD=BC and AB=DC,
and BD com (Hyp)
$\therefore$ $\angle$ABD=$\angle$BDC and
$\angle$ADB=$\angle$DBC (I. 8)
$\therefore$ AB is parallel to DC
and AD is parallel to BC (I 27).
Hence ABCD is a parallelogram



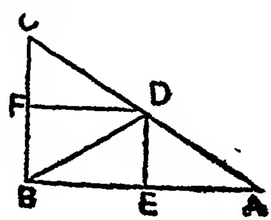
4 Euclid I. 32.

5 Euclid (1st and 2nd corollary of the 32 prop. Book I.)

6. Euclid II. 9.

7. Euclid III 22

8. Let ABC be the $\triangle$, ABC being rt. $\angle$.
At B make $\angle$ABD=$\angle$A meeting AC at D.
$\because$ $\angle$DAB=$\angle$ABD $\therefore$ BD=DA
Again $\angle$C+$\angle$A=$\angle$CBA
$\therefore$ $\angle$CBD=$\angle$C $\therefore$ BD=CD
$\therefore$ AD=BD=CD. $\qquad\therefore$ CA is double of BD, and D is the middle of AC.



9. Euclid IV. 12.

1865.—MORNING.

Examiners,— { REV J BARTON, M A
MR J REES }

1 Find the value of $11\frac{1}{8}+14\frac{5}{8}+21\frac{7}{40}+32\frac{3}{625}$, both by vulgar fractions and by decimals, showing that the two results coincide, and reduce $25^{\circ}\ 36'\ 45''$ to the decimal of 75°.

2 Find the product of the sum and difference of ·0421 and ·0029, and divide one-tenth of the square root of that product by ten times the continued product of ·02, ·03, and ·07.

3 How many yards of matting 3·5 feet wide will cover the floor of a room 85·3 feet long and 40·5 feet broad; and how much will it cost, at 2 rupees 10 annas and 8 pie per *square* yard?

4. If the wages of 25 men amount to 766 rupees 10 annas 8 pie in 16 days, how many men must work 24 days to receive 1035 rupees, the daily wages of the latter being one-half those of the former?

5 What principal in 3 years 73 days will amount to 100 rupees 15 annas, at $6\frac{1}{4}$ per cent simple interest? A bill for 5035 rupees 4 annas, drawn on September 12th at 5 months, was discounted on January 16th at 4 per cent, what was the discount charged?

6 Divide the continued product of $1+x+y$, $1+x-y$, $1-x+y$ and $x+y-1$ by $1-2xy-x^2-y^2$, and resolve $4(uz-xy)^2-(u^2-x^2-y^2+z^2)^2$ into four factors.

7 Find the greatest common measure of $2x^5-11x^2-9$ and $4x^5+11x^4+81$, and reduce $\dfrac{x^3-6x^2-37x+210}{x^3+4x^2-47x-210}$ to its lowest terms.

8 Simplify as much as possible *any one* of the following:—

(i) $\dfrac{x^3}{(x-y)(x-z)}+\dfrac{y^3}{(y-x)(y-z)}+\dfrac{z^3}{(z-x)(z-y)}$.

(ii) $\dfrac{1}{x(x-y)(x-z)}+\dfrac{1}{y(y-x)(y-z)}+\dfrac{1}{z(z-x)(z-y)}$.

(iii) $\dfrac{x^2-yz}{(x-y)(x-z)}+\dfrac{y^2+zx}{(y+z)(y-x)}+\dfrac{z^2+xy}{(z-x)(z+y)}$.

9 Find the value of $\dfrac{x+2a}{x-2a}+\dfrac{x+2b}{x-2b}$ when $x=\dfrac{4ab}{a+b}$.

10. Solve *any two* of the following equations:—

(i) $\dfrac{x-a}{b}+\dfrac{x-b}{c}+\dfrac{x-c}{a}=\dfrac{x-(a+b+c)}{abc}$.

(ii) $\frac{1}{2}\left(x-\frac{a}{3}\right)-\frac{1}{3}\left(x-\frac{a}{4}\right)+\frac{1}{4}\left(x-\frac{a}{5}\right)=0$.

(iii) $\frac{x-1}{x-2}-\frac{x-2}{x-3}=\frac{x-5}{x-6}-\frac{x-6}{x-7}$

(iv) $\left(\frac{a^2}{x}+b\right)^{\frac{1}{2}}-\left(\frac{a^2}{x}-b\right)^{\frac{1}{2}}=c^{\frac{1}{2}}$.

1865.—AFTERNOON.

Examiners,— { REV J CARBONELLE
REV. MR. D CARNDUFF.

1 What are parallel straight lines? When is a straight line said to be perpendicular to another? What are the complements of a parallelogram? What is a gnomon?

2 Define a circle, a sector of a circle, a segment of a circle; when is a sector also a segment? When is a circle said to be *inscribed* in a plane rectilineal figure? When is a plane rectilineal figure said to be inscribed in a circle?

3 If two triangles have two sides of the one equal to two sides of the other, each to each, but the angle contained by the two sides of the one greater than the angle contained by the two sides, equal to them of the other, the base of that which has the greater angle shall be greater than the base of the other, required a proof

4 Resolve *any one* of the following —

(*a*) Given one of the sides of a right-angled triangle containing the right angle and the sum of the other two sides, to construct the triangle

(*b*) Given one of the sides of a right-angled triangle containing the right angle and the difference of the other two sides construct the triangle

(*c*) The straight line drawn from the right angle of a right-angled triangle to the middle of the opposite side, is equal to the half of that side.

5. If a straight line be divided into two equal parts, and also into two unequal parts, the rectangle contained by the unequal parts together with the square of the line between the points of section is equal to the square of half the line; required a proof.

6 Resolve *either* of the following —

(*a*) Divide a given straight line into two parts, so that the rectangle contained by them shall be equal to a given square

(*b*) Produce a given straight line so that the rectangle contained by the whole line thus produced and the part of it produced shall be equal to a given square

7 The angles in the same segment of a circle are equal to one another, required a proof

8 Demonstrate *either* of the following —

(*a*) If a rectilineal figure of an even number of sides be inscribed in a circle, the first, third, fourth, &c angles are together equal to the second, fourth, sixth, &c. angles taken together, any angle being assumed as the first.

(*b*) If a circle be inscribed in any triangle, the points of contact shall divide the sides into segments such that any one side together with the remote segment of either of the other two sides shall be equal to half the sum of the three sides

SOLUTIONS

1865.—MORNING.

1. $11\tfrac{3}{5}+14\tfrac{5}{8}+21\tfrac{7}{40}+32\tfrac{3}{625}$

$$=78+\frac{3000+3125+875+24}{5000}=78+\tfrac{7024}{5000}$$

$$=78+1\tfrac{2024}{5000}=79\tfrac{253}{625}=79\cdot4048$$

or the sum $=11\cdot6+14\cdot625+21\cdot175+32\cdot0048=79\cdot4048$.

And $25°, 36', 45''=25\tfrac{49}{80}$

$$\therefore \text{the fraction required}=\frac{25\frac{49}{80}}{75}=\frac{2049}{80\times75}=\tfrac{683}{2000}=\cdot3415$$

2 $(\cdot0421+\cdot0029)(\cdot0421-\cdot0029)=\cdot045\times\cdot0392=\cdot001764$

$\therefore \tfrac{1}{10}\sqrt{(\cdot001764)}\div(10\times\cdot02\times\cdot07\times\cdot03)$

$=\tfrac{1}{10}\times\cdot042\div\cdot00042=\cdot0042\div\cdot00042=10$

3 The area of the room $=85\tfrac{3}{10}$ ft $\times\ 40\tfrac{1}{2}$ ft $=\dfrac{853\times81}{10\times2}$ sq ft.

$\therefore$ the length of the matting required

$$=\frac{853\times81\times2}{10\times2\times7}\text{ ft}=\frac{69093}{70}\text{ ft}=\frac{23031}{70}\text{ yds}=329\tfrac{1}{70}\text{ yds}$$

Rs 2 10as 8 pie $=$ Rs $2\tfrac{2}{3}=$ the cost per sq yd.

$$\therefore \text{the whole cost}=\frac{853\times\overset{3}{\overset{9}{\cancel{81}}}}{\underset{5}{\cancel{10}}\times\cancel{2}}\times\frac{1}{\cancel{9}}\times\frac{\overset{2}{\overset{4}{\cancel{8}}}}{\cancel{3}}=\text{Rs }\tfrac{5118}{5}$$

$=$ Rs 1023 9as $7\tfrac{1}{5}$ pie

4. Rs $766\tfrac{2}{3}$: Rs 1035 ; 24 days : 16 days ; $\tfrac{1}{2}$: 1 $\Big\}$ :: 25 men : x

$$\therefore x=\frac{1035\times16\times25\times3\times2}{2300\times24}\text{ men}$$

$$=\frac{45\times23\times4\times4\times25\times6}{23\times4\times25\times4\times6}\text{ men}=45\text{ men}.$$

5 The interest on Rs 100 for $3\tfrac{1}{5}$ yrs at $6\tfrac{1}{4}$ p c

$=$ Rs $\dfrac{25\times16}{4\times5}=$ Rs 20, and Rs. $(100+20)=$ Rs 120.

Rs. 120 Rs. $100\frac{15}{16}$. Rs 100 x

$\therefore x = \text{Rs. } \frac{1615 \times 100}{16 \times 120} = \text{Rs } \frac{8075}{96} = \text{Rs } 84 \text{ 1a. 10p}$

(a) The bill is due on the 15th of February

From Jan. 16, to Feb 15, there are $(15+15)=30$ days

The int on Rs 100 at 4 p c for 30 days $=\frac{24}{73}$ Rs

$\therefore 100\frac{24}{73} \quad 5035\frac{1}{4}. \quad \frac{24}{73}$ discount

$\therefore$ discount $= \text{Rs } \frac{24 \times 20141 \times 73}{73 \times 4 \times 7324} = \text{Rs. } 16 \text{ 8as}$

6 Since $1+2xy-x^2-y^2=1-(x^2+y^2-2xy)=1-(x-y)^2$

$=(1+x-y)(1-x+y)$

$\therefore$ Ans. $=(1+x+y)(x+y-1)=(x+y)^2-1=x^2+y^2+2xy-1.$

(b) $4(uz-xy)^2-(u^2-x^2-y^2+z^2)^2$

$=\{2(uz-xy)\}^2-(u^2-x^2-y^2+z^2)^2$

$=(2uz-2xy+u^2-x^2-y^2+z^2)(2uz-2xy-u^2+x^2+y^2-z^2)$

$=\{(u+z)^2-(x+y)^2\}\{(x-y)^2-(u-z)^2\}$

$=(u+z+x+y)(u+z-x-y)(x-y+u-z)(x-y-u+z)$

7. $2x^5-11x^2-9 \,)\, 4x^5+11x^4+81 \,(\, 2$

$4x^5-22x^2-18$

$11 \,|\, (11x^4+22x^2+99) = x^4+2x^2+9$

$x^4+2x^2+9 \,)\, 2x^5-11x^2-9 \,(\, 2x$

$2x^5+4x^3+18x$

$4x^3+11x^2+18x+9 \,)\, 4x^4+8x^2+36 \,(\, x-11$

$4x^4+11x^3+18x^2+9x$

$-11x^3-10x^2-9x+36$

4

$-44x^3-40x^2-36x+144$

$-44x^3-121x^2-198x-99$

$81 \,(\, 81x^2+162x+243$

x^2+2x+3

$x^2+2x+3 \,)\, 4x^3+11x^2+18x+9 \,(\, 4x+3$

$4x^3+8x^2+12x$

$3x^2+6x+9$

$3x^2+6x+9$

$\therefore x^2+2x+3$ is the G. C M.

(b) Here G C M $=x^2-x-42$

$\therefore$ Ex. $=\dfrac{(x^2-x-42)(x-5)}{(x^2-x-42)(x+5)}=\dfrac{x-5}{x+5}$

8. (1) Fraction

$$=\frac{x^3}{(x-y)(x-z)}-\frac{y^3}{(x-y)(y-z)}+\frac{z^3}{(x-z)(y-z)}$$

$$=\frac{x^3(y-z)-y^3(x-z)+z^3(x-y)}{(x-y)(x-z)(y-z)}$$

Numerator $=x^3y-x^3z-y^3x+y^3z+z^3x-z^3y$

$=x^3(y-z)+yz(y^2-z^2)-x(y^3-z^3)$

$=(y-z)\{x^3+yz(y+z)-x(y^2+yz+z^2)\}$

$=(y-z)\{x(x^2-y^2)-yz(x-y)-z^2(x-y)\}$

$=(y-z)(x-y)\{x^2+xy-yz-z^2\}$

$=(y-z)(x-y)\{(x^2-z^2)+y(x-z)\}$

$=(y-z)(x-y)(x-z)(x+y+z)$ Ans $=x+y+z$.

(2) Fraction

$$=\frac{1}{x(x-y)(x-z)}-\frac{1}{y(y-z)(x-y)}+\frac{1}{z(x-z)(y-z)}$$

$$=\frac{yz(y-z)-xz(x-z)+xy(x-y)}{xyz(x-y)(x-z)(y-z)}.$$

Numr $=y^2z-yz^2-x^2z+xz^2+x^2y-xy^2$

$=(x-y)(x-z)(y-z)$,

$\therefore$ Ans $=\dfrac{1}{xyz}$.

(3) Fraction

$$=\frac{x^2-yz}{(x-y)(x-z)}-\frac{y^2+zx}{(y+z)(x-y)}-\frac{z^2+xy}{(x-z)(y+z)}$$

$$=\frac{(y+z)(x^2-yz)-(y^2+xz)(x-z)-(x-y)(z^2+xy)}{(x-y)(x-z)(y+z)}$$

Numr $=x^2y+x^2z-y^2z-yz^2-y^2x-x^2z+zy^2+xz^2-xz^2$
$-x^2y+z^2y+xy^2=0$ $\therefore$ Ans. $=0$.

9. $\dfrac{x+2a}{x-2a}+\dfrac{x+2b}{x-2b}$

$$=\frac{(x+2a)(x-2b)+(x-2a)(x+2b)}{(x-2a)(x-2b)}$$

$$=\frac{x^2+2(a-b)x-4ab+x^2-2(a-b)x-4ab}{x^2-2(a+b)x+4ab}$$

$$=\frac{2(x^2-4ab)}{x^2-8ab+4ab}=\frac{2(x^2-4ab)}{x^2-4ab}=2,$$

since $(a+b)x=4ab$

10 (1) $\frac{x-a}{b}+\frac{x-b}{c}+\frac{x-c}{a}=\frac{x-(a+b+c)}{abc}$.

$\therefore ac(x-a)+ba(x-b)+bc(x-c)=x-(a+b+c)$

$\therefore (ac+ab+bc-1)x=-a-b-c+a^2c+ab^2+bc^2$

$\therefore x=\frac{a^2c+ab^2+bc^2-a-b-c}{ac+ab+bc-1}$

(2) See Ex 5 (1), 1859

(3) $\frac{x-1}{x-2}-\frac{x-2}{x-3}=\frac{x-5}{x-6}-\frac{x-6}{x-7}$

$\therefore \left(1+\frac{1}{x-2}\right)-\left(1+\frac{1}{x-3}\right)=\left(1+\frac{1}{x-6}\right)-\left(1+\frac{1}{x-7}\right)$

or $\frac{1}{x-2}-\frac{1}{x-3}=\frac{1}{x-6}-\frac{1}{x-7}$

$\therefore \frac{-1}{x^2-5x+6}=\frac{-1}{x^2-13x+42}$

$\therefore x^2-5x+6=x^2-13x+42$

$\therefore 13x-5x=42-6, \quad \therefore 8x=36, \quad \therefore x=4\frac{1}{2}.$

(4) $\left(\frac{a^2}{x}+b\right)^{\frac{1}{2}}-\left(\frac{a^2}{x}-b\right)^{\frac{1}{2}}=c^{\frac{1}{2}}$

sq $\frac{a^2}{x}+b+\frac{a^2}{x}-b-2\sqrt{\left(\frac{a^2+bx}{x}\times\frac{a^2-bx}{x}\right)}=c$

or $-\frac{2}{x}\sqrt{(a^4-b^2x^2)}=c-\frac{2a^2}{x}=\frac{cx-2a^2}{x}$

Multiply by x and square

$\therefore 4(a^4-b^2x^2)=c^2x^2+4a^4-4a^2cx$

$\therefore c^2x^2+4b^2x^2=4a^2cx \qquad \therefore x=0$

or $x(c^2+4b^2)=4a^2c \qquad \therefore x=\frac{4a^2c}{4b^2+c^2}$

1865 —AFTERNOON

1 Euclid I Def 37, Def, 10; Euclid II Def 2

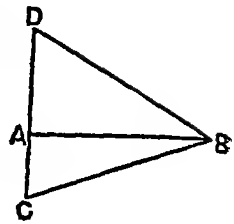
If a point be taken in the diagonal of a parallelogram and from it straight lines be drawn parallel to the sides, then the parallelogram is divided into four parallelograms of which the two through which the diagonal passes are parallelograms about the diagonal; and the other two which complete the whole figure are called complements of the parallelograms about the diagonal

2 Euclid I Def. 15 Euclid III Def. 10, Def 6. Euclid IV. Def 5, Def. 3

A sector is a segment when the two bounding radii of the sector are in one straight line, i e, when it becomes a semicircle

3 Euclid I 24

4 (*a*) and (*b*) Let the two str. lines, AB, one of the sides, and AC, the sum of difference of the other two sides, meet at A and be at rt $\angle$s to each other



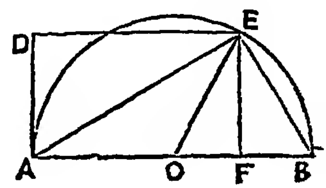
Join CB
Then as AC is $>$ or $<$ AB (I 20)
the $\angle$ at B is $>$ or $>$ $\angle$ at C (I 19)
At the point B in the str. line CB
make the $\angle$ CBD$=\angle$ at C, meeting CA or AC prod in D (I. 23)
then CD$=$BD (I. 6)
$\therefore AC=DC\pm DA=BD\pm AD.$

Hence ABD is the reqd $\triangle$, having the rt. $\angle$ at A.

(*c*) See Solution of question 8 of 1864

5. Euclid II. 5

6 (*a*) Let AB be the given st line to be divided



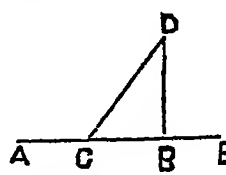
Draw AD $\perp$ AB and$=$the side of the given square
Desc a $\frac{1}{2}$ $\odot$ upon AB
From D draw DE $\parallel$ AB cutting the $\odot$ in E
From E draw EF$\perp$AB.
Then F is the reqd point of division.

Let C the centre of the circle and join CE.

$\because AF.FB+CF^2=CB^2=CE^2=CF^2+EE^2$ (II. 5)

$\therefore AE\ FB=EF^2=AD^2=$the given square. (I. 47).

(*b*) Let AB be the given str line
Bisect it in the point C



At B draw BD$\perp$AB and make BD$=$the side of the given square

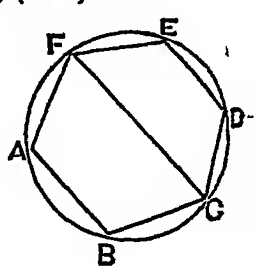
Join CD
Produce AB to E making CE$=$CD
Then E is the point

$\because AE\ EB+CB^2=CE^2=CD^2=CB^2+BD^2$, (I 47) II. 6.

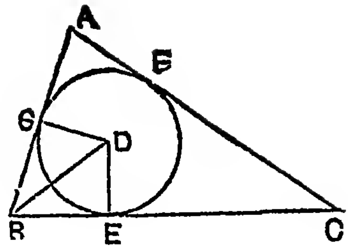
$\therefore AE\ EB=BD^2=$the given square

7. Euclid III. 21

8 (*a*) Let ABCDEF be the rectilineal fig inscribed within a $\odot$.



Join FC
$\angle FAB+\angle BCF=2$ rt $\angle$s
$=\angle ABC+\angle AFC$ (III 22)
Also $\angle FCD+\angle DEF=2$ rt $\angle$s
$=\angle CDE+\angle EFC$ (III 22)
$\therefore \angle FAB+\angle BCD+\angle DEF=\angle ABC$
$+\angle CDE+\angle EFA.$



(*b*) Let ABC be the $\triangle$ and GEF the $\odot$ touching AB, BC and CA at G, E and F.

Let D be the centre.

Join GD, BD, DE

Then $\angle$DGB and $\angle$DEB each = a rt $\angle$ (III 10)

$\therefore BD^2 = BG^2 + GD^2 = BE^2 + ED^2$ (I 47)

But GD = DE.

$\therefore BG = BE^2 \therefore BG = BE$.

Likewise EC = CF and AF = AG

Wherefore AB + FC = BC + AF.

I. 1866 —MORNING

Examiners,— { Mr. R Thwaytes, B.A.
Mr J S Rees }

1 Add together $\frac{5}{16}$, $\frac{7}{32}$, $\frac{3}{8}$, ·046875 and 1·23

Simplify $\dfrac{\cdot0075+2\cdot1}{\cdot0175}$ and $\dfrac{4\cdot255+\cdot0064}{\cdot00032}$

2 Find by Practice the value of 1 ton 5 cwt 2 qrs. 14lbs at £3 15s 7*d* per cwt.

3 Find the square root of 1524·9025 and of 152·49025 to three places of decimals and the value of $\dot{0}09\dot{9}$ of £1 5s 3*d*

4. Three gardeners working all day can plant a field in ten days but one of them having other employment can only work half the time. How long it will take them to complete the work?

5 Find the Compound Interest of £55 for one year, payable quarterly at five per cent per annum

6 If $a=1$, $b=2$, $c=-\frac{1}{2}$, $d=0$, find the value of

$$\frac{a-b+c}{a-b-c}-\frac{ad-bc}{bd-ac}-\sqrt{\left(\frac{b^3}{a^3}-\frac{a^3}{c^3}\right)}$$

7. Divide $6a^3-17a^2b+7ab^2-5b^3$ by $2a-5b$ and x^5-x^{-5} by $x-x^{-1}$ and find the continued product of $ax-1$, $x^2+\frac{1}{a^2}$, and $ax+1$.

8 Find the G C M of $x^3+6x^2+11x+6$ and $x^3+9x^2+27x+27$. and the L C M of xy, $x-y$ and y^3-x^2y.

9. Simplify the following —

(1) $$\frac{x+3y}{4(x+y)(x+2y)}+\frac{x+2y}{(x+y)(x+3y)}-\frac{x+y}{4(x+2y)(x+3y)}$$

(2) $$\frac{a^2+3a+2}{a^2+2a+1}\times\frac{a^2+5a+4}{a^2+7a+10}$$

10. Extract the cube root of

$$x^6+6x^5+21x^4+44x^3+63x^2+54x+27.$$

11. Solve the equations —

$$(1)\quad \frac{x-3}{7}-\frac{\frac{x}{2}-3}{3}=\frac{\frac{x}{6}+2}{2}-\frac{x-6}{3}+\frac{x}{8}.$$

$$(2)\ \frac{x-a}{b-a}=\frac{x-b}{a-b}. \qquad (3)\ \frac{p-q}{qx+r}=\frac{p+q}{px-r}$$

12. A is twice as old as B and 4 years older than C. The sum of the ages of A, B, and C is 96 years Find B's age

I. 1866 —AFTERNOON.

Examiners,— { REV K S MACDONALD, M A / MR. J SIME, B A

1 Define superficies, rhomboid, plane angle, parallel straight lines, centre of a circle, acute-angled triangle

2 Any two sides of a triangle are together greater than the third

3. The straight lines which join the extremities of two equal and parallel straight lines towards the same parts, are also themselves equal and parallel

AB is parallel to CD and unequal to it, and they are joined towards the same parts by the straight lines AC and BD. If AC is equal to BD, show that AD is equal to BC

4 If any two points be taken in the circumference of a circle, the straight line which joins them shall fall within the circle

5 If two straight lines cut one another within a circle, the rectangle contained by the segments of one of them is equal to the rectangle contained by the segments of the other

6 In a circle, the angle in a semicircle is a right angle . but the angle in a segment greater than a semicircle is less than a right angle and the angle in a segment less than a semicircle is greater than a right angle

7 Inscribe a circle in a given triangle

Describe a circle which shall touch a given straight line at a given point, and pass through another given point

8 Inscribe a circle in a given equilateral and equiangular pentagon

9 Enumerate the Propositions from which the following corollaries are derived —

(1) Every equiangular triangle is also equilateral

(2) All the exterior angles of any rectilineal figure are together equal to four right angles

(3) The difference of the squares of two unequal lines is equal to the rectangle contained by their sum and difference

II 1866 —MORNING.

1 Reduce $3^\circ\ 45'\ 36''.25$ to the decimal of 36°

Simplify $\left(\frac{1}{3}+\frac{3}{1\frac{3}{5}}+\frac{11}{14}+\frac{5}{9}-1\right)-\frac{5}{9}$ of $\frac{6}{8}$ of $2\frac{1}{7}$.

2 Find the value of 6 cwt. 2 qrs 7lbs at £3 4s $6\frac{1}{2}d$ per cwt

3. Find the square of 0 0204 and the square root of 81·757764; and divide one-tenth of the latter result by one hundred times the former

Divide 0·1001 by 0 000390625, and 10 01 by 390 625

5 What is the expense of paving a rectangular verandah whose length is 42 feet and breadth 15 feet, with Burdwan paving stones 18 inches square, and which cost 15 Rs per score?

6 The 3 per cents are at $85\frac{1}{8}$, what price should the $3\frac{1}{2}$ per cents bear, that an investment may be made with equal advantage in either stock? And what interest would be derived by so investing £5,000?

7. Find the product of the four factors—

$$x+y+z,\ x+y-z,\ x+z-y,\ z+y-x$$

Multiply $x^{\frac{2}{3}}+x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}}$ by $x^{\frac{1}{3}}-y^{\frac{1}{3}}$.

Divide $(x+y+z)(xy+xz+yz)-xyz$ by $x+y$

8 Reduce to its simplest form —

$$\frac{x^2-(y-z)^2}{(x+z)^2-y^2}+\frac{y^2-(x-z)^2}{(x+y)^2-z^2}+\frac{z^2-(x-y)^2}{(y+z)^2-x^2}$$

Find the greatest common measure of

$$2x^5-11x^2-9 \text{ and } 4x^5+11x^4+81.$$

9. Extract the square root of

$$\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12,$$

and show that

$$(x-y)^3+(y-z)^3+(z-y)^3=3(x-y)(y-z)\,z-x)$$

10 Solve the equations :—

$$\frac{5-3x}{4}+\frac{5x}{3}=\frac{3}{2}-\frac{3-5x}{3}$$

$$5x+\frac{\cdot 02x+\ 07}{03}-\frac{x+2}{9}=9\ 5$$

II 1866.—AFTERNOON.

1 Define a *plane rectilineal angle*, a *right angle*, and a *rectangle*. What is *a segment of a circle*, what *a sector*? What are *similar segments of circles*? Give Euclid's definition of a *regular polygon*

2 Upon the same base and on the same side of it, there cannot be two triangles that have their sides which are terminated in one extremity of the base equal to one another and likewise those which are terminated in the other extremity

Why does Euclid not prove the remaining case of this proposition?

3 The complements of the parallelograms which are about the diagonal of any parallelogram are equal to one another.

4 If a straight line be divided into two equal parts and also into two unequal parts, the rectangle contained by the unequal parts together with the square on the line between the points of section is equal to the square on half the line State and prove this algebraically *only*

5 (*a*) Divide a given straight line into two parts, so that the rectangle contained by the whole line and one of the parts shall be equal to the square on the other part.

(*b*) Produce a given straight line to a point such that the rectangle contained by the whole line thus produced, and the part produced, shall be equal to the square on the given straight line

6 One circle cannot touch another in more points than one whether it touches it on the inside or outside

7 Describe an isosceles triangle having each of the angles at the base double of the third angle

8 ABC is an isosceles triangle of which B is the vertex; BA, BC are bisected in D and E respectively, AE, CD intersect in F Show that the triangle BDE is equal to three times the triangle DEF.

9. Construct a rectangle that shall be equal to a given square, the difference of two adjacent sides being given

10 If a tangent of a circle be parallel to a chord, prove that the intercepted arc is bisected in the point of contact of the tangent

11 To describe a circle that shall touch a given line and also touch a given circle.

SOLUTIONS

I 1866.—MORNING.

1 $\frac{5}{16}+\frac{7}{32}+\frac{3}{8}+046875+1\ 23$

$= 3125+21875+375+{\cdot}046875+1\ 23=2\ 183125$

(*a*) Ans $= \frac{2\ 1075}{0175} = \frac{21075}{175} = 120\frac{3}{7} = 120\ 428571$

(*b*) Ans $= \frac{4\ 2614}{00032} = \frac{426140}{32} = 13316\ 875$

2

	£	s	d	
2 qrs = ½ of 1 cwt	3	15	7	= value of 1 cwt
			5	
	18	17	11	= value of 5 cwt
			4	
	75	11	8	= value of 20 cwt or 1 ton
	18	17	11	= value of 5 cwt
14 lbs. = ¼ of 2 qrs	1	17	9½	= value of 2 qrs
		9	$5\frac{3}{8}$	= value of 14 lbs.
	£96	16	$9\frac{7}{8}$	= value of 1 ton 5 cwt. 2 qrs, 14lbs.

3 $\sqrt{(1524\ 9025)} = 39\ 05$

$\sqrt{(\dot{1}5\dot{2}\ 4\dot{9}0\dot{2}5\dot{0})} = 12\ 348$ . .

and $\dot{0}09\dot{9}$ of £1 5*s* 3*d* $= \frac{33}{3333}$ of 303*d.* = 3*d*

4\. 2½ men : 3 men : 10 days no of days

$$\therefore \text{no. of days} = \frac{10 \times 3 \times 2}{5} = 12$$

5 The quarterly interest of £1 at 5 per cent = $£\frac{1}{80}$.

£
8,0) 55 = 1st quarter's principal.

6875 = 1st quarter's interest

8,0) 55 6875 = 2nd quarter's principal

69609375 = 2nd quarter's interest.

8,0) 56 38359375 = 3rd quarter's principal.

·704794921875 = 3rd quarter's interest.

8,0) 57 08388671875 = 4th quarter's principal.

7136048583984375 = 4th quarter's interest

57 8019935302734375 = 4th quarter's amount
55

£2 8019935302734375 = reqd interest.
20

s 16 03987060546875
12

d 0 478447265625
4

q 1 9137890625

Hence reqd. interest = £2 16*s* 0¼*d* 9137890625*q*.

6\. Ans $= \frac{1-2-\frac{1}{2}}{1-2+\frac{1}{2}} - \frac{2 \times \frac{1}{2}}{1 \times \frac{1}{2}} - \sqrt{\left(\frac{8}{1} - \frac{1}{-\frac{1}{8}}\right)}$

$$= \frac{-1\frac{1}{2}}{-\frac{1}{2}} - 2 - \sqrt{(8+8)} = 3 - 2 - 4 = -3.$$

6. (a) $2a-5b\,)\,6a^3-17a^2b+7ab^2-5b^3\,(\,3a^2-ab+b^2$

$$\begin{array}{r}
6a^3-15a^2b \\
\hline
-2a^2b+7ab^2 \\
-2a^2b+5ab^2 \\
\hline
2ab^2-5b^3 \\
2ab^2-5b^3 \\
\hline
\end{array}$$

(b) $x-x^{-1}\,)\,x^5-x^{-5}\,(\,x^4+x^2+1+x^{-2}+x^{-4}$

$$\begin{array}{l}
x^5-x^3 \\
\hline
x^3-x^{-5} \\
x^3-x \\
\hline
x-x^{-5} \\
x-x^{-1} \\
\hline
x^{-1}-x^{-5} \\
x^{-1}-x^{-3} \\
\hline
x^{-3}-x^{-5} \\
x^{-3}-x^{-5} \\
\hline
\end{array}$$

(c) Ans. $=(a^2x^2-1)\left(x^2+\frac{1}{a^2}\right)=a^2x^4-x^2+x^2-\frac{1}{a^2}=a^2x^4-a^{-2}.$

8. $x^3+6x^2+11x+6\,)\,x^3+9x^2+27x+27\,(\,1$

$$\begin{array}{r}
x^3+6x^2+11x+6 \\
\hline
3x^2+16x+21\,)\,x^3+6x^2+11x+6 \\
3 \\
\hline
3x^3+18x^2+33x+18\,(\,x \\
3x^3+16x^2+21x \\
\hline
2x^2+12x+18 \\
3 \\
\hline
6x^2+36x+54\,(\,2 \\
6x^2+32x+42 \\
\hline
4)\,4x+12 \\
\hline
x+3
\end{array}$$

$x+3\,)\,3x^2+16x+21\,(\,3x+7$

$$\begin{array}{r}
3x^2+9x \\
\hline
7x+21 \\
7x+21 \\
\hline
\end{array}$$

$\therefore$ G. C. M. $=x+3$.

(a) Since $y^3 - x^2y = y(y^2 - x^2) = -y(x+y)(x-y)$

$\therefore$ L C M. $= xy(y^2 - x^2)$

9. (1) Ans $= \dfrac{(x+3y)^2 + 4(x+2y)^2 - (x+y)^2}{4(x+y)(x+2y)(x+3y)}$.

Num $= x^2 + 6xy + 9y^2 + 4x^2 + 16xy + 16y^2 - (x^2 + 2xy + y^2)$

$= 4(x^2 + 5xy + 6y^2) = 4(x+2y)(x+3y)$

$\therefore$ Ans $= \dfrac{1}{x+y}$.

(2) Ans $= \dfrac{(a+1)(a+2)}{(a+1)^2} \times \dfrac{(a+1)(a+4)}{(a+2)(a+5)} = \dfrac{a+4}{a+5}$

10. $x^6 + 6x^5 + 21x^4 + 44x^3 + 63x^2 + 54x + 27 \;(\; x^2 + 2x + 3$

x^6

$3x^4 + 6x^3 + 4x) \quad 6x^5 + 21x^4 + 44x^3$

$\qquad\qquad\qquad\quad 3x^6 + 12x^4 + 8x^3$

$x3^4 + 12x^3 + 21x^2 + 18x + 9) \quad 9x^4 + 36x^3 + 63x^2 + 54x + 27$

$\qquad\qquad\qquad\qquad\qquad\quad 9x^4 + 36x^3 + 63x^2 + 54x + 27$

11. (1) $\dfrac{x-3}{7} - \dfrac{\frac{x}{2} - 3}{3} = \dfrac{\frac{x}{6} + 2}{2} - \dfrac{x-6}{3} + \dfrac{x}{8}$

or $24x - 72 - 28x + 168 = 14x + 168 - 56x + 336 + 21x$

or $-4x + 96 = -21x + 504$

$\therefore 17x = 408$, $\qquad \therefore x = 24$

(2) $\dfrac{x-a}{b-a} = \dfrac{x-b}{a-b}$,

or $\dfrac{x-a}{b-a} = \dfrac{b-x}{b-a}$, $\qquad \therefore x - a = b - x$

$\therefore 2x = a + b$, $\qquad \therefore x = \frac{1}{2}(a+b)$

(3) $\dfrac{p-q}{qx+r} = \dfrac{p+q}{px-r}$,

or $(px - r)(p - q) = (p + q)(qx + r)$

or $p^2x - pr - pqx + qr = pqx + q^2x + pr + qr$

$\therefore (p^2 - 2pq - q^2)x = 2pr$, $\qquad \therefore x = \dfrac{2pr}{p^2 - 2pq - q^2}$

12 Let x = age of B,

then $2x$ = age of A,

and $2x - 4$ = age of C

By the question,

$x + 2x + 2x - 4 = 96$, $\qquad \therefore 5x = 100$, $\qquad \therefore x = 20$.

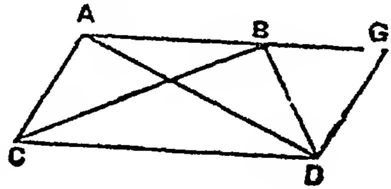
I. 1866.—AFTERNOON.

1 Euclid I Def 5; Def 35 Def 8, Def 37, Def 31

The centre of a circle is that point within the circle, from which straight lines drawn to the circumference, are equal

2 Euclid I 20

3 Euclid I 33



(*a*) Prod AB to G making AG =CD.

Join GD

∵ AC=GD (Hyp)

∴ AG=GD (I 33);

but AC=BD (Hyp) ∴ BD = GD

and ∠DBG=∠BGD (I 5)

But ∠ACD=∠AGD (I 34)

∴ ∠ACD=∠DBG ;

also ∠GBD=∠BDC (I. 29), ∴ ∠ACD=∠BDC ;

also ∵ AC=BD and CD com ; ∴ AD=BC (I 4)

4 Euclid III. 2

5 Euclid III 15

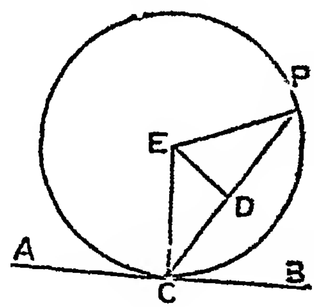
6 Euclid III 31

*7 Euclid IV, 4

(a) Let AB be the given str line and C a given point in it

Let P be the other given point

It is required to describe a ⊙ which shall touch the given str line at C and shall pass through the pt P



Join CP and bisect it at D

Draw DE⊥CP and draw CE⊥AB intersecting DE at E

Join EP

∵ CD=DP and ED com

and ∠EDC=∠EDP ;

∴ EC=EP (I 4)

∴ a ⊙ desc with E as centre and EC as radius will pass through P,

and ∵ AB⊥EC

AB touches the ⊙ (Cor III. 16)

8 Euclid IV 13

9 (1) Euc 1 6 (2) Euc 1. 32 (3) Euc. 11 5

II 1866.—MORNING.

1 3° 45′ 36″ 25

60|36″ ·25

60|45′ 60416

36 = 6×6 6|3° 7600694

6| 6266782407̇

104446373456...

(a) $\left(\frac{1}{3}+\dfrac{\frac{3}{7}}{1\frac{3}{5}}+\frac{11}{14}+\frac{5}{9}-1\right)\div\frac{5}{9}$ of $\frac{5}{8}$ of $2\frac{5}{7}$

$=\left(\frac{1}{3}+\frac{3}{7}\times\frac{5}{8}+\frac{11}{14}+\frac{5}{9}-1\right)\div\dfrac{5\times5\times19}{9\times8\times7}$

$=\left(\frac{1}{3}+\frac{15}{56}+\frac{11}{14}+\frac{5}{9}-1\right)\div\frac{475}{504}$

$=\left(\dfrac{168+135+396+280-504}{504}\right)\div\frac{475}{504}$

$=\dfrac{979-504}{504}\div\frac{475}{504}=\frac{475}{504}\times\frac{504}{475}=1$

	£.	s	d	
2 qrs $=\frac{1}{2}$ of 1 cwt	3	4	$6\frac{1}{2}$	= cost of 1 cwt
			6	
	19	7	3	= cost of 6 cwts.
7lbs $=\frac{1}{8}$ of 2 qrs	1	12	$3\frac{1}{4}$	= cost of 2 qrs
		4	$0\frac{13}{32}$	= cost of 7lbs
	£21	3s	$6\frac{1}{2}d.\ \frac{5}{8}q$	= cost of 6 cwts 2 qrs 7lbs.

3 $\cdot0204\times\cdot0204=\cdot00041616$

$\sqrt{81\cdot757764}=9\cdot042$

and $\dfrac{9\cdot042}{10}-(100\times\cdot00041616)=\cdot9042-\cdot041616=21\ 7272\ldots\ldots$

4 $1\cdot0010000-\cdot000390625=256\ 526$

$1001000-390\ 625=0256256$

5 The area of the verandah $=(42\times15)$ sq. ft. and that of each stone $=(18\times18)$ sq. in. $=\dfrac{18\times18}{12\times12}$ sq. ft. $=\frac{9}{4}$ sq. ft.

$\therefore$ the no. of stones $=\dfrac{42\times15\times4}{9}=280=14$ score.

$\therefore$ cost reqd. = Rs. $15\times14=$ Rs. 210.

6 £3 : £$3\frac{1}{2}$ :: £$85\frac{1}{8}$: x

$\therefore x=£\dfrac{681\times7}{8\times2\times3}=£\frac{1589}{16}=£99\frac{5}{16}$

and £$85\frac{1}{8}$: £5000 :: £3 : x

$\therefore x=£\dfrac{3\times5000\times8}{681}=£\dfrac{40\,000}{227}=£176\ 4s.\ 2\frac{170}{227}d.$

7. $(x+y+z)(x+y-z)(x+z-y)(z+y-x)$

$=\{(x+y+z)\}\{(x+y)-z\}\{z+(x-y)\}\{z-(x-y)\}$

$=\{(x+y)^2-z^2\}\{z^2-(x-y)^2\}$
$=(x^2+y^2+2xy-z^2)(z^2-x^2-y^2+2xy)$
$=\{2xy+(x^2+y^2-z^2)\}\{2xy-(x^2+y^2-z^2)\}$
$=4x^2y^2-(x^2+y^2-z^2)^2$
$=4x^2y^2-(x^4+y^4+z^4+2x^2y^2-2x^2z^2-2y^2z^2)$
$=2x^2y^2+2x^2z^2+2y^2z^2-x^4-y^4-z^4.$

(a)
$$\begin{array}{l} x^{\frac{2}{3}}+x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}} \\ x^{\frac{1}{3}}-y^{\frac{1}{3}} \\ \hline x+x^{\frac{2}{3}}y^{\frac{1}{3}}+x^{\frac{1}{3}}y^{\frac{2}{3}} \\ \quad -x^{\frac{2}{3}}y^{\frac{1}{3}}-x^{\frac{1}{3}}y^{\frac{2}{3}}-y \\ \hline x-y \end{array}$$

(b) $\dfrac{(x+y+z)(yz+zx+xy)-xyz}{x+y}$

$=\dfrac{\{(x+y)+z\}\{z(x+y)+xy\}-xyz}{x+y}$

$=\dfrac{z(x+y)^2+z^2(x+y)+yx(x+y)}{x+y}$

$=(x+y)z+xy+z^2$ Ans

8 Fraction

$=\dfrac{(x+y-z)(x-y+z)}{(x+z-y)(x+z+y)}+\dfrac{(y+x-z)(y-x+z)}{(y+x-z)(x+y+z)}$

$+\dfrac{(z+x-y)(z-x+y)}{(y+z+x)(y+z-x)}$

$=\dfrac{x+y-z}{x+z+y}+\dfrac{y-x+z}{x+y+z}+\dfrac{z+x-y}{x+y+z}=\dfrac{x+y+z}{x+y+z}=1.$

(b) See Ex 2 of 1865.

9 The expression $=\left(x^2+\dfrac{1}{x^2}\right)^2-4\left(x^2+\dfrac{1}{x^2}\right)-8+12.$

$=\left(x^2+\dfrac{1}{x^2}\right)^2-4\left(x^2+\dfrac{1}{x^2}\right)+4=\left(x^2+\dfrac{1}{x^2}-2\right)^2$

$\therefore$ sq root $=x^2+\dfrac{1}{x^2}-2$

(a) $(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x).$

Take $x-y=a$, $y-z=b$, and $z-x=c$.

Then $a+b+c=0$

Since $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ac)=0$

$$\therefore a^3+b^3+c^3=3ab^3.$$

Substituting back, we have

$$(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x).$$

10 (a) $\frac{5-3x}{4}+\frac{5x}{3}=\frac{3}{2}-\frac{3-5x}{3}$

$\therefore 15-9x+20x=18-12+20x$

$\therefore 9x=9$ or $x=1$

(b) $5x+\frac{02x+07}{03}-\frac{x+2}{9}=9\ 5$

$\therefore 4\ 5x+6x+21-x-2=9\ 5\times 9$

$9\ 5x=9\ 5\times 9-19$

$=9\ 5\ (9-2)=9\ 5\times 7.$

$\therefore x=7$

II. 1866.—AFTERNOON.

1 Euc I Def. 9, Def 10, Def. 23 Euc. III Def. 6, Def 10. Euc, I Def. 24

A polygon that is both equilateral and equiangular, is said to be regular

2 Euc. 1 7. Because it is self evident for the whole is greater than its part

3. Euclid I 43

4 See Question 8 (2) of I 1859

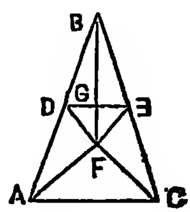
5 (a) Euclid II 11

(b) See Solution of Question 6 (b) of 1865 The only difference is that the given square, in this proposition, is the square on the given line.

6. Euclid III 13.

7 Euclid IV. 10

8 Join BF cutting DE in G



$\because$ BD=AD (Hyp)

$\therefore \triangle BDE=\triangle ADE=\triangle DEF+\triangle ADF$ (I 38)

$=\triangle ADF+\triangle BDF=\triangle DEF+\triangle DFG+\triangle BDG$ (I 38)

Take away this common $\triangle BDG$,

$\therefore \triangle BGE=\triangle DEF+\triangle DGF$

Likewise $\triangle BDG=\triangle DEF+\triangle ECF$

Add these equals,

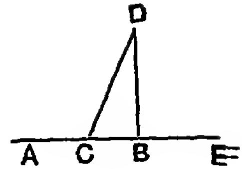
$\therefore \triangle BGE+\triangle BDG=2\triangle DEF+\triangle DGF+EGF$,

or $\triangle BDE=3\triangle DEF$

9 Let CB=half in difference.

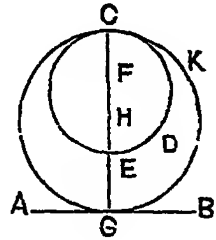
Draw BD$\perp$CB and make it=to the side of the given square.

Join CD.



Prod CB both ways
and cut CA, CE making each of them $=$ CD
$\therefore$ AB BE$+$CB$^2=$CE^2 (II 6)
$=$CD$^2=$CB$^2+$BD^2 (I 47)
$\therefore$ AB BE$=$DB$^2=$the given square
And AB$-$BE$=$AC$+$CB$-$BE$=$CB $+$ CE $-$BE$=$2CB

10 See Solution of Question 9 of 1863



11 Let AB be the given line and CDE be the given ⊙
Find the centre F (III I)
Draw FG$\perp$AB
Prod GF meeting the ⊙ in C
Bisect GC in H
With centre H and rad HG desc the ⊙GCK.
$\therefore$ $\angle$HGB is a rt $\angle$
The ⊙ CGK touches the line AB (III 16)
also it touches the ⊙CDE (III 11)

1867.—MORNING.

Examiners,—{ Mr J M Scott, B A
Mr W. G Wilson, B A }

1 The driving wheel of a locomotive is 226 inches in circumference and makes 91 revolutions per minute, at what rate per hour is the engine travelling?

2 Divide the least common multiple of 156, 260, 720, and 429 by their greatest common measure, and find the square root of the quotient

3 If a butcher buy 10 cwt of beef at 44*s* 4*d* per cwt and sell it at the rate of $4\frac{1}{2}d$ per lb, how much does he lose or gain?

4 Find the value of the following expressions —

$$\frac{5\frac{1}{6}\times 2\frac{2}{7}\times 9\frac{1}{9}\times 3\frac{10}{11}}{1\frac{5}{6}\times 67} \text{ and}$$

$$\frac{0\,625 \text{ of } £143\ 12s\ 0d + 0\,625 \text{ of } £71\ 16s\ 0d}{\frac{5}{8} \text{ of } 5175d}$$

5 Reduce £1 5*s* 6*d*. to the fraction of £1,000 and 6*s*. $1\frac{3}{5}d$ to the fraction of £150 10*s*, and express the results both as vulgar and decimal fractions

6 If £450 amount to £523 10*s* in 1 year 8 months, calculate the rate per cent

7 Reduce to its lowest terms $\dfrac{x^4-x^3-x+1}{x^4+x^3-x-1}$ and find the greatest common measure of $2x^3+9x^2+4x-15$ and $4x^3+8x^2+3x+20$.

8 Simplify $\left(\frac{x^2+y^2}{x^2-y^2} - \frac{x^2-y^2}{x^2+y^2}\right) \div \left(\frac{x+y}{x-y} - \frac{x-y}{x+y}\right)$ or show that

$$\left(\frac{b}{c}+\frac{c}{b}\right)^2+\left(\frac{c}{a}+\frac{a}{c}\right)^2+\left(\frac{a}{b}+\frac{b}{a}\right)^2=4+\left(\frac{b}{c}+\frac{c}{b}\right)\left(\frac{a}{c}+\frac{c}{a}\right)\left(\frac{a}{b}+\frac{b}{a}\right)$$

9. Prove either of the identities—

$$(ay-bz)^2+(cx-az)^2+(bz-cy)^2=$$
$$(x^2+y^2+z^2)(a^2+b^2+c^2)-(ax+by+cz)^2$$

$$16s(s-a)(s-b)(s-c)=2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4,$$

where $2s=a+b+c$

10. Solve either of the equations—

$$(x+\tfrac{5}{2})(x-\tfrac{3}{2})-(x+5)(x-3)+\tfrac{3}{4}=0$$

$$\frac{\sqrt{x}+a}{(\sqrt{x}-b)(\sqrt{x}-c)}+\frac{\sqrt{x}+b}{(\sqrt{x}-a)-(\sqrt{x}-c)}+\frac{\sqrt{x}+c}{(\sqrt{x}-a)(\sqrt{x}-b)}=0.$$

11 Solve the simultaneous equations—

$$\left.\begin{aligned} ax+by+c&=0\\ a_1x+b_1y+c_1&=0\end{aligned}\right\} \text{ and } \left.\begin{aligned} x+5y-4z&=5\\ 3x-2y+2z&=14\\ -10x+8y+z&=6\end{aligned}\right\}$$

12 Extract the square root of—

$$x^6+8x^4-2x^3+16x^2-8x+1$$

$$\text{or } a^4+b^4+c^4+d^4-2(a^2+c^2)(b^3+d^2)+2a^2c^2+2b^2d^2.$$

1867.—AFTERNOON.

Examiners,—{ Mr. C. Marris, B.A.
Mr. M. Mowat, M.A.

1 (*a*) Define (i) a plane rectilineal angle, (ii) parallel straight lines, (iii) a rhombus

(*b*) Construct an isosceles triangle having each of the *sides* double of the base

2 (*a*) If two angles of a triangle be equal to each other, the sides which subtend or are opposite to the equal angles shall be equal to one another

(*b*) The straight line which bisects the vertical angle of an isosceles triangle bisects the base perpendicularly

3. Parallelograms upon the same base and between the same parallels are equal to one another In what different senses is the word *equal* used by Euclid when treating of the equality of triangles?

4 If a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts together with twice the rectangle contained by the parts Prove this and show *from it* how x^2+2xy may be made a complete square.

5 Describe a square equal to a given rectilineal figure Define a *gnomon*

6 The angle at the centre of a circle is double of the angle at the circumference upon the same base, that is, the same part of the circumference Prove this, not omitting the case when the angle at the centre is equal to or greater than, two right angles.

7 The opposite angles of a quadrilateral figure inscribed in a circle are together equal to two right angles, the angle in a segment of a circle greater than a semi-circle is less than a right angle and the angle in a segment less than a semi-circle is greater than a right angle Prove these propositions from question 6

8 If two chords in a circle intersect one another, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other

9 Inscribe a circle in a given square What proposition in the 4th book enables us to divide a right angle into ten equal parts? Give the number or enunciation

10 Describe a rhombus equal to a given square.

SOLUTIONS

1867.—MORNING.

1 In 91 revolutions, the engine goes over (226×91) in.

$\therefore 1' \quad 60' \quad (226 \times 91)$ in $\quad x$

$$\therefore x = \frac{226 \times 91 \times 60}{12 \times 3 \times 1760} \text{ miles} = 19 \text{ miles } 336 \text{ yds } 2 \text{ ft}$$

2 The L C M $= 2 \times 2 \times 3 \times 13 \times 5 \times 12 \times 11 = 102960$

and G C. M $= 1$

$\therefore$ the quotient $= 102960$

and $\sqrt{(102960)} = 328\ 87\ldots \ . \ \ldots$

3 The selling price per cwt $= 28 \times 4 \times \frac{9}{2}d = 42s$

and the cost price per cwt $= 44s\ 4d$

$\therefore 44s\ 4d - 42s = 2s\ 4d$ is the loss in every cwt

$\therefore$ his loss reqd. $= 10 \times (2s\ 4d)$ £1 $3s\ 4d$

$$4 \quad \frac{5\frac{1}{6} \times \frac{22}{31} \times 9\frac{1}{2} \times 3\frac{10}{19}}{1\frac{5}{6} \times 67} = \frac{\frac{31}{6} \times \frac{22}{31} \times \frac{19}{2} \times \frac{67}{19}}{\frac{11}{6} \times 67} = \frac{\frac{11 \times 67}{6}}{\frac{11 \times 67}{6}} = 1.$$

625 of £143 $12s$ + 625 of £71 $16s$ = 625 of (£143 $12s$. + £71 $16s$) $= \frac{5}{8}$ of £215 $8s$ $=$ £$\frac{1077}{8}$

and $\frac{5}{8}$ of $5175d = \frac{5}{8}$ of £$\frac{5175}{240}$

$\therefore$ the value of the expression $= \dfrac{\frac{5}{8} \text{ of £}215\ 8s.}{\frac{5}{8} \text{ of £}\frac{5175}{240}}$.

$$= \frac{359 \times 48}{1725} = 9\tfrac{569}{575}.$$

5 $£1\ 5s\ 6d = £1\frac{11}{40} = £\frac{51}{40}$

$$\therefore \text{fraction required} = \frac{51}{40 \times 1000} = \frac{51}{40000} = .001275$$

And $5s\ 1\frac{3}{8}d = \frac{491}{96}s.$ and $£150\ 10s = 3010s.$

$$\therefore \text{fraction reqd} = \frac{491}{96 \times 3010} = \frac{491}{288960} = .001699 \ldots\ldots$$

6 The interest $= £523\ 10s. - £450 = £73\ 10s.$

$£450 \times 1\frac{2}{3} : £100 \times 1 :: £73\frac{1}{2} :$ rate per cent

$$\therefore \text{Rate} = \frac{147 \times 100 \times 3}{2 \times 450 \times 5} = 9\tfrac{4}{5}\ \%$$

7 $$\text{Ans.} = \frac{x^3(x-1) - (x-1)}{x^3(x+1) - (x+1)} = \frac{(x^3-1)(x-1)}{(x^3-1)(x+1)} = \frac{x-1}{x+1}.$$

(a)
$$\begin{array}{r|l|l}
2x^3+9x^2+4x-15 & 4x^3+8x^2+3x+20 & 2 \\
 & 4x^3+18x^2+8x-30 & \\ \hline
 & -5 \quad -10x^2-5x+50 & \\ \hline
 & 2x^2+x-10 &
\end{array}$$

$$\begin{array}{r|l|l}
2x^2+x-10 & 2x^3+9x^2+4x-15 & x+4 \\
 & 2x^3+x^2-10x & \\ \hline
 & +8x^2+14x-15 & \\
 & 8x^2+4x-40 & \\ \hline
 & 5\,|\,10x+25 & \\ \hline
 & 2x+5 &
\end{array}$$

$$\begin{array}{r|l|l}
2x+5 & 2x^2+x-10 & x-2 \\
 & 2x^2+5x & \\ \hline
 & -4x-10 & \\
 & -4x-10 & \\ \hline
\end{array}$$

$\therefore$ $2x+5$ is the G. C. M.

8 $$\left(\frac{x^2+y^2}{x^2-y^2} - \frac{x^2-y^2}{x^2+y^2}\right) \div \left(\frac{x+y}{x-y} - \frac{x-y}{x+y}\right)$$

$$= \left(\frac{(x^2+y^2)^2 - (x^2-y^2)^2}{(x^2-y^2)(x^2+y^2)}\right) \div \left(\frac{(x+y)^2-(x-y)^2}{x^2-y^2}\right)$$

$$= \frac{4x^2y^2}{(x^2+y^2)(x^2-y^2)} \times \frac{x^2-y^2}{4xy} = \frac{xy}{x^2+y^2}$$

(b) $$\left(\frac{b}{c}+\frac{c}{b}\right)^2 + \left(\frac{c}{a}+\frac{a}{c}\right)^2 + \left(\frac{a}{b}+\frac{b}{a}\right)^2$$

$$=\frac{b^2}{c^2}+\frac{c^2}{b^2}+2+\frac{c^2}{a^2}+\frac{a^2}{c^2}+2+\left(\frac{a}{b}+\frac{b}{a}\right)^2$$

$$=4+\frac{a^2+b^2}{c^2}+c^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)+\left(\frac{a}{b}+\frac{b}{a}\right)^2$$

$$=4+\frac{ab}{c^2}\left(\frac{a^2+b^2}{ab}\right)+\frac{c^2}{ab}\left(\frac{a^2+b^2}{ab}\right)+\left(\frac{a}{b}+\frac{b}{a}\right)^2$$

$$=4+\frac{ab}{c^2}\left(\frac{a}{b}+\frac{b}{a}\right)+\frac{c^2}{ab}\left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{a}{b}+\frac{b}{a}\right)^2$$

$$=4+\left(\frac{a}{b}+\frac{b}{a}\right)\left(\frac{ab}{c^2}+\frac{c^2}{ab}+\frac{a}{b}+\frac{b}{a}\right)$$

$$=4+\left(\frac{a}{b}+\frac{b}{a}\right)\left\{\frac{b}{c}\left(\frac{a}{c}+\frac{c}{a}\right)+\frac{c}{b}\left(\frac{a}{c}+\frac{c}{a}\right)\right\}$$

$$=4+\left(\frac{a}{b}+\frac{b}{a}\right)\left(\frac{a}{c}+\frac{c}{a}\right)\left(\frac{b}{c}+\frac{c}{b}\right)$$

9 (a) Left side

$$=a^2y^2+b^2z^2+c^2x^2+a^2z^2+b^2x^2+c^2y^2-2abyz-2acxz-2bcyx$$

$$=a^2(y^2+x^2+z^2)+b^2(x^2+y^2+z^2)+c^2(x^2+y^2+z^2)$$

$$-(a^2x^2+b^2y^2+c^2z^2)+(2abyx+2acxz+2bcyz)$$

$$=(a^2+b^2+c^2)(x^2+y^2+z^2)-(ax+by+cz)^2$$

(b) $16s(s-a)(s-b)(s-c)$

$$=2s(2s-2a)(2s-2b)(2s-2c)$$

$$=(a+b+c)(b+c-a)(a+c-b)(a+b-c)$$

$$=\{(a+b)+c\}\{(a+b)-c\}\times\{c+(a-b)\}\{c-(a-b)\}$$

$$=\{(a+b)^2-c^2\}\{c^2-(a-b)^2\}$$

$$=(a^2+b^2+2ab-c^2)(c^2-a^2-b^2+2ab)$$

$$=\{2ab+(a^2+b^2-c^2)\}\{2ab-(a^2+b^2-c^2)\}$$

$$=4a^2b^2-(a^2+b^2-c^2)^2$$

$$=4a^2b^2-(a^4+b^4+c^4-2a^2c^2-2b^2c^2+2a^2b^2)$$

$$=2a^2b^2+2c^2b^2+2a^2c^2-a^4-b^4-c^4$$

10. $(x+\frac{5}{2})(x-\frac{3}{2})-(x+5)(x-3)+\frac{3}{4}=0$

or $x^2+x-\frac{15}{4}-(x^2+2x-15)+\frac{3}{4}=0$

or $x^2+x-\frac{15}{4}-x^2-2x+15+\frac{3}{4}=0$

or $-x=\frac{15}{4}-15-\frac{3}{4}=-12$ $\therefore x=12$

(b) $$\frac{\sqrt{x}+a}{(\sqrt{x}-b)(\sqrt{x}-c)}+\frac{\sqrt{x}+b}{(\sqrt{x}-a)(\sqrt{x}-c)}+\frac{\sqrt{x}+c}{(\sqrt{x}-a)(\sqrt{x}-b)}=0.$$

L. C. M. of the denrs $=(\sqrt{x}-a)(\sqrt{x}-b)(\sqrt{x}-c)$

$\therefore (\sqrt{x}+a)(\sqrt{x}-a)+(\sqrt{x}+b)(\sqrt{x}-b)+(\sqrt{x}+c)(\sqrt{x}-c)=0$

$\therefore x-a^2+x-b^2+x-c^2=0$, or $3x=a^2+b^2+c^2$

$\therefore x=\frac{1}{3}(a^2+b^2+c^2)$

11 $\left.\begin{array}{l} ax+by+c=0 \\ a_1x+b_1y+c_1=0 \end{array}\right\}$ or $\begin{array}{ll} ax+by=-c & \ldots (1) \\ a_1x+b_1y=-c_1 \ldots & (2) \end{array}$

Multiplying (1) by a_1, $a_1ax+a_1by=-a_1c$

and (2) by a, $a_1ax+ab_1y=-ac_1$

$\therefore$ By subtr $y(a_1b-ab_1)=ac_1-a_1c$

$$y=\frac{ac_1-a_1c}{a_1b-ab_1}$$

From (1) $ax=-c-\dfrac{b(ac_1-a_1c)}{a_1b-ab_1}=\dfrac{a(b_1c-bc_1)}{a_1b-ab_1}$

$$\therefore x=\frac{b_1c-bc_1}{a_1b-ab_1}.$$

(b) $\left.\begin{array}{lr} x+5y-4z=5 \ldots\ldots & (1) \\ 3x-2y+2z=14 \ldots \quad \ldots\ldots & (2) \\ -10x+8y+z=6 & \ldots (3) \end{array}\right\}$

Multiplying (1) by 3, $3x+15y-12z=15$

Subtracting (2) $\quad 3x-2y+2z=14$

$\therefore$ we get $17y-14z=1 \ldots \quad (4)$

Multiplying (1) by 10, $10x+50y-40z=50$

Adding (3) $-10x+8y+z=6$

we get $58y-39z=56 \quad (5)$

$\times$ (4) by 38 and (5) by 17

$986y-812z=58$

$986y-663z=952$

Subtracting $-149z=-894$ $\quad\therefore z=6$

From (4) $17y=14z+1=85$ $\quad\therefore y=5$

From (1) $x=5+4z-5y=5+24-25=4$

12 $\quad x^6+8x^4-2x^3+16x^2-8x+1\ (x^3+4x-1=$ sq root

x^6

$$\begin{array}{l|l} 2x^3+4x & 8x^4-2x^3+16x^2 \\ & 8x^4+16x^2 \\ \hline 2x^3+8x-1 & -2x^3-8x+1 \\ & -2x^3-8x+1 \\ \hline \end{array}$$

(a) $a^4+b^4+c^4+d^4-2(a^2+c^2)(b^2+d^2)+2a^2c^2+2b^2d^2$

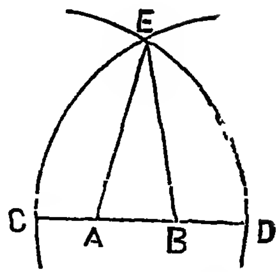
$=(a^4+c^4+2a^2c^2)+(b^4+d^4+2b^2d^2)+2(a^2+c^2)(b^2+d^2)$

$$=(a^2+c^2)^2+(b^2+d^2)^2-2(a^2+c^2)(b^2+d^2)$$

$$=\{(a^2+c^2)-(b^2+d^2)\}^2$$

$\therefore$ square root $=(a^2+c^2)-(b^2+d^2)=a^2-b^2+c^2-d^2$.

1867.—AFTERNOON.



1 (a) Euc I Def 9, Def 37, Def 34

(b) Let AB be the str line, prod BA both ways to C and D (I 3) making CA, BD each equal to BA

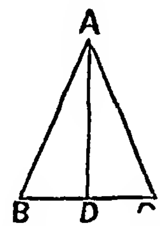
With centre A and rad AD desc a ⊙ also with centre B and rad BC desc. a ⊙ cutting the former ⊙ at E

Join AE, BE

Then ABE is the reqd isos △

$\because$ AE=AD=2BA and BE=BC=2AB

$\therefore$ AE=BE and each is double of AB



2 (a) Euclid I 6

(b) Let ABC be an isos △,

AD bisects the vertical ∠BAC

$\because$ AB=AC and AD com

also ∠BAD=CAD,

$\therefore$ BD=DC and the ∠ADB=∠ADC (I 4),

but they are adjacent ∠s

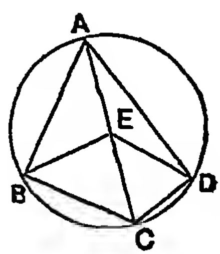
$\therefore$ AD⊥BC

3 Euclid I 35 (1) Equal in area only (2) Equal in every respect (i e.) in area, angles and sides

4 Euclid II 4 By adding y^2 to the expression, for it will then become $(x+y)^2$

5 Euclid II 14 Def 2

6 Euclid III 20



7 (a) Let the quad ABCD be inscribed within the ⊙ ABCD

$\because$ ∠BED=2∠BAD and the reflex ∠BED (i e) ∠AEB+∠AED=2∠BCD (III 20)

$\therefore$ the ∠s at A and C=½ ∠s at E

(I 15 Cor)=½ of 4 rt ∠s=2 rt ∠s

(b) Let BAD be the segment >½ ⊙.

The ∠BED is <2 rt ∠s ((III 20)

and $\therefore$ its half ∠BAD <a rt ∠

Let BCD be the segment <½ ⊙

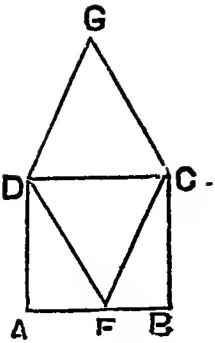
The reflex ∠BED=2 ∠BCD (III 20)

$\therefore$ ∠BCD=½ reflex ∠BED (i e)> a rt. ∠.

8 Euclid III 35

9. Euclid IV. 8. Euclid IV. 10.

10 Let ABCD be the given square
Bisect AB in the point E.
Join DE and CE.
On CD and on the side remote from E
desc the $\triangle DCG = \triangle DEC$
Then DECG is the reqd rhombus
∵ AE=BE, AD=BC and $\angle DAE = \angle EBC$
∴ DE=EC (I 4)
Hence D[illegible]C=CG=GD (Cons)
8) Again ∵ [illegible]C=[illegible]CE=$\angle$AED=EDC (I. 22)
∵ DE=CE
∴ DECG is a rhombus
The $\triangle DEC = \frac{1}{2}$ of fig DECG (I 34)
and also=$\frac{1}{2}$ of fig ABCD (I 4)
Wherefore the rhombus DECG=the given sq ABCD



1868.—MORNING.

Examiners,— { MR M MOWAT, M A.
MR J M SCOTT, B.A.

1. Find the difference between 1 6 of 3 4 of £1 125 and $\frac{1}{5}$ of 3 6 of £9 1125, and find the value of—

$$\frac{6\ 27 \times 0\ 5}{(\frac{1}{2} \text{ of } \frac{2}{7}) \times 8\ 36} - \frac{(\frac{1}{4} \text{ of } \frac{1}{10}) \times (\frac{3}{4} \text{ of } 21\frac{1}{4})}{(\frac{1}{3} \text{ of } \frac{6}{8}) \times 1\ 4}.$$

2. Extract the square root of 153 140624, and of 3 3, each to three places of decimals.

3. If one man walks 165 miles in 6 days, how far will another man walk in 15 days if the first man walks $3\frac{3}{4}$ miles in the same time that the other man walks 4 miles ?

4 Three equal glasses are filled with a mixture of spirits and water, the proportion of spirits to water in each glass is as follows — in the first glass as 2 3, in the second as 3 4, and in the third as 4 5 The contents of the three glasses are poured into a single vessel : what is the proportion of spirits to the water in it ?

5 Find the interest on £350 from 3rd March to 28th December at $4\frac{1}{2}$ per cent per annum

6. How many yards of carpet 15 inches wide will be required for a room 19 feet 7 inches long, and 18 feet 9 inches wide ?

7. Given $a=\sqrt{2}$, $b=\sqrt{3}$, $c=4$, and $d=0$, find the value of

$$\sqrt{(a^2+b^2+c^2)(b^2+c^2)(b^2+d^2)};$$

and extract the square root of

$$a^2+b^2+c^2+d^2-2a(b-c+d)-2b(c-d)-2cd.$$

8 Simplify $\left(\frac{a+b}{a-b}+\frac{a^2+b^2}{a^2-b^2}\right)\div\left(\frac{a-b}{a+b}-\frac{a^3-b^3}{a^3+b^3}\right)$ and show that

$$1-\left(\frac{b^2+c^2-a^2}{2bc}\right)^2=\frac{(a+b+c)(a+c-b)(b+c-a)(a+b-c)}{4b^2c^2}$$

9 Solve the equations $\frac{4x+3}{9}+\frac{29-7x}{12-5x}=\frac{8x+19}{18}$

and $\frac{8x+4}{\sqrt{(x+5)}}=4\sqrt{x+5}$

10 Find the greatest common measure of x^3+4x^2-5 and x^3-3x-2, and the lowest common multiple of $x^5-5x^3+x^2+4x-4$ and $x^4+x^3-6x^2-4x+8$

11 Solve the simultaneous equations —

$$\left.\begin{aligned} \tfrac{1}{2}x+\tfrac{1}{3}y&=12-\tfrac{1}{6}z \\ \tfrac{1}{2}y+\tfrac{1}{4}z-\tfrac{1}{6}x&=8 \\ \tfrac{1}{3}x+\tfrac{1}{3}z&=10 \end{aligned}\right\}$$

12 There is a number, the sum of whose digits is 5, and if 10 times the digit in the place of ten be added to 4 times the digit in the place of units, the number will be inverted. What is the number?

1868.—AFTERNOON.

Examiners, | Mr C. Martin, B A.
| Mr R. Dick, M A

1 Define plane superficies, acute-angled triangle, sector and segment of a circle, noting the case in which the two last coincide. What are the points of resemblance and difference in the following — chord and diameter, segment of a circle, and semi circle, square and rhombus? and distinguish between postulate and axiom, problem and theorem

2 To make a triangle of which the sides shall be equal to the three given straight lines, but any two whatever of these must be greater than the third

Prove that the two circles required in this proposition must cut each other

3 (a) *Enunciate* the propositions of the first book in which Euclid proves the equality of two triangles in every respect, and those in which two triangles are proved equal in area only

(b) *Enunciate* the only other case in which two triangles can be equal in every respect

(c) What inference would you draw as to the equality of two triangles which are equiangular to each other

4 (a) Prove that the three interior angles of every triangle are equal to two right angles, *without producing* a side of the triangle

(b) Show from I. 47 how to find a square which shall be equal to the difference of two given squares

5 If a straight line be divided into any two parts, four times the rectangle contained by the whole line and one of the parts, together with the square on the other part, is equal to the square on the straight line which is made up of the whole and that part

State and prove this algebraically only, and illustrate by a numerical example.

6 (a) To divide a straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part Give *only* the construction in this proposition, and show how many points of section can be obtained.

(b) Prove by means of II 12 and II 13 that if any side of a triangle be bisected, the square on the other two sides are together equal to twice the square on half the line bisected, and twice the square on the line drawn from the point of bisection to the opposite angle

7 From a given point either without a given circle or in its circumference, to draw a straight line touching the circle Give the construction *only*, and shew, by assuming the angle in a semi-circle to be a right angle, how the first case of this proposition may be more simply effected

8 On a given straight line to describe a segment of a circle containing an angle equal to a given rectilineal angle, in two following cases only —

(i) When the given angle is a right angle

(ii) When the given angle is the angle of an equilateral triangle

9 (a) Inscribe a circle in a given triangle

(b) If the three points be joined in which the inscribed circle meets the sides of the triangle, show that the resulting triangle is acute-angled

10 Perform *one only* of following deductions :—

(1) Construct a right angled triangle, having given the hypotenuse and the sum of the sides.

(2) Two circles have the same centre, show that all chords of the outer circle which touch the inner circle are equal

SOLUTIONS

1868.—MORNING.

1 1.6 of 3.4 of £1.125 = £6.12.

$\frac{1}{5}$ of $3\frac{2}{3}$ of £9.1125 = £6.6875

$\therefore$ Difference = £$(6.6875 - 6.12)$ = £$.5625$ = $11s\ 3d$.

(a) 1st fraction $= \dfrac{\frac{1}{2} \times 6.27}{\frac{3}{8} \times 8.36} = \dfrac{3.135}{3.135} = 1.$

2nd fraction $=\dfrac{\frac{1}{4\pi}\times 16}{\frac{[illegible]}{9}+1\frac{4}{9}}=\frac{2}{5}\times\frac{1}{2}=\frac{1}{5}$

$\therefore$ Ans $=1-\frac{1}{5}=5$

2. $\sqrt{(3\ 300000)}=1\ 816$

$\sqrt{(153\ 140625)}=12\ 875$

3 $3\frac{3}{4}$ mi 4 mi 165 mi x

6 days 15 days

$x=\dfrac{165\times 4\times 15\times 4}{15\times 6}=440$ miles

4. First glass contains $\frac{2}{5}$ spirit $+\frac{3}{5}$ water
Second „ „ $\frac{3}{7}$ „ $+\frac{4}{7}$ „
Third „ „ $\frac{4}{9}$ „ $+\frac{5}{9}$ „

$\therefore$ spirit in three glasses $=\frac{2}{5}+\frac{3}{7}+\frac{4}{9}=\frac{401}{315}$,

and water $=\frac{3}{5}+\frac{4}{7}+\frac{5}{9}=\frac{544}{315}$

$\therefore$ spirit : water :: 401 : 544

5 The no of days from 3rd March to 28th Dec $=300$

Interest of £350 at $4\frac{1}{2}$ p c for 1 year $=£15\frac{3}{4}$

$\therefore$ the int reqd $=£\dfrac{63\times 300}{4\times 365}=£12\ 18s.\ 10\frac{62}{73}d$

6. The area of the floor $=(19\frac{7}{12}\times 18\frac{3}{4})$ sq ft $=\dfrac{235\times 75}{48}$ sq ft.

$\therefore$ Carpet reqd to cover the floor $=\dfrac{235\times 75}{48}\times\frac{12}{15}$ ft

$=\frac{1175}{4}$ ft $=293\frac{3}{4}$ ft $=97$ yds 2ft 9in

7 Ans $=\sqrt{\{(2+3+16)(3+16)(3+0)\}}=\sqrt{(21\cdot 19\cdot 3)}$

$=3\sqrt{(133)}=34\ 59$

(a) The expression

$=(a^2+b^2-2ab)+(c^2+d^2-2cd)+(2ac-2bc-2ad+2bd)$

$=(a-b)^2+(c-d)^2+2(a-b)(c-d)$

$=\{(a-b)+(c-d)\}^2$ $\therefore$ sq root $=a-b+c-d.$

8. $\left(\dfrac{a+b}{a-b}+\dfrac{a^2+b^2}{a^2-b^2}\right)\div\left(\dfrac{a-b}{a+b}-\dfrac{a^3-b^3}{a^3+b^3}\right)$

$=\left\{\dfrac{(a+b)^2+a^2+b^2}{a^2-b^2}\right\}\div\left\{\dfrac{(a-b)(a^2-ab+b^2)-(a^3-b^3)}{a^3+b^3}\right\}$

$=\dfrac{2(a^2+ab+b^2)}{(a+b)(a-b)}\times\dfrac{(a+b)(a^2-ab+b^2)}{2ab(b-a)}$

$=\dfrac{(a^2+ab+b^2)(a^2-ab+b^2)}{ab(a-b)(b-a)}=-\dfrac{a^4+a^2b^2+b^4}{ab(a-b)^2}.$

(a) $1-\left(\frac{b^2+c^2-a^2}{2bc}\right)^2=\left(1+\frac{b^2+c^2-a^2}{2bc}\right)\left(1-\frac{b^2+c^2-a^2}{2bc}\right)$

$=\left\{\frac{(b+c)^2-a^2}{2bc}\right\}\left\{\frac{a^2-(b-c)^2}{2bc}\right\}$

$=\frac{(b+c+a)(b+c-a)(a+b-c)(a-b+c)}{4b^2c^2}.$

9 $\frac{4x+3}{9}+\frac{29-7x}{12-5x}=\frac{8x+19}{18}$

$\frac{4x}{9}+\frac{1}{3}+\frac{29-7x}{12-5x}=\frac{4x}{9}+\frac{19}{18}$

$\therefore \frac{29-7x}{12-5x}=\frac{19}{18}-\frac{1}{3}=\frac{13}{18}$

$\therefore (12-5x)\times 13=18(29-7x)$

$\therefore 156-65x=522-126x,$ $\therefore 126x-65x=522-156.$

$\therefore 61x=366.$ $\therefore x=6$

(b) Multiply the equation by $\sqrt{x}+5$,

then $8x+4=4x+20$;

$\therefore 4x=16$ $\therefore x=4$

10 $x^3-3x+2)\ x^3+4x^2-5\ (1$

x^3-3x+2

$4x^2+3x-7)\ 4x^3-12x+8\ (x-3$

$4x^3+3x^2-7x$

$-3x^2-5x+8$

4

$-12x^2-20x+32$

$-12x^2-9x+21$

$-11)\ -11x+11$

$x-1$

$x-1)\ 4x^2+3x-7\ (4x+7$

$4x^2-4x$

$7x-7$

$7x-7$

$\therefore x-1$ is the G C M.

(a) The G C M of the expressions $= x^2-4$

L C M $\dfrac{(x^4-x^3-4x^2-4x+8)(x^5-5x^3+x^2+4x-4)}{x^2-4}$

$=(x^2+x-2)(x^3-x+1)(x^2-4)$

11 From (1) × by 3 $\frac{3}{2}x+y+\frac{1}{2}z=36$

" (1) × by 2 $y+\frac{2}{3}z-\frac{1}{3}x=16$

By subtr $\frac{11}{6}x-\frac{1}{6}z=20$

From (3) ÷ by 2 $\frac{1}{4}x+\frac{1}{6}z=5$

By addt $\frac{25}{12}x=25$ $\therefore x=12$

From (3) $\frac{1}{3}z=10-\frac{1}{2}x=4$ $\therefore z=12$

From (1) $\frac{1}{3}y=12-\frac{1}{6}z-\frac{1}{2}x=12-2-6=4$ $\therefore y=12$

12 Let x be the digit in the place of tens

and y " " of units

then No $=10x+y$

Now, by the question,

$x+y=5$ (1), $10x+4y=10y+x$ ....(2)

From (2) $9x=6y$ or $x=\frac{2}{3}y$

Therefore (1) $\frac{2}{3}y+y=5$ $\therefore \frac{5}{3}y=5$ $\therefore y=3$

And from (1) $x=2$ Hence no $=23$

1868 —AFTERNOON

1 Euclid I Def 7, Def 31, Euclid III Def 10, Def. 6, in the case of a semi-circle.

A diameter is a chord passing through the centre of the circle.

A semi circle is a segment of a circle, whereof the chord passes through the centre

A square is both equilateral and rectangular, while a rhombus is only equilateral

Postulates are problems, the possibility of which is admitted to be self evident.

Axioms are self evident theorems.

Problems are propositions in which some geometrical construction is to be made while theorems are propositions in which some geometrical truth is to be proved.

2 Euclid I 22 Prove indirectly. If not one straight line shall be greater than the sum of other two straight lines

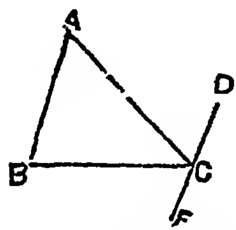
3 (a) The propositions 4th, 8th, and 26th treat of the equality of two triangles in every respect, and propositions, 37th and 38th of the equality of two triangles in area only

(b) If two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise the angles opposite to one of those sides equal to that in the other, also the angles, opposite to the

remaining equal side, are of the same species, that is, they are acute or right; then the triangles will be equal in every respect.

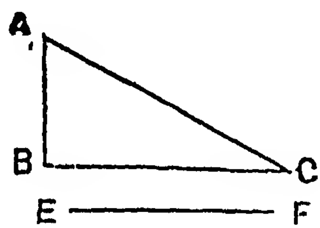
(c) Equiangular triangles are not necessarily equal in area nor in every respect

4 Let ABC be a $\triangle$
Though C draw DCF $\parallel$ AB
$\therefore \angle BAC = \angle ACD$ and $\angle ABC = \angle BCF$ (I. 29)
and to these add the $\angle ACB$
$\therefore \angle BAC + \angle ABC + \angle ACB = ACD + \angle ACB + \angle BCF$
$= 2$ rt. $\angle$s (I 13).



(b) Let AB and EF be the two sides of two given squares also let EF > AB

From B draw BC$\perp$AB.
With A as centre and EF as rad desc $\odot$ cutting BC in the point C.
Join AC and BC
(I 47) $\because AC^2 = AB^2 + BC^2$;
take AB^2 from both these equals,
$\therefore AC^2 - AB^2 = BC^2$.



5. Euclid II. 8.

Let the str. line AB contain a linear units, of which the parts AC, CB contain m, n units respectively.

Then $m+n=a$ or $m=a-n$,

and $m^2=a^2-2an+n^2$, adding $4an$ to these equals,

$4an+m^2=a^2+2an+n^2=(a+n)^2$.

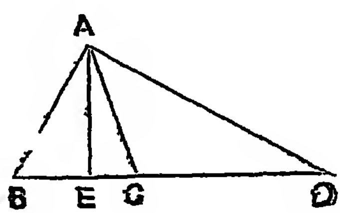
Let $m=3$, and $n=2$, then $a=5$

$4\times5\times2+9=(5+2)^2$ or $49=(7)^2$

6 (a) Euclid II 11. CD, CF, and KG are similarly divided in the points K, A, and H respectively

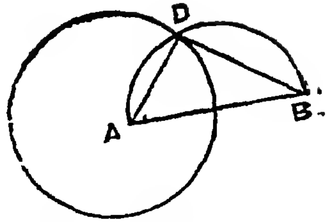
Also EB in the point where it cuts KG.

(b) Let ABD be a $\triangle$
Bisect BD in C
From A draw AE$\perp$BD and join AC
$\because AD^2 = AC^2 + CD^2 + 2DC\ CE.$ (I. 12)
and $AB^2 = AC^2 + BC^2 - 2BC\ CE$ (II. 13)
$\therefore AD^2 + AB^2 = 2AC^2 + 2BC^2$
$\because BC = CD.$



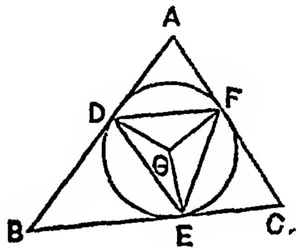
7 Euclid II. 17

(a) Let A be the centre of the $\odot$ and B the point without the $\odot$.
Join AB.
On AB desc $\frac{1}{2}$ $\odot$ ADB cutting the $\odot$ at D
Join AD and BD
The $\angle$ADB=a rt $\angle$ (Hyp)
$\therefore$ BD touches the $\odot$ (III. 16)

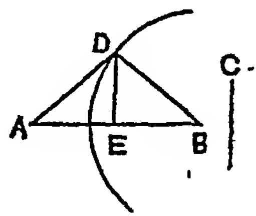


8. Euclid III 33
(a) (1) First case
(2) The $\angle$BAD is to be made equal to the $\angle$ of an equilateral triangle.

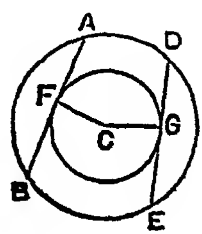
9 (a) Euclid IV 4
(b) Let ABC be a $\triangle$ and G the centre of the inscribed $\odot$ touching AB, BC, CA in D, E, F respectively.
Join DE, EF and DF.
Then $\triangle$ DEF is acute angled.
Join DG, GE and GF
$\because$ the $\angle$s of the quad. GECF=4 rt. $\angle$s (I. 32, Cor 1)
and $\angle$GFC+$\angle$GEC=2 rt $\angle$s
$\therefore$ EGF+$\angle$ECF=2 rt $\angle$s (III. 18)
Hence $\angle$EGF < 2 rt. $\angle$s;
but $\angle$EGF=2 EDF (III 20);
$\therefore$ $\angle$EDF < a rt $\angle$ or $\angle$EDF is acute.
Likewise $\angle$DFE and $\angle$FED may be proved to be acute



10 (1) Let AB be the sum of the sides and C be the hypotenuse
At the pt A in the str line AB make the $\angle$BAD=$\frac{1}{2}$ a rt $\angle$ (I 23).
From the centre B and at the distance C desc a $\odot$ cutting or touching AD at D
Draw DE$\perp$AB Join DB
Then BDE is the required $\triangle$
$\because$ $\angle$DAE=$\frac{1}{2}$ a rt $\angle$ and $\angle$DEA=rt $\angle$
$\therefore$ $\angle$ADE=$\frac{1}{2}$ a rt $\angle$ (I 32)
$\therefore$ AE=DE (I 6)
$\therefore$ DE+EB=AB and DB=C.



(2) Let AB and DE be the chords of the outer $\odot$ which touch the inner $\odot$ in the points F and G
Let C be the common centre
Join FC and CG
Then CF and CG are $\perp$AB and DE respectively (III 18)
But FC=CG (Def 15);
$\therefore$ AB=DE (III 14)



1869—MORNING.

Examiners,— { MR. A. W CROFT, M.A.
MR. EWBANK, B.A.

1 Simplify—$\dfrac{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}-\frac{1}{16}}{\frac{1}{3}+\frac{1}{4}} \div \dfrac{\frac{1}{2}+\frac{1}{4}}{1-\frac{1}{2}\times\frac{1}{3}}$, and reduce 4 hrs. 1 min $10\frac{1}{2}$ sec to the decimal of a week

2 Add together ·062435 of £100 + 7 4375 of 10s + 1 35$\dot{6}$ of 7s 6d + 2 784 of $2\frac{1}{2}d$, and reduce the result to the fraction of £26 10s $7\frac{1}{2}d$

3 Divide ·0007 by ·035 and by 3500 and extract the square root of each quotient to four decimal places

4 A room is 37 ft. 2 in long, 25 ft. 8 in broad, and 22 ft 6 in. high, find the cost of covering its four walls with paper $1\frac{3}{4}$ yd wide, at 1s $1\frac{3}{4}d$. a yard

5 In what time will £563 13s $4\frac{1}{2}d$, amount to £901 17s $5\frac{1}{2}d$. at $3\frac{3}{4}$ per cent?

6. Divide $x^3+y^3+3xy-1$ by $x+y-1$, and extract the square root of $x^4-3x^3+\frac{11}{12}x^2+2x+\frac{4}{9}$

7 Resolve all the following expressions into factors, and thence find the Highest Common Measure of x^4+2x^2+1 $x^5+x^4-x^2-1$, and x^4-1, and the Lowest Common Multiple of $6x^2-x-1$, $3x^2+7x+2$, and $2x^2+3x-2$

8. Simplify—

(a) $$\frac{x}{x-a}-\frac{x}{x+a}-\frac{\dfrac{x+a}{x-a}-\dfrac{x-a}{x+a}}{\dfrac{x+a}{x-a}+\dfrac{x-a}{x+a}}$$

(b) $$\frac{a^2+ac}{a^2c-c^3}-\frac{a-c}{(a+c)c}-\frac{2c}{a^2-c^2}$$

(c) $$\frac{3x^3-2x^2-x}{4x^3-2x^2-3x+1}$$

9 Solve the equations —

(a) $\frac{1}{5}(x-2)-\frac{1}{4}(x-4)=\frac{1}{12}(2x-3)-2\frac{3}{4}$.

(b) $$\left.\begin{aligned}\frac{a}{x}+\frac{b}{y}&=m\\ \frac{b}{x}+\frac{a}{y}&=n\end{aligned}\right\}$$

10 A labourer is engaged for thirty days, on condition that he receives 2s 6d for each day he works, and loses 1s. each day he is idle; he receives 2l., 7s. in all. How many days does he work and how many days is he idle?

1869.—AFTERNOON.

Examiners, { REV J P ASHTON, M A
REV. K S. MACDONALD, M A.

1 Define a circle, trapezium, a gnomon, and a sector when is one rectilineal figure said to be inscribed in another rectilineal figure?

2 To a given straight line to apply a parallelogram, which shall be equal to a given triangle and have one of its angles equal to a given rectilineal angle.

3 If the square described upon one of the sides of a triangle be equal to the squares described upon the other two sides of it, the angle contained by these two sides is a right angle

4 Having given the base of a triangle, the difference of the sides, and the difference of the angles at the base, it is required to describe the triangle

5 In obtuse angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side upon which, when produced, the perpendicular falls, and the straight line intercepted without the triangle between the perpendicular and the obtuse angle.

6 If two circles intersect one another, their common chord, when produced, bisects their common tangent

7 No straight line can make so great an acute angle with the diameter of a circle at its extremity, or so small angle with the line that is perpendicular to it, as not to cut the circle

8 To inscribe a circle in a given triangle

9 To inscribe a circle in a rhombus

SOLUTIONS

1869.—MORNING.

1. $$\frac{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\frac{1}{12}}{\frac{1}{3}+\frac{1}{6}}-\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}=\frac{1+\frac{6+4+3-1}{12}}{\frac{1}{2}}+\frac{\frac{5}{6}}{1-\frac{1}{6}}$$
$$=\frac{2}{\frac{1}{2}}-\frac{\frac{5}{6}}{\frac{5}{6}}=4-1=4$$

4 hrs $1'\ 10\frac{1}{2}''=4\frac{47}{2400}$ hrs $=\frac{9647}{2400}$ hrs.

and 1 week $=7\times 24$ hrs

$\therefore$ the fraction required $=\dfrac{9647}{2400}+\dfrac{1}{7\times 24}=\dfrac{9647}{403200}$
$= 02392609\dot{1}2698\dot{4}.$

2. $\cdot 062435$ of £100 = £6 4*s*. 10 44*d* ;

$7\cdot 4375$ of 10*s* = £3 14*s* 4 5*d*.

1 356 of 7*s* 6*d*. $=1\frac{107}{300}$ of $\frac{15}{2}s.=(\frac{107}{300}\times\frac{15}{2})s$ = 10*s* 2 1

2.784 of $2\frac{1}{2}d = 6.96d$.

$\therefore$ the value reqd. $=$ £6 4s. $10.44d.$ + £3 14s. $4.5d.$
+ 10s. $2.1d.$ + $6.96d.$ = £10 10s.

Also £10 10s. $=$ £$\frac{21}{2}$, and £20 10s. $7\frac{1}{2}d$ $=$ £$20\frac{17}{32}$ $=$ £$\frac{657}{32}$

$\therefore$ the fraction reqd. $= \frac{21}{2} \times \frac{32}{657} = \frac{112}{219}$.

3. $.0007 \div .035 = .02$

$.0007 \div 3500 = .0000002$

and $\sqrt{(.0\dot{2}0\dot{0}0000)} = .1414$ ....

$\sqrt{(.0\dot{0}0\dot{0}0020)} = .0004$ ....

4. The area of the 4 walls $= 22\frac{1}{2}$ ft. $\times 2$(37ft. 2in. + 25 ft. 8 in.)

$= \frac{45}{2} \times 2 \times \frac{377}{6}$ sq. ft. $= \frac{15 \times 377}{2 \times 9}$ sq. yds.

$\therefore$ the length of carpet $= \frac{5 \times 377}{6} \times \frac{4}{5}$ yds. $= \frac{377 \times 2}{3}$ yds.

$\therefore$ the cost reqd. $=$ £$\frac{377 \times 2}{3} \times \frac{11}{48 \times 4}$ $=$ £$\frac{4147}{288}$ $=$ £14 7s. $11\frac{5}{6}d$

5. The int. $=$ £901 17s. $4\frac{1}{8}d.$ $-$ £563 13s. $4\frac{1}{2}d.$ $=$ £338 4s. $0\frac{3}{10}d.$

$\therefore$ £563 13s. $4\frac{1}{2}d.$: £100 :: £338 4s. $0\frac{3}{10}d$: Int. of £100

$\therefore$ Int. of £100 $=$ £$\frac{100 \times 811683 \times 2}{10 \times 270561}$ $=$ £60

$\therefore$ £$3\frac{3}{4}$: £60 :: 1 year : no. of years.

No. of years reqd. $= \frac{1 \times 60 \times 4}{15} = 16$

6. $x+y-1)\; x^3+y^3+3xy-1 \;(\; x^2-xy+y^2+x+y+1$

$$
\begin{array}{l}
x^3+y^3+3xy-1 \\
x^3+x^2y-x^2 \\
\hline
-x^2y+x^2+3xy+y^3-1 \\
-x^2y-xy^2+xy \\
\hline
xy^2+y^3+2xy+x^2-1 \\
xy^2+y^3-y^2 \\
\hline
x^2+2xy+y^2-1 \\
x^2+xy-x \\
\hline
xy+x+y^2-1 \\
xy+y^2-y \\
\hline
x+y-1 \\
x+y-1 \\
\hline
\end{array}
$$

(a)
$$\begin{array}{r|l}
 & x^4-3x^3+\tfrac{11}{12}x^2+2x+\tfrac{4}{9}\ \big)\ x^2-\tfrac{3}{2}x-\tfrac{2}{3} \\
 & x^4 \\ \hline
2x^2-\tfrac{3}{2}x & -3x^3+\tfrac{11}{12}x^2 \\
 & -3x^3+\tfrac{9}{4}x^2 \\ \hline
2x^2-3x-\tfrac{2}{3} & -\tfrac{4}{3}x^2+2x+\tfrac{4}{9} \\
 & -\tfrac{4}{3}x^2+2x+\tfrac{4}{9} \\ \hline
\end{array}$$

7. $x^4+2x^2+1=(x^2+1)^2$

$$x^6+x^4-x^2-1=x^4(x^2+1)-(x^2+1)=(x^2+1)(x^4-1)$$
$$=(x^2+1)(x^2+1)(x^2-1)=(x^2+1)^2(x+1)(x-1)$$
$$x^4-1=(x^2+1)(x^2-1)=(x^2+1)(x+1)(x-1)$$

$\therefore$ G C. M. $=x^2+1$.

(b) $6x^2-x-1=(3x+1)(2x-1)$

$3x^2+7x+2=(3x+1)(x+2)$

$2x^2+3x-2=(2x-1)(x+2)$

$\therefore$ L C M $=(3x+1)(2x-1)(x+2)$.

8 (a) Fraction

$$=\frac{x(x+a)-x(x-a)}{x^2-a^2}-\frac{\dfrac{(x+a)^2-(x-a)^2}{x^2-a^2}}{\dfrac{(x+a)^2+(x-a)^2}{x^2-a^2}}=\frac{2ax}{x^2-a^2}-\frac{2ax}{x^2+a^2}$$

$$=\frac{2ax(x^2+a^2)(-x^2+a^2)}{(x^2+a^2)(x^2-a^2)}=\frac{2a^2\ 2ax}{x^4-a^4}=\frac{4a^3x}{x^4-a^4}.$$

(b) Since 1st $=\dfrac{a(a+c)}{c(a+c)(a-c)}=\dfrac{a}{c(a-c)}$

$$\text{Ans}=\frac{a}{c(a-c)}-\frac{a-c}{(a+c)c}-\frac{2c}{a^2-c^2}$$

$$=\frac{a(a+c)-(a-c)^2-2c^2}{c(a^2-c^2)}=\frac{a^2+ac-a^2-c^2+2ac-2c^2}{c(a^2-c^2)}$$

$$=\frac{3ac-3c^2}{c(a^2-c^2)}=\frac{3c(a-c)}{c(a+c)(a-c)}=\frac{3}{a+c}.$$

(c) Ans $=\dfrac{(x-1)(3x^2+x)}{(x-1)(4x^2+2x-1)}=\dfrac{3x^2+x}{4x^2+2x-1}.$

9. (a) $\frac{1}{8}(x-2)-\frac{1}{7}(x-4)=\frac{1}{12}(2x-3)-2\frac{3}{4}$.

$\therefore \frac{1}{8}x-\frac{1}{4}-\frac{1}{7}x+\frac{4}{7}=\frac{1}{6}x-\frac{1}{4}-\frac{11}{4}$

$\therefore \frac{1}{8}x-\frac{1}{7}x-\frac{1}{6}x=-\frac{1}{4}-\frac{11}{4}+\frac{1}{4}-\frac{4}{7}$

$\therefore -\frac{31}{168}x=-\frac{93}{28}\qquad \therefore x=3\times 6=18.$

(b) $\frac{a}{x}+\frac{b}{y}=m \ldots (1)$, $\frac{b}{x}+\frac{a}{y}=n \ldots (2)$

Multiply (1) by b and (2) by a

Then $\left.\begin{array}{l}\frac{ab}{x}+\frac{b^2}{y}=mb \\ \frac{ab}{x}+\frac{a^2}{y}=an\end{array}\right\}$ Subtracting, we have $\frac{a^2-b^2}{y}=an-bm.$

$\therefore y=\frac{a^2-b^2}{an-bm}$

From (1) $\frac{a}{x}=m-\frac{b}{y}=m-\frac{b(an-mb)}{a^2-b^2}=\frac{a(ma-nb)}{a^2-b^2}$

$\therefore x=\frac{a^2-b^2}{ma-nb}$

10. Let x be the number of days he worked.

Then $30-x=$ No of days he was idle.

By the question,

$2\frac{1}{2}x-(30-x)=47$ or $\frac{7}{2}x=77$

$\therefore 7x=2\times 77$ $\qquad\therefore x=22$

Hence he worked for 22 days and was idle for 8 days.

1869.—AFTERNOON.

1. Euclid I. Def. 15, Def. 30; Euclid II Def. 2, and Euclid III. Def 10, Euclid IV. Def. 1.

2 Euclid I 44

3 Euclid I 48

4 Let AB be the difference of the sides

On AB desc a segment of a ⊙ containing an ∠equal to ½ the diff. of the ∠s at the base

In this segment place the line CA equal to the given base. (IV. 1.)

Join BC

At the point C and in the str. line BC make the ∠BCD=∠CBD.

Then ACD is the required △ (Hyp)

∵ ∠DBC=∠DCB (Hyp) ∴ DC=BD (I. 6)

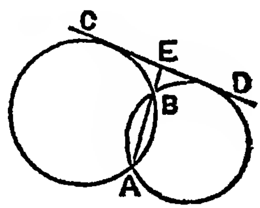
and AB=BD−AD=CD−AD=the difference of the sides AD, CD.

Also ∠CAD−ACD=ABC+ACB−ACD

=DCB+ACB−ACD=2 ACB

=the difference of the angles at the base.

5. Euclid II. 12.



6. Let A and B be the points of intersection of the two ⊙s,

and let CD be the common tangent,

prod AB to cut CD in E

∵ CE^2=AE EB=ED^2 (III. 36)

∴ CE=ED.

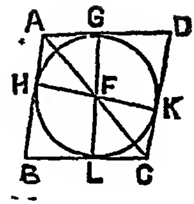
7 Euclid III. 16

8 Euclid IV 4

9. Let ABCD be the given rhombus

Join AC and bisect it in F

From F draw FG and FH $\perp$ AD and AB respectively



Prod GF and HF to meet BC and DC at L and K respectively

$\because$ AD=AB and BC=CD, also AC com

$\therefore$ $\angle$BAC=$\angle$DAC=$\angle$DCA (I 5) (I 8)

$\therefore$ AB $\parallel$ DC (I 27)

Likewise AD $\parallel$ BC

$\therefore$ $\angle$GLC and $\angle$HKC are rt $\angle$s (I 29)

In the four $\triangle$s AGF, AHF, CKF and CLF,

$\angle$AGF=$\angle$AHF=$\angle$CKF=$\angle$CLF (Ax 11)

also $\angle$GAF=$\angle$HAF=$\angle$KCF=$\angle$LCF

and AF or FC is equal or com

$\therefore$ GF=HF=KF=LF=(I 26)

Hence if with centre F and rad any of these equal lines, a $\odot$ be desc that $\odot$ will pass through points G, H, L, and K, and will touch the sides of the rhombus in those points (III 16, Cor)

1870.—MORNING.

Examiners,— { MR THWAITES, M A / MR MOWAT, B A }

1 Find the cost of matting a room whose floor is 8 yards long by $7\frac{1}{4}$ yards wide, with mats 2 feet wide and $9\frac{1}{2}$ feet long, at the rate of 9 annas 2 pies per mat

If the same room be $15\frac{1}{2}$ feet high, find how many cubic feet it will contain

2 Distinguish between a vulgar fraction and a decimal fraction. Multiply $999\frac{1843}{4146}$ by 999

State the rule for the multiplication of decimals, and apply it to point the products in (i) $1{\cdot}23 \times 0011$ and (ii) 29000×01

Divide ·37 by 148, and show that $\frac{123}{4\dot{1}} = \frac{123123}{414141}$

3 Find the square root of $19740\frac{1}{4}$ and $4\frac{3}{50}$, the latter to four places of decimals

4 Two gangs of six men and nine men are set to reap two fields of 35 and 45 acres respectively The first gang complete their work in 12 days, in how many days will the second gang complete theirs?

5 Find which is the better investment, $3\frac{1}{2}$ per cent. stock at $93\frac{3}{4}$ or $3\frac{3}{4}$ per cent at 105

6. Find how many rupees are equivalent to 200*l* at the rate of 1*s* $11\frac{3}{16}d$ per rupee

7. Find the product of $3a+2b$ and $3a+2c-3b$ and test the result by making $a=1$, $b=2$, $c=3$

Divide—

$$x^8+x^6y^2+x^4y^4+x^2y^6+y^8 \text{ by } x^4-x^3y+x^2y^2-xy^3+y^4$$

$$\text{and } \frac{a^3}{b^3}+\frac{b^3}{a^3} \text{ by } \frac{a}{b}+\frac{b}{a}.$$

8 Prove that—

$$\frac{1}{1+\frac{1}{a-x}}+\frac{1}{1-\frac{1}{a-x}}+\frac{2}{1+\frac{1}{a^2-x^2}}=\frac{4a^4}{a^4-x^4}$$

and shew that the notation $\frac{\frac{a}{c}}{c}$ is of ambiguous meaning

Simplify the expressions $\frac{x^{a+b}x^{a-b}x^{c-2a}}{x^{c-a}}$

$$\text{and } \frac{\frac{1+x}{1-x}+\frac{4x}{1+x^2}+\frac{8x}{1-x^2}-\frac{1-x}{1+x}}{\frac{1+x^2}{1-x^2}+\frac{4x^2}{1+x^2}-\frac{1-x^2}{1+x^2}}$$

9. Solve the equations—

$$\frac{x-3}{5}-\frac{x-5}{4}=\frac{2}{3}, \quad \frac{a-x}{a}+\frac{2a-x}{2a}=\frac{3a-x}{3a}$$

$$\left.\begin{aligned} \frac{1}{5x}+\frac{y}{9}&=5 \\ \frac{1}{3x}+\frac{y}{2}&=14 \end{aligned}\right\}$$

10. Find the least common multiple of —

$$x^3+x^2y+xy^2+y^3 \text{ and } x^3-x^2y+xy^2-y^3$$

Reduce $\frac{2x^4-x^3-9x^2+14x-5}{7x^3-19x^2+17x-5}$ to its lowest terms

11. Extract the square root of—

$$4x^4+8ax^3+4a^2x^2+16b^2x^2+16ab^2x+16b^4$$

12 AB is a railway 200 miles long, and three trains (P, Q, R), travel upon it at rates of 25, 20 and 30 miles per hour respectively; P and Q leave A at 7 A M and 8-15 A M respectively, and R leaves B at 10-30 A M When and where will P be equidistant from Q and R?

1870 —AFTERNOON.

Examiners,— { REV J HENRY
MR McLAREN SMITH

1 Define a straight line, a plain superficies, a circle, a rectangle, a segment of a circle, a sector of a circle In what particular case can a sector be called a segment ?

2 Distinguish between angle in a segment and angle of a segment, draw a diagram for each When is a circle said to be described about, and when is it inscribed in, a plain rectilineal figure ? Can the diagonals of any quadrilateral figure be called diameters ? why can they in parallelograms ?

3 Given that two triangles are between the same parallels and equal in area prove that their bases are equal

Show that all the exterior angles of a quadrilateral figure, made by producing the sides successively in the same direction, are together equal to four right angles Will the value be the same for the exterior angles of a pentagon ?

5 The rectangle contained by the sum of two straight lines and their difference is equal to the difference of their squares (The proposition 5th, Book II may be assumed without proving)

6 Divide a straight line into two parts, so that the rectangle contained by the whole and one of the parts shall be equal to the square on other part In the diagram given by Euclid, point out any other straight line, besides the given one, similary divided

7 Enunciate and prove the tenth proposition of the fourth Book How does it enable us to divide a right angle into ten equal parts ?

8 The straight lines OA OB being given, intersecting in O and a point C being given in OA describe a circle touching OA in C and also touching OB

9 Having given that the angle in a semicircle is a right angle, show that the angle in a segment greater than a semicircle is less than a right angle

10 The straight line drawn from the right angle, in any right angled triangle, to the bisection of the hypotenuse, is equalto half the hypotenuse

SOLUTIONS.

1870.—MORNING.

1 The area of floor $= (8 \times \frac{29}{4})$ sq yds

" " mat $= (2 \times \frac{19}{2})$ sq ft $= \frac{19}{9}$ sq yds.

$\therefore$ No of mats $= \dfrac{2 \times 29 \times 9}{19}$

$\therefore$ the cost reqd. $= \dfrac{2 \times 29 \times 9}{19} \times \frac{55}{6}$ as $= \frac{4785}{19}$ as

$=$ Rs. 15 11as $10\frac{2}{19}$ pies

The cubical content $= 8 \times 3 \times \frac{29}{4} \times 3 \times \frac{31}{2}$ cub ft

$= 8091$ cub ft

2 Decimals are fractions having either 10, or some power of 10, for their denominator. For this reason they are call Decimal fractions, in contradistinction to Vulgar Fractions, which are represented by a different notation, and not limited in their denominator to 10 or powers of 10

$(999 + \frac{4945}{4946}) \times 999 = (1000 - \frac{1}{4946}) \times 999$

$= 999000 - \frac{999}{4946} = 998999 + 1 - \frac{999}{4946} = 998999\frac{3947}{4946}$

(1) $1\,23 \times \ 0011 = \ 001353.$ (2) $29000 \times {}^{\cdot}01 = 290.$

(a) $\cdot 37 \div 14\dot{8} = \frac{37}{99} \times \frac{999}{148} = \frac{111}{44} = 2\ 5227$

$\dfrac{\cdot\dot{1}23}{4\dot{1}} = \dfrac{123}{999} \times \dfrac{99}{41} = \dfrac{11}{37}$

$\dfrac{123123}{414141} = \dfrac{11}{37}$

3 $\sqrt{(\dot{1}9740\ 2\dot{5})} = 140\ 5.$ $\sqrt{(4\ 010\dot{1}0101)} = 2\ 0025$

4 6 men $\times 12d$ 9 men $\times xd$ $35ac$ $45ac$

$\therefore x = \dfrac{6 \times 12 \times 45}{9 \times 35}$ days $= 10\frac{2}{7}$ days

5 £$3\frac{1}{2}$ · £$3\frac{3}{4}$ £$98\frac{1}{4}$ x

$\therefore x = £\,\dfrac{393 \times 15 \times 2}{4 \times 4 \times 7} = 105\frac{15}{56}$

$\therefore$ the latter is the better investment

6. 1s $11\frac{3}{16}d.$ £200 1 rupee x

$\therefore x =$ Rs. $\dfrac{1 \times 200 \times 20 \times 192}{371} =$ Rs $2070\frac{30}{371}$

7. $3a + 2c - 3b \qquad = 3 + 6 - 9 = 0$
$3a + 2b \qquad = 3 + 6 \quad = 9$

$9a^2 + 6ac - 9ab \qquad 0 = \text{Prod.}$
$6ab + 4bc - 6b^2$

Prod $= 9a^2 + 6ac - 3ab + 4bc - 6b^2$

$= 9\ 1 + 6\ 1.3 - 3\ 1\ 3 + 4.3\ 3 - 6\ 9$

$= 9 + 18 - 9 + 36 - 54 = 63 - 63 = 0$

$$
\text{(a)}\quad \begin{array}{r} x^4-x^3y+x^2y^2 \\ -xy^3+y^4 \end{array}\Bigg)\begin{array}{l}
x^8+x^6y^2+x^4y^4+x^2y^6+y^8 \\
x^8-x^7y+x^6y^2-x^5y^3+x^4y^4 \\ \hline
x^7y+x^5y^3+x^2y^6+y^8 \\
x^7y-x^6y^2+x^5y^3-x^4y^4+x^3y^5 \\ \hline
x^6y^2+x^4y^4-x^3y^5+x^2y^6+y^8 \\
x^6y^2-x^5y^3+x^4y^4-x^3y^5+x^2y^6 \\ \hline
x^5y^3+y^8 \\
x^5y^3-x^4y^4+x^3y^5-x^2y^6+xy^7 \\ \hline
x^4y^4-x^3y^5+x^2y^6-xy^7+y^8 \\
x^4y^4-x^3y^5+x^2y^6-xy^7+y^8 \\ \hline
\end{array}\Bigg(\begin{array}{l} x^4+x^3y+x^2y^2 \\ +xy^3+y^4 \end{array}
$$

$$
\text{(a)}\quad \frac{a}{b}+\frac{b}{a}\Bigg)\begin{array}{l}
\dfrac{a^3}{b^3}+\dfrac{b^3}{a^3} \\[2ex]
\dfrac{a^3}{b^3}+\dfrac{a}{b} \\ \hline
-\dfrac{a}{b}+\dfrac{b^3}{a^3} \\[2ex]
-\dfrac{a}{b}-\dfrac{b}{a} \\ \hline
\dfrac{b}{a}+\dfrac{b^3}{a^3} \\[2ex]
\dfrac{b}{a}+\dfrac{b^3}{a^3} \\ \hline
\end{array}\Bigg(\frac{a^2}{b^2}-1+\frac{b^2}{a^2}
$$

8 Ans $=\dfrac{1}{1+\dfrac{x}{a}}+\dfrac{1}{1-\dfrac{x}{a}}+\dfrac{2}{1+\dfrac{x^2}{a^2}}$

$$=\frac{a}{a+x}+\frac{a}{a-x}+\frac{2a^2}{a^2+x^2}=\frac{2a^2}{a^2-x^2}+\frac{2a^2}{a^2+x^2}$$

$$=2a^2\left\{\frac{1}{a^2-x^2}+\frac{1}{a^2+x^2}\right\}=\frac{2a^2\,2a^2}{a^4-x^4}=\frac{2a^2 2a^2}{a^4-x^4}=\frac{4a^4}{a^4-x^4}$$

(a) $\dfrac{\dfrac{a}{b}}{c}$ may mean $a\times\dfrac{c}{b}=\dfrac{ac}{b}$, or $\dfrac{a}{b}\times\dfrac{1}{c}=\dfrac{a}{bc}$.

(a) $\dfrac{x^{a+b}x^{a-b}x^{c-2a}}{x^{c-a}} = \dfrac{x^{a-b+a-b+c-2a}}{x^{c-a}}$

$= \dfrac{x^c}{x^{c-a}} = x^{c-(c-a)} = x^a.$

(c) Numr $= \dfrac{(1+x)^2-(1-x)^2}{1-x^2} + \dfrac{8x}{1-x^2} + \dfrac{4x}{1+x^2}$

$= \dfrac{4x}{1-x^2} + \dfrac{8x}{1-x^2} + \dfrac{4x}{1+x^2} = 4x \left\{\dfrac{3}{1-x^2} + \dfrac{1}{1+x^2}\right\}$

$= 4x\left\{\dfrac{3(1+x^2)+1-x^2}{1-x^4}\right\} = \dfrac{8x(2+x^2)}{1-x^4}.$

Denr $= \dfrac{(1+x^2)^2-(1-x^2)^2}{1-x^4} + \dfrac{4x^2}{1+x^4}$

$= \dfrac{4x^2}{1-x^4} + \dfrac{4x^2}{1+x^4} = 4x^2\left(\dfrac{1}{1-x^4} + \dfrac{1}{1+x^4}\right) = \dfrac{8x^2}{(1-x^4)(1+x^4)}$

$\therefore$ Ans $= \dfrac{8x(2+x^2)}{1-x^4} \times \dfrac{(1-x^4)(1+x^4)}{8x^2} = \dfrac{(2+x^2)(1+x^4)}{x}.$

9 (a) $\dfrac{x-3}{5} - \dfrac{x-5}{4} = \dfrac{2}{3}$

or $12x-36-15x+75=40.$

or $-3x=40+36-75=1$ $\quad\therefore x=-\frac{1}{3}$

(b) $\dfrac{a-x}{a} + \dfrac{2a-x}{2a} = \dfrac{3a-x}{3a}$

or $6a-6x+6a-3x=6a-2x$

or $-9x+2x=-6a$ $\quad\therefore x=\frac{6}{7}a$

(c) $\dfrac{1}{5x} + \dfrac{y}{9} = 5$ (1), $\quad \dfrac{1}{3x} + \dfrac{y}{2} = 14 \ldots (2)$

Multiply (1) by $\frac{1}{3}$ and (2) by $\frac{1}{5}$

Then $\left.\begin{array}{l}\dfrac{1}{15x} + \dfrac{y}{27} = \dfrac{5}{3} \\ \dfrac{1}{15x} + \dfrac{y}{10} = \dfrac{14}{5}\end{array}\right\}$ Substraction

$\frac{17}{270}y = \frac{17}{15}$

$\therefore y+\frac{270}{15}=18$, and $\dfrac{1}{15x} = \frac{5}{3} - \frac{2}{3} = 1$ $\quad\therefore x=\frac{1}{15}$

10. 1st quantity $= x^2(x+y)+y^2(x+y) = (x+y)(x^2+y^2)$

2nd quantity $= x^2(x-y)+y^2(x-y) = (x-y)(x^2+y^2)$

$\therefore$ L C M $= (x+y)(x^2+y^2)(x-y) = x^4-y^4$

(a) Here G C. M $= x^2-2x+1$

$\therefore$ Ans $= \dfrac{(x^2-2x+1)(2x^2+3x-5)}{(x^2-2x+1)(7x-5)} = \dfrac{2x^2+3x-5}{7x-5}$

11. $4x^4+8ax^3+4a^2x^2+16b^2x^2+16ab^2x-16b^4 \big(2x^2+2ax+4b^2$ = sq root

$4x^4$

$4x^2+2ax \,|\, 8ax^3+4a^2x^2$

$ 8ax^3+4a^2x^2$

$4x^2+4ax+4b^2 \,|\, 16b^2x^2+16ab^2x+16b^4$

$ 16b^2x^2+16ab^2x+16b^4$

12. Let x = no of hrs P travelled after starting,

then $x-1\frac{1}{4}$ = no. of hrs. Q travelled

and $x-3\frac{1}{2}$ = ——— R ———

$\therefore AP=25x,\ AQ=20(x-1\frac{1}{4}),\ BR=30(x-3\frac{1}{2})$

Now $PQ=AP-AQ=25x-20(x-1\frac{1}{4})$

and $PR=AB-AP-BR=220-25x-30(x-3\frac{1}{2})$

But by question, $PQ=PR$

$\therefore 25x-20(x-1\frac{1}{4})=220-25x+30(x-3\frac{1}{2})$

or $5x+25=325-55x$, $\therefore 60x=300$, $\therefore x=5$

Hence time = 5 hrs or 12 o'clock

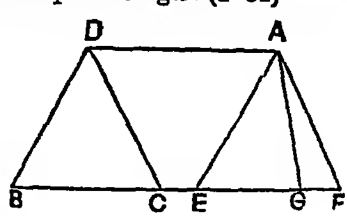
and distance $=AP=25x=125$ miles

1870.—AFTERNOON.

1. Euc. I Defs. 4, 7, 15, 33, Euc III Defs 6, 10. A sector becomes a segment of a circle, when the two radii which compose the sector are in the same straight line, i e. when both of them become a semi circle

2 Euc III. Defs 6 and 7; Euc. IV Defs 4 and 5, Euc III. Def 21 The diagonal of a parallelogram may be called its diameter because it divides the parallelogram into two equal triangles (I 34)

3 Let DBC and AEF be two equal Δs between the ∥ s AD and BF.



Then BC shall be equal to EF.

If not, let EG be equal to BC,

Join AG.

$\therefore \Delta DBC=\Delta AEG$, (I 38)

$\therefore \Delta AEG=\Delta AEF$, (Ax 9)

which is absurd,

$\therefore BC=EF$.

4 Euclid (I 32, Cor II) Yes, the value will be the same.

5 Let AD be the sum of the two str lines AB and BD.

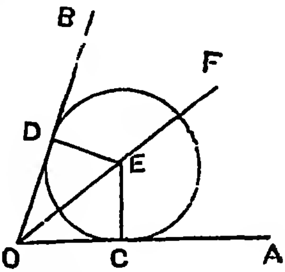
From BD produced cut off BC=AB

Then CD is the difference of BC and BD or AB and BD

$\because$ AD DC+BD²=BC² (II 6)
Take BD² from both these equals;
$\therefore$ AD DC=BC²−BD²=AB²−BD²
$\therefore$ (AB+BD)(AB−BD)=AB²−BD²

6. Euclid II. 11 See the solution of question 6 (*a*) of 1868 Bisect the vertical angle and again bisect one of the parts.

7 The vertical $\angle=\frac{1}{4}$ of the $\angle$ at the base
$\therefore$ it is $=\frac{1}{5}$ of all the $\angle$s of the $\triangle$ (*i e*) of 2 rt $\angle$s (I 32)
Then each of the last divided part is equal to one tenth of a right angle.



8. Bisect the $\angle$AOB by OE
From C draw CE$\perp$OA cutting OE in E.
From E draw ED$\perp$OB
$\because$ $\angle$ODE=$\angle$OCE (Ax 11)
$\angle$DOE=$\angle$COE and OE common.
$\therefore$ DE=EC, (I 26)
$\therefore$ the $\odot$ desc with centre E and rad EC will pass through the pt. D
Again $\because$ OD and OC are $\perp$DE and CE
$\therefore$ OD and OC touch the $\odot$ at D and C (III 16, Cor.)

9 Euclid III 31

10. See solution of question 8 of 1864

1871.—MORNING.

Examiners,— { MR EUBANK, B A / MR GRIFFITHS, B A }

1 6625 railway tickets were sold at a station $\frac{2}{5}$ths of which were 9 annas each and the rest 5 annas each What was the amount received from the tickets?

2. Find the greatest and least of the fractions $\frac{1}{7}$, $\frac{5}{28}$, $\frac{6}{49}$, $\frac{23}{196}$. Add together $2\frac{2}{3}$ of £2 13*s*. $6\frac{1}{3}$*d* and £3 15*s* $9\frac{1}{2}$*d*$\div 6\frac{3}{7}$, and

Simplify—

$$\frac{2\frac{1}{2}-1\frac{3}{4}}{4 \text{ of } 1\frac{1}{3}+6\times\frac{1}{2}}\times\frac{5-\frac{5}{9}}{\frac{1}{2}+\frac{2}{3}}+\frac{5}{9}$$

3. Divide 027 by 14·4 and 1208 04 by 017.

Find the value of 11 1375 of Rs 6-8 as − 56 of Rs 7 8 as, and reduce 8 as 6 pie to the decimal of Rs 3 7 as.

4 If the carriage of $9\frac{3}{4}$ mds for a distance of 80 miles be Rs 3 how many miles should 130 mds be carried for Rs. 27-8 as.

5 What sum of money will amount to Rs 3761 14 as in $3\frac{1}{2}$ years at $4\frac{1}{2}$ per cent per annum simple interest?

6 Multiply $x^3-\frac{1}{2}x^2y-3y^3$ by $2x^2-\frac{1}{3}y^2$, and find the square root of $x^4-2x^2-\frac{2}{x^2}+\frac{1}{x^4}+3$.

7 Reduce $\frac{10x^3+19x^2-9}{25x^3-19x+6}$ to its lowest terms, and find the least common multiple of $2(x-2)^2$, $2x^2-8$, x^3+2x^2, $2x^2-4x$

8 Simplify (i) $1+\frac{a}{b}-\frac{b}{a+b}-\frac{a^2}{ab-b^2}+\frac{2a^3}{a^2-b^2}$

(ii) $\left(1-\frac{1}{1+x}\right)\left(x+\frac{1}{2+x}\right)\times\frac{\frac{1}{x^2}-x}{1+\frac{1}{x}}\div\left(1+x+\frac{1}{x}\right)$

9 Solve the equations —

(i) $\frac{6}{3x-5}+\frac{1}{x-5}=\frac{2}{2x-5}$

(ii) $4x-\frac{2}{5}(2y-3)=6\frac{1}{5}$

$3y-\frac{2}{3}(3x-1)=7.$

10 A and B compared their monthly incomes and found that A's income was to that of B as 7 to 9, and that the third of A's income was Rs 30 greater than the difference of their incomes. Find what each received

1871.—AFTERNOON.

Examiners,— { MR MOWAT, M A.
MR WILSON, B A

1 Define a plane rectilineal angle, a gnomon, the distance of a point from a right line the distance of a point from a circle, an angle in a segment of a circle, similar segments, regular polygon When is a right line said to be placed in a circle?

2. (a) Prove that in any triangle if a side be produced, the external angle so formed is equal to the sum of the two internal and opposite angles.

(b) In a polygon of n sides the sum of all the internal angles equals $(2n-4)$ right angles

3 (a) If the square on one side of a triangle equal the sum of the squares on the other two sides, then the angle contained by these two sides will be a right angle

(b) Find a line whose square shall be equal to the sum of the squares on three given right lines

4. ABC is a right-angled triangle AD, the perpendicular from A upon the hypotenuse BC, is produced in the direction DA till it meets a side produced of the square on AC in O. Prove that it will meet a side produced of the square on AB in the same point O, that AO shall be equal to BC, and that if O be joined with B, and A with E, the extremity of the side BE of the square on BC the figure OAEB shall be a parallelogram equal in area to the square on AB.

5. The rectangle under two lines, one of which is divided into any number of parts, is equal to the sum of the rectangles under the undivided line and the several parts of the divided line Prove this proposition, and deduce from it the following rule for finding the area of a triangle multiply any side by the perpendicular upon it from the opposite angle and take half the product for the area

6 If a tangent be drawn to a circle at any point, and from the point of contact a right line be drawn cutting the circle, the angles made by this line with the tangent are equal to the angles in the alternate segment of the circle.

7 Given the base, the vertical angle, and the perpendicular let fall from the vertex on the base, construct the triangle and show that *in general* there can be two triangles constructed satisfying the given conditions

8 If from any point without a circle two right lines be drawn, one of which cuts the circle and the other touches it; the rectangle contained by the whole line which cuts the circle and the part of it without the circle shall be equal to the square on the line which touches it.

9. (*a*) To describe a circle about a given triangle.

(*b*) The perpendiculars erected at the middle points of the sides of a triangle meet in a point

SOLUTIONS

1871.—AFTERNOON.

1 $\frac{6625 \times 2}{5} \times 9$ as $= 23850$ as

and, $\frac{6625 \times 3}{5} \times 5$ as $= 19875$ as

∴ the amount received $= 43725$ as. $=$ Rs 2732 13 as

2 The L C. M $= 196$.

$\therefore \frac{1}{7} = \frac{28}{196}, \frac{5}{28} = \frac{35}{196}, \frac{6}{49} = \frac{24}{196}$ and $\frac{21}{196} = \frac{21}{196}$

∴ $\frac{5}{28}$ is the greatest, and $\frac{21}{196}$ is the least.

(*a*) $2\frac{2}{5}$ of £2 13*s* $6\frac{1}{2}d. = \frac{12 \times (£2\ 13s\ 6\frac{1}{2}d)}{5} = £6\ 8s\ 6d.$

$£\,3\ 15s\ 9\frac{1}{2}d - 6\frac{3}{7} = \dfrac{(£3\ 15s\ 9\frac{1}{2}d)\times 7}{45} = 11s.\ 9\frac{43}{90}d$

$\therefore$ the value $= £6\ 8s.\ 6d + 11s.\ 9\frac{43}{90}d. = £7\ 0s\ 3\frac{43}{90}d.$

(b) $\dfrac{2\frac{1}{2}-1\frac{3}{4}}{4 \text{ of } 1\frac{1}{3}+6\times\frac{1}{2}} \times \dfrac{5-\frac{5}{9}}{\frac{1}{2}+\frac{2}{5}} + \frac{5}{9} = \dfrac{\frac{5}{2}-\frac{7}{4}}{4 \text{ of } \frac{4}{3}+3} \times \dfrac{4\frac{4}{9}}{\frac{5+4}{10}} + \frac{5}{9}$

$= \dfrac{\frac{10-7}{4}}{\frac{16}{3}+3} \times \dfrac{\frac{40}{9}}{\frac{9}{10}} + \frac{5}{9} = \dfrac{\frac{3}{4}}{\frac{25}{3}} \times \frac{40}{9} \times \frac{10}{9} + \frac{5}{9} = \frac{3}{4} \times \frac{3}{25} \times \dfrac{40\times 10}{81} + \frac{5}{9}$

$= \frac{4}{9} + \frac{5}{9} = 1$

3 $\quad .027 - .144 = .001875$

$\dfrac{1208.04}{.017} = \dfrac{\frac{120804}{100}}{\frac{15}{900}} = \dfrac{120804}{100} \times \dfrac{900}{16} = 67952.25$

11.1375 of Rs $6\frac{1}{2} = \frac{890}{80}$ of $\frac{13}{2}$ Rs. = Rs. 72 6 3 as

.56 of Rs. $7\frac{1}{2}$ = Rs $\frac{17}{4}$ = Rs 4 4 as

$\therefore$ value = Rs 72 6 3 as. − Rs 4 4 as = Rs. 68 2 3 as

= Rs 68 2 as 3 6 p

And 8 as 6 pies $= \frac{17}{2}$ as and Rs 3 7 as. = 55 as.

$\therefore$ the fraction reqd $= \dfrac{17}{2\times 55} = \dfrac{17}{110} = .154.$

4. $9\frac{3}{4}$ m $\times$ 80 mi : 130 m $\times$ x mi :: Rs 3 : Rs. $27\frac{1}{2}$.

$\therefore\ x = \dfrac{39\times 80\times 55}{4\times 2\times 130\times 3}$ miles = 55 miles.

5 Let Rs 100 be the sum,

$\therefore$ the amount of it in $3\frac{1}{2}$ yrs at $4\frac{1}{2}$ p c

= Rs 100 + Rs $\frac{63}{4}$ = Rs $\frac{463}{4}$

$\therefore$ Rs $\frac{463}{4}$: Rs $3761\frac{7}{9}$ :: Rs 100 : reqd. sum.

$\therefore$ the sum reqd = Rs. $\dfrac{100\times 30095\times 4}{8\times 468}$ = Rs 6250.

6 $\quad x^3 - \frac{1}{2}x^2y - 3y^3$

$2x^2 - \frac{1}{3}y^2$

$2x^5 - x^4y - 6x^2y^3 - \frac{1}{3}x^3y^2 + \frac{1}{6}x^2y^3 + y^5$

$= 2x^5 - x^4y - \frac{1}{3}x^3y^2 - \frac{35}{6}x^2y^3 + y^5$

(a) $x^4-2x^2+3-\frac{2}{x^2}+\frac{1}{x^4}\left(x^2-1+\frac{1}{x^2}\right.=$ sq root.

$$\begin{array}{l|l} & x^4 \\ \hline 2x^2-1 & -2x^2+3 \\ & -2x^2+1 \\ \hline 2x^2-2+\frac{1}{x^2} & 2-\frac{2}{x^2}+\frac{1}{x^4} \\ & 2-\frac{2}{x^2}+\frac{1}{x^4} \\ \hline \end{array}$$

7. Here G C. M. $=5x^2+2x-3$

$$\therefore \text{Ans.}=\frac{(2x+3)(5x^2+2x-3)}{(5x-2)(5x^2+2x-3)}+\frac{2x+3}{5x-2}$$

(a) $2(x-2)^2=2(x-2)(x-2)$

$2x^2-8=2(x^2-4)=2(x-2)(x+2)$

$x^3+2x^2=x^2(x+2)$, $2x^2-4x=2x(x-2)$

L C. M. $=2x^2(x-2)^2(x+2)$.

8. (i) $1+\frac{a}{b}-\frac{b}{a+b}-\frac{a^2}{ab-b^2}+\frac{2a^2}{a^2-b^2}$

$$=\frac{b+a}{b}-\frac{b}{a+b}-\frac{a^2}{b(a-b)}+\frac{2a^2}{(a+b)(a-b)}$$

$$=\frac{(a^2-b^2)(a+b)-b^2(a-b)-a^2(a+b)+2a^2b}{b(a^2-b^2)}$$

Numr. $=(a+b)(a^2-b^2-a^2)-b^2(a-b)+2a^2b$

$=-b(ab+b^2+ab-b^2-2a^2)=2ab(a-b)$.

$$\text{Ans.}=\frac{2ab(a-b)}{b(a+b)(a-b)}=\frac{2a}{a+b}$$

(ii) The expression $=$

$$\left(\frac{1+x-1}{1+x}\right)\times\left(\frac{2x+x^2+1}{2+x}\right)\times\frac{\frac{1-x^3}{x^2}}{\frac{x+1}{x}}-\left(\frac{x+x^2+1}{x}\right)$$

$$=\frac{x}{1+x}\times\frac{x^2+2x+1}{2+x}\times\frac{1-x^3}{x^2}\times\frac{x}{x+1}\times\frac{x}{x^2+x+1}$$

$$=\frac{x^2(x+1)^2(1-x)(1+x+x^2)}{(1+x)(2+x)(x+1)(x^2+x+1)}=\frac{x(1-x)}{2+x}.$$

9. (1) $\frac{6}{3x-5}-\frac{1}{x-5}=\frac{2}{2x-5}$

Multiply each side by $2x-5$

$\therefore \frac{12x-30}{3x-5}-\frac{2x-5}{x-5}=2$

or $\left(4-\frac{10}{3x-5}\right)-\left(2+\frac{5}{x-5}\right)=2$

$\therefore \frac{2}{3x-5}=-\frac{1}{x-5}$, or $2x-10=-3x+5$

$\therefore 5x=15$ $\qquad x=3$

(2) $4x-\frac{3}{5}(2y-3)=6\frac{1}{5}$ (1) $3y-\frac{2}{3}(3x-1)=7$ ..(2)

From (1) $4x-\frac{6}{5}y+\frac{9}{5}=\frac{31}{5}$ or $4x-\frac{6}{5}y=\frac{22}{5}$

From (2) $3y-2x+\frac{2}{3}=7$ or $-2x+3y=\frac{19}{3}$

Multiplying (2) by -2, $4x-6y=-\frac{38}{3}$

Subtracting (1) $\frac{24}{5}y=\frac{256}{15}$, $\qquad \therefore y=\frac{256}{15}\times\frac{5}{24}=3\frac{5}{9}$

and $4x=\frac{22}{5}+\frac{6}{5}y=\frac{22}{5}+\frac{64}{15}=\frac{130}{15}$, $\qquad \therefore x=\frac{13}{6}=2\frac{1}{6}$.

10 Let x be the income of A in Rs

Then B's income $=\frac{9}{7}$ of A's $=\frac{9}{7}x$

Now by the question,

$\frac{1}{3}x=\frac{9}{7}x-x+30=\frac{2}{7}x+30$, or $\frac{1}{21}x=30$

$\therefore x=$ Rs 630 A's income and B's $=\frac{9}{7}x=$ Rs. 810.

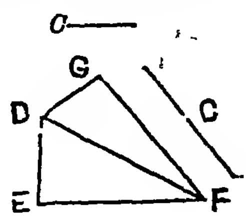
1871.—AFTERNOON.

1. Euclid I Defs 9 and 10, Euclid II Def. 2; Euclid III Defs. 8 and 11, Euclid 1 Def 24, Euclid IV Def. 7

2 (*a*) Euclid I Prop 32 1st case.

(*b*) I 32, Cor 1 Sum of the int $\angle$s+4 rt $\angle$s$=2n$ rt $\angle$s, $\therefore$ sum of the int $\angle$s$=(2n-4)$ rt $\angle$s

3 (*a*) Euclid I 48



(*b*) Let DE, EF, and C be the three str lines

Place DE and EF at rt $\angle$s to each other join DF.

Then $DF^2=DE^2+EF^2$ (I 47)

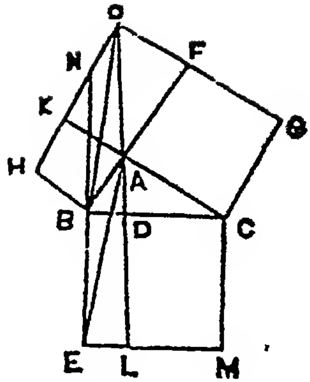
Again draw DG$\perp$DF and make DG=C. Join GF

Then $FG^2=DG^2+DF^2=DG^2+DE^2+EF^2$ (I 47).

4 Let AFGC and AKHB be the squares on AC and AB.

Join OK and prod. EB to meet OH in N

∵ ∠ ABC + ∠ ACB = a rt. ∠
= ∠BAD + ∠ ABD (I. 32)
∴ ∠ BAD = ∠ ACB ·
also ∠ BAD = ∠ OAF (I. 15)
∴ ∠ OAF = ∠ BCA :
also ∠ BAC = ∠ OFA (Ax 11)
and AC = AF,
∴ OA = BC and OF = AB
= AK (I 26)
∵ the ∠s KAF and AFO are rt ∠s
∴ AK ∥ OF (I 28)
but AK = OF,
∴ OK = and ∥ AF, (1 33)



and AKO is a rt ∠ ; also AKH is a rt. ∠.
∴ HK is in the same str line with KO (I 14)
Wherefore HK produced will pass through the pt. O
Again ∵ the ∠ ODB = ∠ DBE = a rt. ∠.
∴ OD ∥ BE ; (I 27)
also AO = BC = BE.
∴ BO = and ∥ EA ; (I. 33)
∴ OAEB is a parallelogram,
and ▱OAEB = ▱OABN = ▱ABHK, the square on AB.

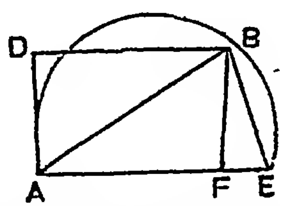
5 Euclid II. 1.

Twice the area of a △ = any ▱ between the same ∥s and upon the same base, also = a rt. ∠d ▱ between the same ∥s and on the same base, which again = product of its sides Hence the rule.

6 Euclid III. 32

7 Let AE be the give base.

On AE desc a seg of a ⊙ containing at ∠ = given vertical ∠.
From A draw AD ⊥ AE and = given perpendicular
From D draw DB ∥ AE and cutting the segment in G and B ;
Join AB and BE.
From B draw BF ⊥ AE.
Then DBFA is a ▱ ; (I. 28)



∴ BF = DA = the given perpendicular and ∠ABE = the given angle

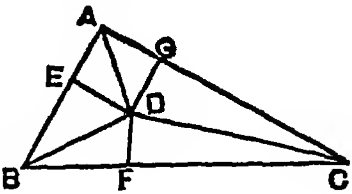
If AG and EG be joined, then another △ is formed which satisfies the conditions

† Only one △ will be formed, if DGB touch the circle

8. Euclid III. 36

9 (a) Euclid IV 5

(a) Let AB, BC and CA, the sides of the $\triangle$ ABC, be bisected in E, F and G respectively



Draw ED and FD$\perp$AB and BC and meeting each other at the point D.
Join AD, BD, DC and DG
$\because$ AE=EB and ED com
and $\angle$ AED=$\angle$ DEB, (Ax 11)
$\therefore$ AD=BD (I 4)
Likewise BD=CD $\therefore$ AD=DC
Again $\because$ AG=GC, DG com and AD=DC,

$\therefore$ $\angle$ AGD=DGC (I 8)

$\therefore$ DG$\perp$AC (Def. 11)

Hence a line drawn from G$\perp$AC, shall pass through D.

1872.—MORNING.

Examiners,—{ Mr Griffths, B A / Mr Mowat, M.A.

1 A merchant bought goods which cost him Rs 9,810. in the first day he sold to the amount of Rs 992-8as 6p ; in the second to that of Rs 1,992 8as 3p and in the next three days to an amount equal to twice the two former Finding that he had one-fourth of the goods left, he calculated his profits in the five days How much were they ?

2. What fraction of Rs 10 is Rs 6 10as 8p ?

Find the value of

$$\tfrac{5}{8} \text{ of Rs. 2 8as} + \tfrac{3}{8} \text{ of Rs 4 11as.} + 2\cdot05 \text{ of Rs 5.}$$

Simplify $4\frac{1}{3}$ of $\frac{7}{20} - \dfrac{\frac{3}{7}}{\frac{26}{33}} + \dfrac{5\frac{1}{3} - 4 \text{ of } \frac{2}{3}}{3\frac{1}{4} - 2\frac{1}{2}}$.

3 Divide 274·72 by ·0544, find the value (correct to six places of decimals) of (i) $\dfrac{\cdot003 \times \cdot05}{\cdot0022}$, (ii) $6\cdot045 - 5\cdot36\dot{7}8$ and extract the square root of 951·1056

4. Find by Practice the cost of 15 mds 25 srs. 11 chs. of oil at Rs 12 10as 3p per maund

5 If the interest of Rs 1000 in five years be Rs 250 what will be the interest of Rs 3,500 for 1 year and 6 months ?

6 Divide $x^4 - 10x^2 + 9$ by $x^2 - 2x - 3$, and find the G C. M. of $3x^3 - 17x^2 + 19x + 11$, and $6x^3 - 25x^2 + 17x - 22$

7 Simplify—

(i) $$\left\{\frac{2a}{x^2-a^2}-\frac{1}{x-a}+\frac{2}{x+a}\right\}\times\frac{x}{x-a+\dfrac{a^2}{x}}$$

(ii) $$\frac{1}{x(x-y)(x-z)}+\frac{1}{y(y-x)(y-z)}+\frac{1}{z(z-x)(z-y)}$$

8 Solve the equations—

(i) $$\frac{7x-1}{4}-\frac{1}{3}\left(2x-\frac{1-x}{2}\right)=6\frac{1}{3}.$$

(ii) $$\left.\begin{aligned}\frac{x-y}{3}&=\frac{y-1}{4}\\ \frac{4x-5y}{7}&=x-7\end{aligned}\right\}$$

9. Rs. 1 100 are so divided among A B and C, that if A were to give B Rs 200 B would then have twice as much as A and three times as much as C. How many rupees did A, B, and C each receive originally?

10. If $a : b :: c : d$ prove that $a : b :: a \pm c : b \pm d$; also show that $a^2 \pm c^2 : b^2 \pm d^2 :: (a \pm c)^2 : (b \pm d)^2$.

1872 —AFTERNOON.

Examiners,— $\left\{\begin{array}{l}\text{Mr McLaren Smith}\\ \text{Rev. J. Fibres}\end{array}\right.$

1 Any two angles of a triangle are together less than two right angles.

State the converse of this proposition. Where is it first used by Euclid?

2 If the square described on one of the sides of a triangle be equal to the squares described on the other two sides of it, the angle contained by these two sides is a right angle

What proposition is tacitly assumed in the proof?

If $a^2+b^2=c^2$ $a+b$ must be greater than c?

3 If a straight line be divided into two equal, and also into two unequal parts, the squares on the two unequal parts are together double of the square on half the line, and of the square on the line between the points of section.

Divide a line so that the rectangle contained by the parts shall be the greatest possible

4 In every triangle, the square on the side subtending either of the acute angles is less than the squares on the sides containing that angle, by twice the rectangle contained by either of these sides, and the straight line intercepted between the acute angle and the perpendicular let fall upon it from the opposite angle

In a triangle ABP, the square on AP is less than the square on PB by a constant quantity. Prove that P must be on a certain straight line

5 If in a circle two straight lines cut one another, which do not pass through the centre they do not bisect each other.

If the parts of two chords at right angles to one another be given, explain how the length of the radius of the circle may be calculated

6 If from any point without a circle two straight lines be drawn, one of which cuts the circle and the other touches it, the rectangle contained by the whole line which cuts the circle and the part of it without the circle shall be equal to the square on the line which touches it.

7 Inscribe an equilateral and equiangular hexagon in a circle

Compare the area of a regular hexagon inscribed in a circle with that of an equilateral triangle inscribed in the same circle

8. Inscribe an equilateral and equiangular quindecagon in a circle Express each of the angles in your figure in terms of a right angle.

SOLUTIONS

1872 —MORNING.

1 The real cost of $\frac{3}{4}$ of the goods $= (9810 \times \frac{3}{4})$ Rs

$=$ Rs 7357 8as

Amount realized in 5 days

$=$(Rs 992 8as 6 p + Rs 1992 8as 3 p) $\times 3$

$=$ Rs 8955 2as. 3 p.

$\therefore$ profits $=$ Rs 8955 2as. 3 p $-$ Rs 7357 8as.

$=$ Rs 1597 10as 3 p

2 Rs 6 10as 8 pies $=$ Rs $6\frac{2}{3}$ $=$ Rs $\frac{20}{3}$,

$\therefore$ the fraction reqd. $= \frac{20}{3} \times \frac{1}{10} = \frac{2}{3}$.

(*a*) $\frac{5}{6}$ of Rs $2\frac{1}{2}$ $=$ Rs $(\frac{5}{6} \times \frac{5}{2})$ $=$ Rs $\frac{25}{12}$ $=$ Rs 2 1a 4 pie.

$\frac{3}{5}$ of Rs $4\frac{11}{16}$ $=$ Rs $(\frac{3}{5} \times \frac{75}{16})$ $=$ Rs. $\frac{45}{16}$ $=$ Rs 2 13as.

2 05 of 5 Rs $=$ Rs 10 4 as

$\therefore$ value $=$ Rs 2 1a 4 pies + Rs 2 13as. + Rs 10 4as.

$=$ Rs 15 2as 4 pies

$$4\frac{1}{3} \text{ of } \frac{2}{39} - \frac{\frac{3}{7}}{\frac{24}{63}} + \frac{5\frac{1}{8} - 4 \text{ of } \frac{2}{3}}{3\frac{1}{4} - 2\frac{1}{2}} = \frac{13}{3} \text{ of } \frac{2}{39} - \frac{3}{7} \times \frac{63}{24} + \frac{\frac{41}{8} - \frac{8}{3}}{\frac{13}{4} - \frac{5}{2}}$$

$$= \frac{2}{9} - \frac{9}{8} + \frac{\frac{123-64}{24}}{\frac{13-10}{4}} = \frac{2}{9} - \frac{9}{8} + \frac{59}{24} \times \frac{4}{3} = \frac{2}{9} - \frac{9}{8} + \frac{59}{18}$$

$$=\frac{16-81+236}{72}=\frac{171}{72}=2\tfrac{27}{72}=2\tfrac{3}{8}.$$

3 $274\cdot7200 \div \cdot0544 = 5050$

(i) $\dfrac{\cdot003\times\cdot05}{\cdot0022}=\dfrac{\frac{3}{1000}\times\frac{5}{100}}{\frac{22}{10000}}=\dfrac{1}{300}\times\dfrac{1}{20}\times\dfrac{5000}{11}=\dfrac{5}{66}=\cdot075$

(ii) $6\cdot04\dot{5}-5\cdot3\dot{6}7\dot{8}=6\cdot045455-5\cdot367878=\cdot6\dot{7}7\dot{5}$

$\sqrt{(\dot{9}51\dot{1}\,105\dot{6})}=30\cdot84.$

4

	Rs	as	pie	
20 sr $=\frac{1}{2}$ of 1 m.	12	10	3	= value of 1 md.
			15	
	189	9	9	= value of 15 mds
4 sr $=\frac{1}{5}$ of 20 sr.	6	5	$1\frac{1}{2}$	= value of 20 seers.
1 sr $=\frac{1}{4}$ of 4 sr	1	4	$2\frac{7}{10}$	= value of 4 seers
8 ch. $=\frac{1}{2}$ of 1 sr		5	$0\frac{27}{40}$	= value of 1 seer
2 ch. $=\frac{1}{4}$ of 8 ch		2	$6\frac{27}{80}$	= value of 8 ch.
1 ch $=\frac{1}{2}$ of 2 ch.			$7\frac{187}{320}$	= value of 2 ch
			$3\frac{507}{640}$	= value of 1 ch.
Rs.	197	11	$7\frac{397}{640}$	= value of 15 mds 25 srs 11 ch.

5 Rs 1000×5 yrs : Rs. $3500\times1\frac{1}{2}$ yrs. :: Rs. 250 : x

$\therefore x=$ Rs. $\dfrac{250\times3500\times3}{1000\times5\times2}=$ Rs 262 8 as

6
$$
\begin{array}{r|l|l}
x^2-2x-3 & x^4-10x^2+9 & x^2+2x-3=\text{Quot} \\
 & x^4-2x^3-3x^2 & \\ \hline
 & 2x^3-7x^2+9 & \\
 & 2x^3-4x^2-6x & \\ \hline
 & -3x^2+6x+9 & \\
 & -3x^2+6x+9 & \\ \hline
\end{array}
$$

(a)
$$
\begin{array}{r|l|l}
3x^3-17x^2+19x+11 & 6x^3-25x^2+17x-22 & 2 \\
 & 6x^3-34x^2+38x+22 & \\ \hline
 & 9x^2-21x-44 & \\
\end{array}
$$

$$
\begin{array}{r|l|l}
9x^2-21x-44 & 9x^3-51x^2+57x+33 & x-3 \\
 & 9x^3-21x^2-44x & \\ \hline
 & -30x^2+101x+33 & \\
 & -27x^2+63x+132 & \\ \hline
-1 & -3x^2+38x-99 & \\ \hline
\end{array}
$$

$$3x^2-38x+99)\;9x^2-21x-44\;(3$$
$$\underline{9x^2-114x+297}$$
$$31\,|\,\underline{93x-341}$$
$$3x-11)\;3x^2-38x+99\;(x-9$$
$$\underline{3x^2-11x}$$
$$-27x+99$$
$$\underline{-27x+99}$$

$\therefore$ $3x-11$ is the G C M

7 (i) The expression $=$

$$\left\{\frac{2a-(x+a)+2(x-a)}{x^2-a^2}\right\}\times\frac{x^2}{\frac{x^3-ax+a^3}{x}}$$

$$=\frac{x-a}{x^2-a^2}\times x^2\times\frac{x}{x^2-ax+a^2}=\frac{x^3}{x^3+a^3}$$

(ii) The expression $=$

$$\frac{1}{x(x-y)(x-z)}-\frac{1}{y(x-y)(y-z)}+\frac{1}{z(x-z)(y-z)}$$
$$=\frac{yz(y-z)-xz(x-z)+xy(x-y)}{xyz(x-y)(x-z)(y-z)}$$
$$=\frac{(x-y)(x-z)(y-z)}{xyz(x-y)(x-z)(y-z)}=\frac{1}{xyz}$$

8 (1) $\frac{7x-1}{4}-\frac{2}{3}x+\frac{1-x}{6}=\frac{19}{3}$

$21x-3-8x+2-2x=76$

$\therefore$ $11x-1=76$ $\therefore$ $11x=77$. $\therefore$ $x=7$.

(2) $\frac{x-y}{3}=\frac{y-1}{4}$ (1)

$\frac{4x-5y}{7}=x-7$ ..(2)

From (1) $4x-4y=3y-3$

or $4x-7y=-3$ ....(3)

From (2) $4x-5y=7x-49$,

or $3x+5y=49$ .. (4)

Multiply (3) by 3 and (4) by 4

$12x-21y=-9$
$12x+20y=196$

By substraction $41y=205$, $\therefore$ $y=5$

From (3) $4x-7y=-3$ $\therefore$ $x=8$

9 Let A's share be x

Then B's share $+200$ Rs $=2(x-200)$

$\therefore$ B's share $=2x-400-200=2x-600$ Rs.

Again $2(x-200)=3$ C's share.

$$\text{C's share} = \frac{2x-400}{3}.$$

$\therefore$ By the question

$$\therefore\; x+2x-600+\frac{2x-400}{3}=1100$$

$$\therefore\; 9x-1800+2x-400=3300$$

$\therefore\; 11x=5500 \qquad \therefore\; x=500$ Rs. A's share

$\therefore$ B's share $=400$ Rs. $\qquad \therefore$ C's share $=200$ Rs

10 (i) Since $\frac{a}{b}=\frac{c}{d}$, $\therefore\; \frac{a}{c}=\frac{b}{d}$, $\therefore\; \frac{a}{c}\pm 1=\frac{b}{d}\pm 1$

$$\therefore\; \frac{a\pm c}{c}=\frac{b\pm d}{d},\quad \therefore\; \frac{a\pm c}{b\pm d}=\frac{c}{d}.$$

But $\frac{c}{d}=\frac{a}{b}$, $\therefore\; \frac{a}{b}=\frac{a\pm c}{b\pm d}$

or $a \quad b : \; a\pm c \quad b\pm d$

(ii) Since $\frac{a}{b}=\frac{c}{d}$ $\therefore\; \frac{a^2}{b^2}=\frac{c^2}{d^2}$, or $\frac{a^2}{c^2}=\frac{b^2}{d^2}$

$$\therefore\; \frac{a^2}{c^2}\pm 1=\frac{b^2}{d^2}\pm 1,\quad \therefore\; \frac{a^2\pm c^2}{c^2}=\frac{b^2\pm d^2}{d^2}.$$

$\therefore\; \frac{a^2\pm c^2}{b^2\pm d^2}=\frac{c^2}{d^2}=\frac{(a\pm c)^2}{(b\pm d)^2}$ as shown above

$\therefore\; a^2\pm c^2 \,.\, b^2\pm d^2 : \,.\, (a\pm c)^2 \quad (b\pm d)^2.$

1872.—AFTERNOON.

1 Euclid I. 17. Axiom 12th is the converse of prop. 17th, Book I. In prop. 29, Book I

2 Euclid I 48 In prop 48, Book I is assumed the corollary that "the squares described upon two equal straight lines are equal"

If $a^2+b^2=c^2$, then a and b must be the two sides of a right $\angle$d $\triangle$ and then they must be greater than c the hypotenuse (I. 20).

3 Euclie II 9

(*a*) The rectangle contained by the two parts of a line is the greatest possible when they are equal

Let the str line AB be bisected at C
Then AC CB is the greatest rectangle contained by the two lines in which AB may be divided.

Let AB be divided in some other point as D

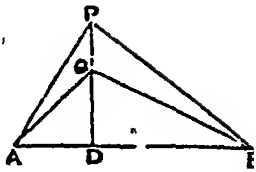
$\therefore$ AD.BD$+$CD$^2=$CB^2 (II 5)

But $CB^2 = AC\ CB$

$\therefore AD.BD + CD^2 = AC\ CD$

Wherefore AC CB is greater than AD DB by CB^2.

4 Euclid II 13



(a) From P draw $PD \perp AB$

Then $PB^2 = AP^2 + AB^2$
$-2\ AB\ AD$ (II. 13)

Take AP^2 from both these equals,

$\therefore PB^2 - AP^2 = AB^2 - 2AB\ AD$.

Let Q be some other position of P

If QR be drawn $\perp$ AB, then we can prove that $QB^2 - AQ^2 = AB^2 - 2AB\ AR$

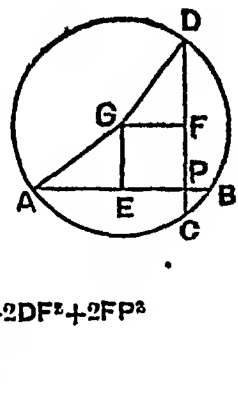
$\therefore AP^2 - 2AB\ AD = AB^2 - 2AB\ AR$,

being each of them equal to a constant quantity;

then $AD = AR$, $\therefore$ QR and PD will coincide.

Wherefore the position of P will always remain on the line draw from $D \perp AB$.

5. Euclid III 4



(a) Let AB and DC be two chords at rt. $\angle$s to each other in P

Bisect AB and CD in E and F.

Let G be the centre of the $\odot$

Join GE, GF;

these lines are $\perp$ to AB and CD respectively (III 3)

Then GFPE is a rectangle and $GF = EP$ also $GE = FP$ (I 28 and I 34)

$$\left.\begin{array}{l} \because AP^2 + PB^2 = 2AE^2 + 2EP^2 \\ \text{and } DP^2 + PC^2 = 2DF^2 + 2FP^2 \end{array}\right\} \text{(II 9)}$$

$$\therefore AP^2 + PB^2 + PD^2 + PC^2 = 2AE^2 + 2EP^2 + 2DF^2 + 2FP^2$$

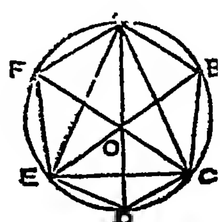
$$= 2AE^2 + 2EG^2 + 2DF^2 + 2GF^2$$

$$= 2AG^2 + 2DG^2 = 4AG^2 \text{ (I 47)} = 4 \text{ (radius)}^2$$

$$\therefore \text{(radius)}^2 = \frac{AP^2 + PB^2 + DP^2 + PC^2}{4}$$

6. Euclid III 36

7. Euclid IV. 15



(a) In the figure of Prop 15th Book IV.

Join AC CE and EA

$\because$ the arcs ABC, CDE and EFA are equal,

$\therefore AC = CE = AE$ (III 29)

$\therefore \triangle ACE$ is equilateral

Again $\because AB = AG$ and $BC = CG$, also AC common.

$\therefore \triangle ABC = \triangle AGO$ (I 4)

Likewise $\triangle$s CDE and AEF$=\triangle$s CGE and AGB respectively:

$\therefore$ the hexagon is double of the $\triangle$ACE.

8 Euclid IV. 16.

(A right angle is equal to 90°). Each angle of the $\triangle$ACD$=\frac{1}{3}$ of 2 rt. $\angle$s$=\frac{1}{3}$ of 180°$=$60°

(1 32, Cor 1) Each angles of the pentagon$=\frac{1}{5}$ (ten rt $\angle$s$-$4rt. $\angle$s)$=\frac{1}{5}$ (6 rt $\angle$s$=\frac{1}{5}$ of 6$\times$90°)$=$108°

Each angle of the quindecagon$=\frac{1}{15}$ (thirty rt. $\angle$s$-$4 rt $\angle$s)$=\frac{1}{15}$ of 26 rt. $\angle$s$=$156°.

1873.—MORNING

Examiners,— { MR THWAYTES, M A. / MR GRIFFITHS, B A. }

1 Find the value of (i) $\dfrac{1+2\frac{1}{2}+3\frac{1}{3}}{\frac{1}{1\frac{1}{2}}+\frac{2}{2\frac{1}{3}}+\frac{8}{3\frac{1}{4}}}\times\dfrac{55\frac{2}{7}\div 11}{1\frac{1}{11}\text{ of }12\frac{5}{6}}$

(ii) $24\frac{5}{6}$ of Rs 103 7 as. 6 p.

If $\frac{5}{16}$ of a maund is worth Rs. 45, what is the price of $\frac{1}{3}$ of a maund.

2 Deduce $\frac{1}{301}$ to a decimal, 019 to a vulgar fraction; and $\dfrac{4\,2-3\,14}{1\,3+2\,\dot{1}02}$ of $\dfrac{1\,3\text{ of }4}{\cdot37\text{ of }8\,\dot{8}1}$ to its lowest terms

3 What is the expense of matting a room 31 ft. 5 in long by 20 ft 4 in wide, the mat costing 14 annas per 12 square *hath* (linear hath = 18 inches)?

4 In what time will Rs 8,500 amount to Rs 15,767 8as. at $4\frac{1}{2}$ per cent. per annum?

5. A person owes the sum of Rs 31,500 and Rs 8,500; and his property only amounts to Rs. 14125 How much is he able to pay in the rupee, and what was the loss upon the second debt?

6. Reduce to their simplest form:—

(i) $\dfrac{3x}{2}\sqrt[4]{\dfrac{400y^2}{81x^2}}$.

(ii) $\dfrac{(x^3-y^3)(x+y)^2}{(x^2+xy+y^2)(x^2-y^2)}$

$\dfrac{2+x}{2(x+1)}+\dfrac{2-x}{2(x-1)}+\dfrac{x}{x^2+1}$.

7. Find the G. C. M. of $x^4-9a^2x^2+10a^3x$ and $ax^3-a^2x^2+a^4$; and the L. C M of $3ax^2-3a^2x$, x^2-a^2, x^2+ax, $\sqrt{(3ax)}$ and $\sqrt{x}-\sqrt{a}$.

8. Solve the equations —

(a) $\frac{12}{x+2} = 6 - 2\left(\frac{3x+2}{x+1}\right)$

(b) $\sqrt{x} - \sqrt{4+x} = \frac{2}{\sqrt{x}}.$

(c) $\left.\begin{array}{l} 2x - \frac{2y-1}{3} = 3\frac{5}{24} + \frac{3x-2y}{4} \\ 4y - \frac{5-2x}{4} = 6 - \frac{3-2y}{5} \end{array}\right\}$

9 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then each of these ratios $= \frac{a+c+c}{b+d+f}$

assuming $\frac{a+b-c}{a+b} = \frac{b+c-a}{b+c} = \frac{c+a-b}{c+a}$ and that

$a+b+c$ in not$=0$, show that $a=b=c$.

10 Two persons started at the same time from A. One rode on horseback at the rate of $7\frac{1}{2}$ miles an hour and arrived at B 30 minutes later than the other, who travelled the ssme distance by train at the rate of 30 miles an hour Find the distanoe between A and B

1873.—AFTERNOON

Examiners,—{REV J P ASHTON, M A
MR MOWAT, M A.

1. Enunoiate and prove the fourth proposition of the First Book.

2 Describe a rectangle equal to a given rectilineal figure

3. Prove from any proposition in the Second Book that the produot of the sum and difference of any two quantities is equal to the difference of their squares.

4 Enunoiate the thirteenth proposition of the Second Book (in every triangle the square on the side subtending an acute angle, &c.) and prove ONLY *in the case in which the perpendicular falls without the triangle*

5. If one circle touoh another internally, the straight line which joins their centres being produoed shall pass through the point of contact.

6 On a given straight line describe a segment of a oircle containing an angle twice as great as the angle of an equilateral triangle

7 Inscribe a circle in a given triangle, and show that the straight lines bisecting the three angles of a triangle meet in a point.

8 If the middle points of the three sides of a triangle be joined, the triangle so formed shall be equiangular to the given triangle and equal to one-fourth of it

9 The exterior angles DBC and ECB of the triangle ABC are bisected by BF and CF, FG and FH are drawn perpendicular to AD and AE, prove that FG is equal to FH and AG to AH

10. AB is a chord of a circle, C a point in the circumference of the smaller segment, find a point D in the circumference of the larger segment so that AB shall bisect the angle DBC.

SOLUTIONS.

1873.—MORNING.

1. (i) $\text{Ans} = \dfrac{6+\frac{1}{2}+\frac{1}{3}}{\frac{2}{3}+\frac{2}{1}\times\frac{3}{7}+\frac{3}{1}\times\frac{4}{13}} \times \dfrac{\frac{167}{3}\times\frac{1}{11}}{\frac{13}{11}\times\frac{41}{3}}$

$$= \frac{6\frac{5}{6}}{\frac{2}{3}+\frac{6}{7}+\frac{12}{13}} \times \frac{167}{33} \times \frac{33}{533} = \frac{\frac{41}{6}}{\frac{182+234+252}{273}} \times \frac{167}{533}$$

$$= \frac{\cancel{41}}{\underset{2}{\cancel{6}}} \times \frac{\overset{\overset{7}{\cancel{21}}}{\cancel{273}}}{\underset{4}{\cancel{668}}} \times \frac{\cancel{167}}{\underset{\cancel{13}}{\cancel{533}}} = \frac{7}{8}.$$

(ii) $24\frac{5}{8}$ of Rs. 103 7as 6 pie = (Rs 103 7as 6 pie) × 24 + $\frac{5}{8}$ of Rs. 103 7as. 6 pie = Rs 2483 4 as. + Rs 86 3as. 7pie = Rs 2569 7 as. 7 pie

(a) $\frac{5}{16}$ mds $\frac{1}{3}$ md : 45 Rs x

$$\therefore x = \text{Rs } \frac{45\times 1\times 16}{3\times 5} = \text{Rs } 48$$

2 1 000 ÷ 303 = 0033

$$019\dot{9} = \frac{19-1}{900} = \frac{18}{900} = \frac{1}{50}$$

(a) $\text{Ans } \dfrac{4\,2222 \quad -3\,144\ldots}{1\,3333 \quad +2\cdot102102\ldots} \times \dfrac{1\frac{3}{10}\times 4}{\frac{37}{100}\times 8\frac{81}{99}}$

$$= \frac{1\,07\dot{7}}{3\,435} \times \frac{\frac{13}{10}\times 4}{\frac{37}{100}\times 8\frac{9}{11}} = \frac{1\frac{7}{90}}{3\frac{435}{999}} \times \frac{\frac{26}{5}}{\frac{37}{100}\times\frac{97}{11}}$$

$$= \frac{97}{90}\times\frac{999}{3432}\times\frac{26}{5}\times\frac{100}{37}\times\frac{11}{97} = \frac{1}{2} = 5$$

3. The area of the floor = 31 ft 5 in × 20 ft 4 in.

$= 377$ in $\times 244$ in $= (\frac{377}{18}\times\frac{244}{18})$ sq hâths $= (\frac{377}{18}\times\frac{122}{9})$ sq hâths

12 sq hâths $\dfrac{377\times 122}{18\times 9}$ sq hâths Rs $\frac{7}{8}$ x

$$\therefore x = \text{Rs } \frac{7\times 377\times 122}{8\times 18\times 9\times 12} = \text{Rs } \frac{160979}{7776} = \text{Rs } 20 \text{ 11as } 2\tfrac{64}{81} \text{ pie}$$

4. The int $=$ Rs 15767 8as $-$ Rs 8500 $=$ Rs 7267 8as.

Rs 100×1 yr Rs $8500 \times x$ yrs $4\frac{1}{2}$ Rs. $7267\frac{1}{2}$

$$\therefore x = \frac{100 \times 14535 \times 2}{8500 \times 9 \times 2} = 19 \text{ years}$$

5 The person owes Rs $(31500 + 8500) =$ Rs 40000

$\therefore$ he can pay $\dfrac{14125 \times 16}{40000}$ as $= \frac{113}{20}$ as $=$ 5as $7\frac{1}{5}$ pies, in the Re.

The loss in the rupee $=$ (Re $1 -$ Re $\frac{113}{320}$) $=$ Re $\frac{207}{320}$

$\therefore$ the loss upon the 2nd debt $=$ Rs $\frac{207}{320} \times 8500$

$$= \text{Rs } \frac{207 \times 425}{16} = \text{Rs } \frac{87975}{16} = \text{Rs. } 5498 \text{ 7as}$$

6 (i) Ans $= \dfrac{3x}{2}\sqrt[4]{\left(\dfrac{20}{9}\,\dfrac{20}{9}\,\dfrac{y}{x}\,\dfrac{y}{x}\right)} = \dfrac{3x}{2}\sqrt{\left(\dfrac{20y}{9x}\right)}$

$$= \sqrt{\left(\frac{3x}{2} \times \frac{3x}{2} \times \frac{20y}{9x}\right)} = \sqrt{(5xy)}$$

(ii) The expr. $= \dfrac{(x-y)(x^2+xy+y^2)(x+y)(x+y)}{(x^2+xy+y^2)(x+y)(x-y)} = x+y.$

(iii) $\dfrac{2+x}{2(x+1)} + \dfrac{2-x}{2(x-1)} + \dfrac{x}{1+x^2}$

$$= \frac{1}{2}\left\{\frac{2+x}{x+1} + \frac{2-x}{x-1}\right\} + \frac{x}{1+x^2}$$

$$= \frac{1}{2}\left\{\frac{(2+x)(x-1) + (2-x)(x+1)}{x^2-1}\right\} + \frac{x}{1+x^2}$$

$$= \frac{1}{2} \times \frac{2x}{x^2-1} + \frac{x}{1+x^2}$$

$$= \frac{x}{x^2-1} + \frac{x}{x^2+1} = x \times \frac{2x^2}{x^4-1} = \frac{2x^3}{x^4-1}$$

7. Rejecting common quantities from both the expressions we have

$$x^3 - 9a^2x + 10a^3 \text{ and } x^3 - ax^2 - 4a^3$$

$$
\begin{array}{r|l|l}
x^3 - ax^2 - 4a^3 & x^3 - 9a^2x + 10a^3 & 1 \\
 & x^3 - ax^2 - 4a^3 & \\
\hline
 & ax^2 - 9a^2x + 14a^3 & \\
\hline
 & x^2 - 9ax + 14a^2 &
\end{array}
$$

$$x^2-9ax+14a^3\,\big)\,x^3-ax^2-4a^3\,\big(\,x+8a$$
$$x^3-9ax^2+14a^2x$$
$$\overline{8ax^2-14a^2x-4a^3}$$
$$8ax^2-72a^2x+112a^3$$
$$\overline{58a^2\mid 58a^2x-116a^3}$$
$$\overline{\quad x-2a\quad}$$

$$x-2a\,\big)\,x^2-9ax+14a^2\,\big(\,x-7a$$
$$x^2-2ax$$
$$\overline{-7ax+14a^2}$$
$$-7ax+14a^2$$
$$\overline{\qquad\qquad}$$

$\therefore\ x-2a$ is the G C M.

(b) The expressions are $3ax(x-a)$, $(x+a)(x-a)$, $x(x+a)$, $\sqrt{(3ax)}$ and $\sqrt{x}-\sqrt{a}$.

$\therefore$ L C. M. $=3ax(x-a)(x+a)=3ax(x^2-a^2)$.

(1) $\dfrac{12}{x+2}=6-\dfrac{6x+4}{x+1}=\dfrac{6x+6-6x-4}{x+1}=\dfrac{2}{x+1}$

divided by 2 and cross multiply

$6(x+1)=x+2$, $\quad\therefore\ 5x=-4$, $\quad\therefore\ x=-\frac{4}{5}$.

(2) $\sqrt{x}-\sqrt{(4+x)}=\dfrac{2}{\sqrt{x}}$

Multiplying both sides by $\sqrt{4x}$

$x-\sqrt{(4x+x^2)}=2$, or $-\sqrt{(4x+x^2)}=2-x$

sq. $4x+x^2=x^2+4-4x$ $\quad\therefore\ 8x=4$ $\quad\therefore\ x=\frac{1}{2}$

(3) Multiply the 1st equation by 24; then,

$48x-8(2y-1)=77+6(3x-2y)$

or $48x-16y+8=77+18x-12y$

$\therefore\ 30x-4y=69.\ \ldots\ (3)$

Multiply the second equation by 20; then,

$80y-5(5-2x)=120-4(3-2y)$

$\therefore\ 80y-25+10x=120-12+8y$

$\therefore\ 72y+10x=133 \quad (4)$

Multiply (4) by 3, then, $216y+30x=399$

Subtracting (3) from (4), $220y=330$.

$\therefore\ y=\frac{3}{2}=1\frac{1}{2}$.

From (3) $30x-4\times\frac{3}{2}=69$ $\quad\therefore\ 30x=75$ $\quad\therefore\ x=2\frac{1}{2}$.

9 Suppose $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$,

then $a=bk$, $c=dk$, $e=fk$.

$\therefore a+c+e=(b+d+f)k$

$\therefore \frac{a+c+e}{b+d+f}=k=$ each of the ratios

(*a*) Hence $1-\frac{c}{a+b}=1-\frac{a}{b+c}=1-\frac{b}{c+a}$

$\therefore \frac{c}{a+b}=\frac{a}{b+c}=\frac{b}{c+a}$ or $\frac{a+b}{c}=\frac{b+c}{a}=\frac{c+a}{b}$

$\therefore \frac{a+b+c}{c}=\frac{a+b+c}{a}=\frac{a+b+c}{b}$

$\therefore \frac{1}{c}=\frac{1}{a}=\frac{1}{b}$, for $a+b+c$ is not$=0$. $\therefore a=b=c$.

10 Let x be the distance between A and B

Then the rider reaches B in $\frac{2}{15}x$ hr.

" passenger " in $\frac{x}{30}$ hrs

Now, by the question $\frac{x}{30}+\frac{1}{2}=\frac{2}{15}x$

$\therefore \frac{1}{10}x=\frac{1}{2}$ $\therefore x=5$ miles

1873.—AFTERNOON

1 Euclid I. 4

2. Euclid I. 45 The given $\angle$ is a rt $\angle$.

3 See Cor. to Prop 5 of Book II., or see Solution of question 5 of 1870

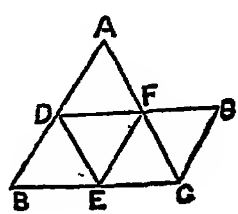
4. Euclid II. 13, 2nd case

5 Euclid III 11.

6 Euclid III. 33 The given $\angle$ is equal to twice the angle of an equilateral $\triangle$ *i e* two thirds of two right angles or 120°.

7 See Solution of Ques. 6 of 1. 1859.

8 Let D, E, F be three middle points of the three sides of the $\triangle$ABC



Join DE, EF and DF

The $\triangle$ DEF is equiangular to $\triangle$ABC and equal to one fourth of it

Produce DF to G making FG=DF.

Join GC

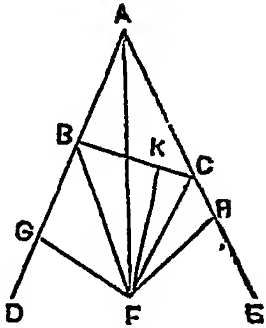
$\because$ AF=FC, DF=FG

and $\angle$AFD=$\angle$CFG (I 15)

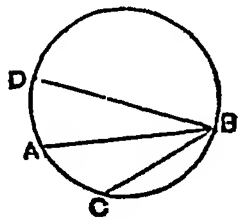
$\therefore$ CG=AD and $\angle$ADF=$\angle$FGC, (I 4).

$\therefore$ AB $\parallel$ CG and DB=AD=GC (I. 27)
$\therefore$ DG $\parallel$ and=BC. (I 33)
Likewise FE $\parallel$ AB and DE $\parallel$ AC.
Because DF $\parallel$ BC, $\therefore \angle ACB=\angle AFD$ } (I 29)
and $\because$ DE $\parallel$ AC $\therefore \angle AFD=\angle FDE$; }
wherefore $\angle ACB=\angle FDE$.
Likewise the $\angle$ at A may be proved= $\angle$ DEF and the $\angle$ at B $=\angle DFE$.
Consequently, $\triangle DEF$ is equiangular to $\triangle ABC$.
Again $\because$ DBEF is a parallelogram,
$\therefore \triangle BDE=\triangle DEF$ · (I 34)
$\because$ DF $\parallel$ BC, $\therefore \triangle DBE=\triangle FEG$ (I 38)
Likewise the $\triangle ADF=\triangle DBE$.
Wherefore $\triangle DEF=\frac{1}{4}\triangle ABC$.

9. Draw FK$\perp$BC and join AF
$\therefore \angle GBF=\angle KBF$ (Ax II) and $\angle BKF=\angle BGF$ also BF common.
$\therefore$ GF=KF (I 4)
Likewise KF=FH
$\therefore$ FG=FH
Again $\because AF^2=AG^2+GF^2=AH^2+FH^2$ (I. 47)
But $FG^2=FH^2 \therefore AG^2=AH^2$ (Ax 3)
Wherefore AG=AH



10. Join BC At the point B in the str line AB make the $\angle ABD=\angle ABC$ (I 23)
and let BD intersect the larger segment in the point D



1874.—MORNING.

Examiners,— { MR THWAYTES, M.A. / MR. GRIFFITHS, M.A. }

1. What fraction of $\frac{2}{3}$ of a rupee is $\frac{4}{9}$ of Rs 5, and what proportion does their difference bear to their sum ?

Divide 999 666 by 30036 and 2 $3\dot{5}7142\dot{8}$ by 10 $2\dot{1}4285\dot{7}$

2 When rice is 10 seers the rupee nine persons can be fed for 30 days at a certain cost For how many days can six persons be fed at the same cost when rice is 14 seers the rupee ?

3 A wooden box 3 ft 8 in long, 2 ft 3 in. high, and 2 ft 4 in. wide, is made of board one inch thick Find the quantity of wood used, and the cubical contents of the box

9 Suppose $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$;

then $a=bk$, $c=dk$, $e=fk$.

$\therefore a+c+e=(b+d+f)k$

$\therefore \frac{a+c+e}{b+d+f}=k=$ each of the ratios.

(*a*) Hence $1-\frac{c}{a+b}=1-\frac{a}{b+c}=1-\frac{b}{c+a}$

$\therefore \frac{c}{a+b}=\frac{a}{b+c}=\frac{b}{c+a}$ or $\frac{a+b}{c}=\frac{b+c}{a}=\frac{c+a}{b}$

$\therefore \frac{a+b+c}{c}=\frac{a+b+c}{a}=\frac{a+b+c}{b}$

$\therefore \frac{1}{c}=\frac{1}{a}=\frac{1}{b}$, for $a+b+c$ is not $=0$. $\therefore a=b=c$.

10 Let x be the distance between A and B

Then the rider reaches B in $\frac{2}{15}x$ hr.

" passenger " in $\frac{x}{30}$ hrs

Now, by the question $\frac{x}{30}+\frac{1}{2}=\frac{2}{15}x$

$\therefore \frac{1}{10}x=\frac{1}{2}$ $\therefore x=5$ miles

1873.—AFTERNOON.

1 Euclid I. 4.

2. Euclid I. 45 The given $\angle$ is a rt $\angle$.

3. See Cor. to Prop 5 of Book II., or see Solution of question 3 of 1870

4. Euclid II 13, 2nd case

5. Euclid III 11.

6 Euclid III. 33 The given $\angle$ is equal to twice the angle of an equilateral $\triangle$ i e two thirds of two right angles or 120°.

7 See Solution of Ques. 6 of 1 1859.

8 Let D, E, F be three middle points of the three sides of the $\triangle$ABC

Join DE, EF and DF

The $\triangle$ DEF is equiangular to $\triangle$ABC and equal to one fourth of it

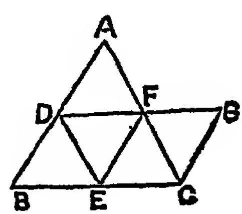
Produce DF to G making FG=DF.

Join GC

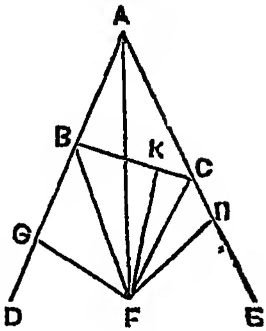
$\because$ AF=FC, DF=FG

and $\angle$AFD=$\angle$CFG (I 15)

$\therefore$ CG=AD and $\angle$ADF=$\angle$FGC, (I 4).

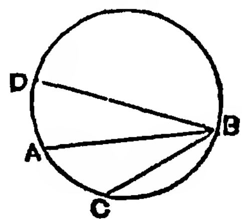


$\therefore$ AB $\parallel$ CG and DB$=$AD$=$GC (I 27)
$\therefore$ DG $\parallel$ and$=$BC. (I 33)
Likewise FE $\parallel$ AB and DE $\parallel$ AC.
Because DF $\parallel$ BC, $\therefore \angle ACB = \angle AFD$ and $\because$ DE $\parallel$ AC. $\therefore \angle AFD = \angle FDE$; } (I 29)
wherefore $\angle ACB = \angle FDE$
Likewise the $\angle$ at A may be proved$= \angle$ DEF and the $\angle$ at B $= \angle DFE$
Consequently, $\triangle DEF$ is equiangular to $\triangle ABC$.
Again $\because$ DBEF is a parallelogram,
$\therefore \triangle BDE = \triangle DEF$ (I 34)
$\because$ DF $\parallel$ BC, $\therefore \triangle DBE = \triangle FEG$ (I 38)
Likewise the $\triangle ADF = \triangle DBE$.
Wherefore $\triangle DEF = \frac{1}{4} \triangle ABC$.



9. Draw $FK \perp BC$ and join AF.
$\therefore \angle GBF = \angle KBF$ (Ax. II) and $\angle BKF = \angle BGF$ also BF common.
$\therefore$ GF$=$KF (I 4)
Likewise KF$=$FH
$\therefore$ FG$=$FH
Again $\because AF^2 = AG^2 + GF^2 = AH^2 + FH^2$ (I. 47)
But $FG^2 = FH^2$ $\therefore AG^2 = AH^2$ (Ax 3)
Wherefore AG$=$AH

10 Join BC At the point B in the str line AB make the $\angle ABD = \angle ABC$ (I 23) and let BD intersect the larger segment in the point D



1874.—MORNING.

Examiners,—{ Mr. Thwaites, M.A. / Mr Griffiths, M.A.

1. What fraction of $\frac{3}{4}$ of a rupee is $\frac{4}{9}$ of Rs 5, and what proportion does their difference bear to their sum?

Divide 999 666 by 30036 and 2 3571428 by 10 2142857

2 When rice is 10 seers the rupee nine persons can be fed for 30 days at a certain cost For how many days can six persons be fed at the same cost when rice is 14 seers the rupee?

3 A wooden box 3 ft 8 in long, 2 ft 3 in high, and 2 ft 4 in wide, is made of board one inch thick Find the quantity of wood used, and the cubical contents of the box.

4 It is said that 240,000 letters are posted in Berlin daily, 16 $\frac{2}{3}$ per cent of which are town letters This gives one letter for every three persons in Berlin, what is its population?

5 What sum will amount to a *lakh* of rupees in ten years at 5 per cent simple interest?

Find the discount on Rs 1,308 due two years hence at $4\frac{1}{2}$ per cent per annum

6 Simplify (i) $\dfrac{x^{m+2n}x^{3m-3n}}{x^{5m-5n}}$

$$\text{(ii)}\ \frac{a}{a+b}-\frac{a+b}{2b}+\frac{a^2+b^2}{2b(a-b)}.$$

$$\text{(iii)}\ \frac{\dfrac{a^3}{b^3}-\dfrac{b^3}{a^3}}{\left(\dfrac{a}{b}-\dfrac{b}{a}\right)\left(\dfrac{a}{b}+\dfrac{b}{a}-1\right)}\times\frac{\dfrac{1}{b}-\dfrac{1}{a}}{\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{ab}}$$

7 Find the Least Common Multiple of $1+4x+4x^2-16x^4$ and $1+2x-8x^3-16x^4$

Extract the square root of—

$$9x^4-2x^3y+1\tfrac{8}{9}x^2y^2-2xy^3-9y^4.$$

8 Solve the equation—

$$\frac{15-\frac{1}{3}x}{5}-\frac{2x+5}{2\frac{1}{2}}=\frac{17-1\frac{4}{5}x}{3}$$

The expression $ax-3b$ is equal to 30 when x is 3, and to 42 when x is 7, what is its value when x is 4 3, and for what value of x is it zero

9 Show that if $a : b \quad c \quad d$,

(i) $a\pm b \quad a\mp b \qquad c\pm d \quad c\mp d$

(ii) $4(a+b)(c+d) = bd \quad \left(\dfrac{a+b}{b}+\dfrac{c+d}{d}\right)^2$

10 A certain number consists of two digits the left-hand digit is double the right-hand digit, and if the digits be inverted the ratio of the number thus formed to 60 is 4 5 Find the number

1874 —AFTERNOON.

Examiners,— { MR GEORGE THOMSON
MR M MOWAT, M.A

1 State what is meant by (i) angle in a segment of a circle; (ii) similar segments, (iii) straight line in a circle

When is one rectilineal figure said to be inscribed in another rectilineal figure?

Two straight lines may be produced ever so far both ways without meeting and yet not be parallel. Mention one familiar instance of this

2 Enunciate the fifth proposition of the first Book, and prove *only* that the angles on the other side of the base are equal to one another

3 If two triangles have two sides of the one equal to two sides of the other, each to each, but the angle contained by the two sides of one of them greater than the angle contained by two sides equal to them of the other, the base of that which has the greater angle shall be greater than the base of the other. Prove this proposition, and write down the numbers of all the propositions in the first four Books in which it is used (If you do not know the numbers, give the first line of the ennunciation of each)

4 If a straight line be divided into any two parts, the square on the whole line is equal to the square on the two parts together with twice the rectangle contained by the two parts *Give the construction and only as much of the demonstration of this proposition as will prove that the parallelograms about the diagonal of a square are squares*

Deduce from this proposition that the square on the whole line is four times the square on half the line

5 Prove that if an angle of a triangle be two-thirds of a right angle the square on the side opposite to it is equal to the sum of the squares on the sides containing it, diminished by the rectangle contained by them

6 One circle cannot touch another in more points than one Prove this *only* when the one circle is within the other Prove that the angle in a segment of a circle less than a semi-circle is greater than a right angle, having given that the angle in a segment greater than a semi-circle is less than a right angle

7 To inscribe an equilateral and equiangular pentagon in a given circle *Give the construction, and prove that the pentagon is equiangular assuming that it is equilateral*

To inscribe an equilateral and equiangular hexagon in a given circle *Give the construction only*

8 State *without proving*, the conditions which must be fulfilled in order that a circle may be described so as to pass (i) through two given points (ii) through three given points, (iii) through four given points

9 A circle is described so as to touch the side BC of the triangle ABC in D, AB produced in E, and AC produced in F, shew that the triangle EDF is obtuse angled

10 QA and QB are two straight lines in a circle at right angles to one another, QD is diameter P any point in the circumference of the smaller segment cut off by QA show that the area of the triangle APQ together with the triangle BPQ is equal to the area of the triangle QPD.

SOLUTIONS

1874.—MORNING.

1 $\frac{4}{9}$ of Rs 5 = Rs $\frac{20}{9}$, $\frac{9}{4}$ of a Re = Rs $\frac{9}{4}$

∴ fraction required = $\frac{20}{9} \times \frac{4}{9} = \frac{80}{81}$

$\frac{9}{4} - \frac{20}{9} = \frac{1}{36}$, and $\frac{9}{4} + \frac{20}{9} = \frac{161}{36}$ ∴ $\frac{1}{36} : \frac{161}{36} = 1 : 161$

(a) $999{\cdot}666 \div 30036 = \frac{999\,66600}{30036} = 3328\,226128 \ldots$

$$2\,3571428 \div 10\,2142857 = \frac{23571428 - 23}{9999990} \div \frac{102142857 - 102}{9999990}$$

$$\frac{23571405}{9999990} \times \frac{9999990}{102142755} = \frac{3}{13} = 230769$$

2 6 men : 9 men, 10 seers : 14 seers } :: 30 days : x

$$\therefore x = \frac{9 \times 14 \times 30}{6 \times 10} \text{ days} = 63 \text{ days}$$

3. Magnitude of the inner part 3 ft 6 in, 2ft, 1 in., 2 ft. 2 in.

Quantity of wood required

= outer volume − cubical content

$= (3\frac{2}{3} \times 2\frac{1}{4} \times 2\frac{1}{3} - 3\frac{1}{2} \times 2\frac{1}{12} \times 2\frac{1}{6})$ cub ft

$= \frac{11}{3} \times \frac{9}{4} \times \frac{7}{3} - \frac{7}{2} \times \frac{25}{12} \times \frac{13}{6}$

$= \frac{77}{4} - \frac{2275}{144}$

$= 19\frac{1}{4} - 15\frac{115}{144}$

$= 3 + 1\frac{1}{4} - \frac{115}{144} = 3\frac{65}{144}$ cub ft

Cub content $= 15\frac{115}{144}$ cub ft

4 100 : $16\frac{2}{3}$ :: 240000 : the number of town letters

$$\text{the number of town letters} = \frac{50 \times 240000}{3 \times 100} = 40000$$

∴ the population = 40000 × 3 = 120000.

5 The interest on Rs 100 for 10 years at 5 per cent. = Rs 50; and Rs 100 + Rs 50 = Rs 150.

∴ Rs 150 : Rs. 100000 :: Rs 100 : the required sum

$$\text{the required sum} = \frac{100000 \times 100}{150} = \text{Rs } 66666\tfrac{2}{3}$$

= Rs 66666 10as 8 pie

The int on Rs 100 for 2 years at 4½ per cent = Rs 9.

Rs 100 + Rs 9 : Rs. 1308 :: Rs 9 : x (discount)

$$\therefore x = \text{Rs } \frac{1308 \times 9}{109} = \text{Rs } 108$$

6 (i) The expr $=x^{m+3m-5m+2n-3n+5n}=x^{4n-m}$.

(ii) Ans $\dfrac{2ab(a-b)-(a^2-b^2)(a+b)+(a+b)(a^2+b^2)}{2b(a^2-b^2)}$

Numr $=2ab(a-b)-(a+b)(a^2-b^2-a^2-b^2)$

$=2ab(a-b)+2b^2(a+b)$

$=2b(a^2-ab+ab+b^2)=2b(a^2+b^2)$

$\therefore$ Ans. $=\dfrac{a^2+b^2}{a^2-b^2}$

(iii) Ans $=\dfrac{\dfrac{a^6-b^6}{a^3b^3}}{\dfrac{a^2-b^2}{ab}\times\dfrac{a^2+b^2-ab}{ab}}\times\dfrac{\dfrac{a-b}{ab}}{\dfrac{b^2+a^2+ab}{a^2b^2}}$

$=\dfrac{(a^3+b^3)(a^3-b^3)}{a^3b^3}\times\dfrac{a^2b^2}{(a^3+b^3)(a-b)}\times\dfrac{a-b}{ab}\times\dfrac{a^2b^2}{a^2+b^2+ab}=a-b.$

7. Here G C M $=1+2x+4x^2$

$\therefore$ L C. M $=\dfrac{(1+4x+4x^2-16x^4)(1+2x-8x^3-16x^4)}{1+2x+4x^2}$

$=(1+2x-4x^2)(1+2x-8x^3-16x^4)$

$=1+4x-16x^3-32x^4+64x^6$

(a) $9x^4-2x^3y+18\frac{1}{9}x^2y^2-2xy^3+9y^4\,(3x^2-\frac{1}{3}xy+3y^2$
$9x^4$

$6x^2-\frac{1}{3}xy$ $\quad -2x^3y+18\frac{1}{9}x^2y^2$
$\quad\quad -2x^3y+\frac{1}{9}x^2y^2$

$6x^2-\frac{2}{3}xy+3y^2$ $\quad 18x^2y^2-2xy^3+9y^4$
$\quad\quad 18x^2y^2-2xy^3+9y^4$

8 $\dfrac{15-\frac{2}{3}x}{5}-\dfrac{2x+5}{2\frac{1}{2}}=\dfrac{17-1\frac{5}{9}x}{3}$

$\therefore \dfrac{45-2x}{15}-\dfrac{4x+10}{5}=\dfrac{51-14x}{9}$

Multiplying both sides by 45,

$135-6x-36x-90=255-70x$

$\therefore 45-42x=255-70x \qquad \therefore 210=28x \qquad \therefore x=7\frac{1}{2}.$

(a) When $x=3$, $ax-3b=3a-3b=30$
$x=7$, $ax-3b=7a-3b=42$

By subtraction, $4a=12$, $\therefore a=3$

From (1) $b=a-10=-7$

Hence when $x=4\frac{1}{3}$, $ax-3b=3\times4\frac{1}{3}+7\times3=34$

Also $ax-3b=0=3x+21$, $\therefore 3x=-21$, $\therefore x=-7$

9. (1) Since $\frac{a}{b}=\frac{c}{d}$

$$\therefore \frac{a\pm b}{b}=\frac{c\pm d}{d} \;\ldots(1)\quad \frac{a\mp b}{b}=\frac{c\mp d}{d}\;\ldots\ldots\;(2)$$

Hence dividing (1) by (2) $\frac{a\pm b}{a\mp b}=\frac{c\pm d}{c\mp d}$

(ii) Since $\frac{a}{b}=\frac{c}{d}$. $\therefore \frac{a+b}{b}=\frac{c+d}{d}$

$$\therefore \frac{a+b}{b}+\frac{c+d}{d}=2\left(\frac{a+b}{b}\right)=2\left(\frac{c+d}{d}\right)$$

Hence $\left(\frac{a+b}{b}+\frac{c+d}{d}\right)^2=4\left(\frac{a+b}{b}\right)\left(\frac{c+d}{d}\right)$

or $bd\left(\frac{a+b}{b}+\frac{c+d}{d}\right)^2=4(a+b)(c+d)$

10 Let x denote the left-hand digit

Then the right-hand digit $=\frac{1}{2}x$

and the number $=10x+\frac{1}{2}x$

Then by the question,

$\frac{1}{2}x\times10+x$ 60 4 5, or $6x$. 60 . 4 5

$\therefore 30x=240$, and $x=8$ Hence No $=84$

1874—AFTERNOON

1 (1) Euclid III Def 8, (2) Def 11, (3) A chord or a straight line in a circle is such that it is terminated by the circumference of the circle.

Euclid IV Def 1 The two straight lines are not to be in the same plane. Place crosswise one pencil above the other, their ends may be produced ever so far both ways without meeting

2. Euclid I. 5 2nd case

3. Euclid I 24 (I 25) and (III. 7 and 8)

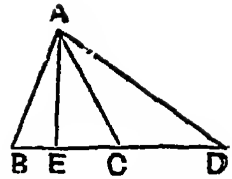
4. Euclid II 4

Because $AB^2=AC^2+CB^2+2AC\ CB$ (II 4)

$=2AC^2+2AC\ AC=4AC^2$ for $AC=BC$.

5. Let ABD be the $\triangle$, in which the $\angle ABD$ $=\frac{2}{3}$ of a right angle.

Then shall $AD^2=AB^2+BD^2-AB\ BD$



Draw $AE \perp BD$.

From ED or ED produced cut off $EC=BE$

Join AC

$\because BE=EC$, AE com

and $\angle AEB=\angle AEC$, (Ax. II)

$\therefore AB=AC$, $\angle ABE=ACE$ (I. 4)

$\therefore \angle ACB=\frac{2}{3}$ of rt $\angle$.

Hence the rem. $\angle BAC$ of the $\triangle ABC=\frac{2}{3}$ of a rt $\angle$. (I 32)

$\therefore \angle BAC=ACB$

Hence $AB=BC$ (I 6) $=2BE$

Now $AD^2=AB^2+BD^2-2BE\ BD$ (II 13)

$=AB^2+BD^2-AB\ BD$

6 Euclid III 13; 31, 3rd case.

7. Euclid IV. 13 (*a*), IV 15.

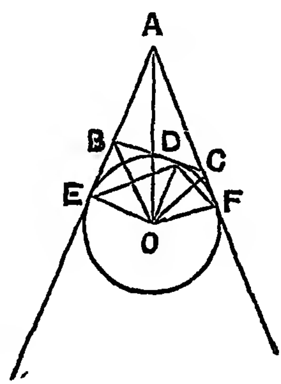
Proof —Of such equal parts as the whole circumference contains five each angle of the pentagon stands on an arc which is equal to three of them, hence all the angles are equal $\therefore$ it is equiangular

8 (1) Always

(2) They must not be in the same straight line

(3) Euclid III 35 and converses of III 21 and III. 22

9 Find the centre O



Join BO, DO, EO, CO, FO

$\because EO^2+EB^2=BO^2=BD^2+DO^2$ (I 47)

But $DO^2=OE^2$ [$\because DO=EO$]

$\therefore EB^2=BD^2$ (Ax 3) and $EB=BD$

Likewise, $DC=CF$

Again $\because BE=BD$

$\therefore \angle BED=\angle BDE$ (I 5)

The $\angle ABC=\angle BED+BDE$ (I 32)

$=2\angle BDE$

Likewise $\angle ACB=2\angle CDF$,

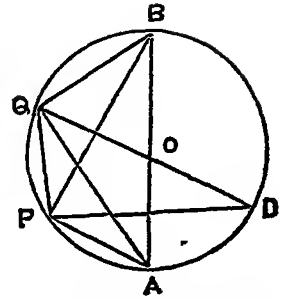
$\therefore$ the $\angle$s $ABC+ACB=2\ \angle$s BDE $+CDF$

But $\angle$s $ABC+ACB<2$ rt $\angle$s (I 17)

$\therefore 2\angle$s $BDE+CDF<2$ rt $\angle$s

$\therefore \angle$s $BDE+CDF<1$ rt $\angle$

Hence $\angle EDF >$ a rt $\angle$ (I. 13).



10 Join AB.

∵ ∠AQB is a rt angle.

∴ AB is the diameter (converse of II 31)

∴ O is the centre

The perpendiculars from A and B on PQ are together equal to twice the perpendicular from O because O is the middle point of AB (property of projection) and the perpendicular from D on PQ is twice the perpendicular from O on PQ (property of projection)

∴ The perpendiculars from A and B on PQ are equal to the perpendicular from D on PQ

Now the triangles APQ, BPQ, and DPQ have the same base PQ and the perpendicular from D is equal to the sum of the perpendiculars from A and B on PQ

∴ $\Delta PQD = \Delta APQ + \Delta BPQ$.

1875 —MORNING.

Examiners,—{Mr W Griffiths, M A. / Rev, J P. Ashton, M A

1. Simplify $\dfrac{17}{7+\dfrac{3}{4-2\frac{3}{4}}} \times \dfrac{2021}{2193} \div \left(1\frac{37}{48} - \dfrac{15}{16}\right)$

Find the value of $\frac{3}{25}$ of 17 Rs 6 as 4 p + $3\frac{3}{7}$ of 12 Rs 5 as $11\frac{1}{3}$ p + Rs 5 49583, *and* extract the square root of 049 to four places of decimals

2 A person received on the death of his aunt $\frac{7}{11}$ of her property and spent 54 of it in paying off his debts; what fraction of his aunt's property did he then possess?

3 A room is 30 ft long, 22 ft wide, $18\frac{1}{2}$ ft high, and has 5 doors and 3 windows; find the expense of colouring the walls at 3 annas per sq yd, deducting 30 sq ft for each door and widow.

4. Find the *present worth* of Rs 19,021 due 4 years hence at $3\frac{3}{4}$ per cent

5 If Rs 16,430 be invested in the Government $4\frac{1}{2}$ per cent. loan at 106, what is the monthly income thence derived?

Supposing that the loan is paid off at par in 10 years, what would be the rate of simple interest (per cent per annum) on the sum invested?

6 Subtract $3a-\frac{2}{3}b+\frac{3}{4}c$ from $2a+\frac{1}{3}b-\frac{1}{4}c$;

multiply $\dfrac{x^2}{y^2}+1+\dfrac{y^2}{x^2}$ by $\dfrac{x}{y}-\dfrac{y}{x}$,

and find the G C M of $6x^4+x^3-6x^2-5x-2$ and $2x^4+3x^3+2x-7x^2-6$.

7. Simplify—

$$\frac{1+x+x^2}{1-x^3}+\frac{1-x+x^2}{1+x^3}-\left(\frac{x}{1+x}+\frac{1-x}{x}\div\frac{1+x}{x}\right)\times\frac{1}{1-x},$$

and show that if $\frac{a-b}{c}+\frac{b-c}{a}+\frac{c+a}{b}=1$

and $a-b+c$ is *not* $=0$, then $\frac{1}{a}=\frac{1}{b}+\frac{1}{c}$.

8. Solve the equations—

(i) $\frac{x-\frac{1}{2}}{5}-\frac{7x-3}{6}+\frac{3}{7}=0$

(ii) $\sqrt{5x-1}=1+\sqrt{5x-2}$

(iii) $\left.\begin{array}{l} 4x-\frac{1}{3}(5y-4)=1, \\ \frac{3y-2x}{4}+\frac{1}{3}x=\frac{1}{2} \end{array}\right\}$

9. If $\frac{a}{b}+\frac{c}{d}=\frac{e}{f}$, prove that each of the fractions is equal to

(i) $\frac{ka+lc+me}{kb+ld+mf}$ (iii) $\left\{\frac{ace}{bdf}\right\}^{\frac{1}{3}}$

10 How many bundles of hay at Rs 5 per thousand must a *ghasrala* mix with 5 600 bundles at Rs 6 per thousand in order that he may gain 20 per cent. by selling the whole at 11 annas per hundred?

1875.—AFTERNOON.

Examiners,— { Mr Mowat, M A
Mr George Thompson

1. (*a*) Define (i) plane superfices, (ii) right angle, (iii) angle of a segment of circle Draw diagrams of the last two

(*b*) In I. 31 (to draw a straight line through a given point parallel to a given straight line), some books give the construction thus —"Let A be the given point, and BC the given straight line . In BC take any point D, and join AD make (I 23) the angle DAE equal to ADC Produce EA to F Then EF is parallel to BC" Show that this is defective

(*c*) Give the construction *only* of I 6 and of I 48.

(*d*) With what sort of properties are the first, second, and third books of Euclid respectively concerned?

2 The straight lines which join the extremities of two equal and parallel straight lines towards the same parts are also themselves equal and parallel Prove this, and show that if the extremities be joined but not towards the same parts, two equal triangles will be formed.

3. Show how the enunciation of II 9 (if a straight line be divided the square on the two unequal parts are together double .) may be made to include II 10, and prove both propositions as one

If you are unable to answer this question, you may, as an alternative of less value, describe a square equal to a given rectilineal figure

N B – No marks will be allowed for writing out II 9 and II. 10, separately, as they are usually found in books

4 (*a*) The angle at the centre of a circle is double of the angle at the circumference upon the same base Prove this only in the case when the angle at the centre falls outside of the angle at the circumference

(*b*) Show that if from any point without a circle straight lines be drawn touching it, the angle contained by the tangents is double the angle contained by the straight line joining the points of contact and the diameter through either of them

5 Describe a regular hexagon about (*not in*) a given circle

SOLUTIONS.

1875.—MORNING.

1 $$\frac{17}{7+\frac{3}{4-2\frac{3}{4}}}\times\frac{2021}{2193}-\left(1\tfrac{37}{48}-\tfrac{15}{16}\right)$$

$$=\frac{17}{7+\frac{3}{1\frac{1}{4}}}\times\frac{2021}{2193}+\left(\tfrac{85}{48}-\tfrac{45}{48}\right)$$

$$=\frac{17}{7+\frac{12}{5}}\times\frac{43\times47}{43\times51}-\tfrac{40}{48}$$

$$=\frac{17\times5}{47}\times\tfrac{47}{51}\times\tfrac{6}{5}=2.$$

(*a*) $\frac{3}{25}$ of Rs 17 6as 4 pie + $3\frac{3}{7}$ of Rs 12 5as. $11\frac{1}{3}$ pie + Rs 5 49583.

$$=\frac{\text{Rs } 52\ 3\text{as}}{25}+\frac{24\,(\text{Rs } 12\ 5\text{as. } 11\frac{1}{3}\text{ p})}{7}+\text{Rs } 5\tfrac{49583}{100000}$$

$=$ Rs 2 $1\frac{2}{5}$ as $+ 24$ (Re 1 12as $3\frac{1}{3}$ p) + Rs. $5\frac{119}{240}$

$=$ Rs 2 $1\frac{2}{5}$ as + Rs 42 $6\frac{2}{3}$as + Rs 5 $7\frac{14}{15}$as = Rs. 50

$\sqrt{(\cdot04\dot{9}0000)}=\cdot2213\ldots.$

2. $\cdot54$ of $\frac{1}{10}=\frac{54}{99}$ of $\frac{1}{10}=\frac{6}{11}$ of $\frac{1}{10}=\frac{3}{55}$,

∴ Property remaining $=\frac{1}{10}-\frac{3}{55}=\frac{5}{110}=\frac{1}{22}$,

3 The area of the walls $= 2\times 18\frac{1}{2}$ ft $\times$ (30 ft $\times$ 22 ft)

$= (2 \times \frac{37}{2} \times 52)$ sq ft

$= 1924$ sq. ft

Area to be coloured $= (1924 - 30 \times 8)$ sq ft

$= 1684$ sq ft $= \frac{1684}{9}$ sq yds

$\therefore$ expense $=$ Rs $\frac{1684}{9} \times \frac{3}{16}$.

$=$ Rs. $\frac{421}{12} =$ Rs 35 1 an 4p

4 The interest on Rs 100 for 4 years at $3\frac{3}{4}$ per cent $=$ Rs. 15 and Rs 100 + Rs 15 = Rs. 115 .

$\therefore$ Rs 115 : Rs. 19021 :: Rs. 100 : x (principal)

$\therefore x =$ Rs. $\dfrac{19021 \times 100}{115} =$ Rs $827 \times 20 =$ Rs 16540.

5 Rs 106 : Rs 16430 :: Rs $4\frac{1}{2}$: x (income)

$\therefore x =$ Rs $\dfrac{16430 \times 9}{106 \times 2} =$ Rs $\dfrac{155 \times 9}{2}$

$\therefore$ monthly income $=$ Rs $\dfrac{155 \times 9}{2 \times 12}$

$=$ Rs $58\frac{1}{8} =$ Rs 58 2as

Again, Rs 106 : Rs 16430 :: Rs 100 : x (stock.)

$\therefore x =$ Rs $\dfrac{16430 \times 100}{106} =$ Rs. 15500

Now 10 years' interest $=$ Rs $(155 \times \frac{9}{2} \times 10) =$ Rs 6975.

$\therefore$ profit in 10 years $=$ Rs $(15500 + 6975 - 16430) =$ Rs 6045.

$\therefore$ yearly profit $=$ Rs 604.5

Hence Rs. 16430 : Rs 100 :: Rs. 604.5 : x

$\therefore x =$ Rs $\dfrac{100 \times 604.5}{16430} =$ Rs. $\frac{195}{53}$ Rs $3\frac{36}{53}$

6. $2a + \frac{1}{3}b - \frac{1}{4}c - 3a + \frac{2}{3}b - \frac{3}{4}c = -a + b - c$

(a) $\left(\dfrac{x^2}{y^2} + 1 + \dfrac{y^2}{x^2}\right)\left(\dfrac{x}{y} - \dfrac{y}{x}\right) = \dfrac{x^3}{y^3} - \dfrac{y^3}{x^3}$

(b)
$$
\begin{array}{l}
2x^4 + 3x^3 + 2x^2 - 7x - 6 \,)\, 6x^4 + x^3 - 6x^2 - 5x - 2 \,(\, 3 \\
\qquad\qquad\qquad\qquad\qquad\;\; \underline{6x^4 + 9x^3 + 6x^2 - 21x - 18} \\
-4 \,)\, \underline{-8x^3 - 12x^2 + 16x + 16} \\
2x^3 + 3x^2 - 4x - 4 \,)\, 2x^4 + 3x^3 + 2x^2 - 7x - 6 \,(\, x \\
\qquad\qquad\qquad\qquad\; \underline{2x^4 + 3x^3 - 4x^2 - 4x} \\
3 \,)\, \underline{6x^2 - 3x - 6}
\end{array}
$$

$$2x^2-x-2)\ 2x^3+3x^2-4x-4\ (x+2$$
$$\underline{2x^3-x^2-2x}$$
$$4x^2-2x-4$$
$$\underline{4x^2-2x-4}$$

$\therefore$ G C. M $=2x^2-x-2$

7. The fraction

$$=\frac{1}{1-x}+\frac{1}{1+x}-\left(\frac{x}{1-x}+\frac{1-x}{1+x}\right)\times\frac{1}{1-x}$$

$$=\frac{2}{1-x^2}-\frac{1}{1-x^2}=\frac{1}{1-x^2}.$$

(a) Here $\left(\frac{a-b}{c}+1\right)-\left(1-\frac{b-c}{a}\right)+\left(\frac{c+a}{b}-1\right)=0.$

or $\frac{a-b+c}{c}-\frac{a-b+c}{a}+\frac{a-b+c}{b}=0.$

Hence $\frac{1}{c}-\frac{1}{a}+\frac{1}{b}=0$, for $a-b+c$ is not $=0$

$\therefore \frac{1}{a}=\frac{1}{b}+\frac{1}{c}$

8. (i) Multiply the equation by $5\times6\times7$ or 210.

$\therefore 42x-21-245x+105+90=0$

$\therefore -203x=-174,$ $\therefore x=\frac{174}{203}=\frac{6}{7}$

(ii) $\sqrt{(5x-1)}=1+\sqrt{(5x-2)}$

Squaring $5x-1=1+5x-2+2\sqrt{(5x-2)}$

$\therefore \sqrt{(5x-2)}=0$. sq $5x-2=0$ $\therefore x=\frac{2}{5}$

(iii) $4x-\frac{1}{3}(5y-4)=1$. (1)

$\frac{3y-2x}{4}+\frac{1}{3}x=\frac{1}{2}$............(2)

Multiply (1) by 3, and (2) by 12

Then $12x-5y+4=3$ or $12x-5y=-1$

and $9y-6x+4x=6$ or $54y-12x=36.$

By addition, $49y=35$ $\therefore y=\frac{5}{7}$

and $12x=5y-1=\frac{18}{7}$, $\therefore x=\frac{3}{14}.$

9 Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=x,$

then $a=bx$, $c=dx$, $e=fx$,

$$\frac{la+lc+me}{lb+ld+mf}=\frac{lbx+ldx+mfx}{bl+ld+mf}=\frac{x(bl+ld+mf)}{bl+ld+mf}$$

$=x=$each of the fractions

(ii) $\left(\frac{ace}{bdf}\right)^{\frac{1}{3}} = \left(\frac{bx\,dx.fx}{bdf}\right)^{\frac{1}{3}} = \left(\frac{x^3(bdf)}{bdf}\right)^{\frac{1}{3}} = (x^3)^{\frac{1}{3}} = x$

= each of the ratios

10. Let x = Number of bundles of hay.

Then by the question,

$$\left(\frac{5}{1000}x + 5600 \times \frac{6}{1000}\right)\frac{120}{100} = \frac{11}{1600}(5600+x)$$

$$\text{or } \left(\frac{1}{200}x + \frac{168}{5}\right)\frac{6}{5} = \frac{11\times 7}{2} + \frac{11}{1600}x$$

$$\therefore \frac{3}{500}x + \frac{1008}{25} = \frac{77}{2} + \frac{11}{1600}x$$

$$\therefore \frac{11}{1600}x - \frac{3}{500}x = \frac{1008}{25} - \frac{77}{2} \text{ or } \frac{7}{8000}x = \frac{91}{50}$$

$\therefore x = 13 \times 160 = 2080$ bundles.

1875.—AFTERNOON.

1. (a) Euc. I Def 7 (2) Def 10, (3) Euc Def. 8.

(b) The line EF may or may not be ∥ BC, if the ∠DAE be made on the same side of DA in which the ∠ADC is. Hence the construction is defective

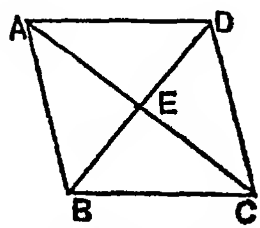
To make it perfect, the construction is to be worded thus. In BC take any point D and join AD, make the ∠DAE=∠ADC on the opposite side of AD (I 13)

(c) See Euclid's Prop 6 and 48, B I.

(d) Properties of str lines, angles and figures, especially of triangles and parallelograms, are concerned in the First Book . those of rectangular parallelograms (rectangles and squares) in the Second Book and those of circles in the Third

2 Euclid I. 33

Let AD and BC be the two=and ∥ str. lines



Let AC and BD be joined cutting each other at the point E

Then △AED=△BEC

∵ AD ∥ BC and BD meets them,

∴ the ∠ADE=EBC (I 29)

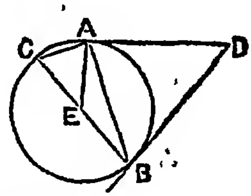
Likewise the ∠DAE=∠ECB also AD=BC (Hyp)

∴ △AED=△BEC (I. 26)

3 If a straight line be divided into two equal parts and also into

two unequal parts (either internally or externally), the squares on the two unequal parts are together double the square on half the line and on the line between the points of section

To prove $AD^2+DB^2=2AC^2+2CD^2$
$\because AD^2=AC^2+CD^2+2AC.CD$ (II 4)
and $DB^2+2BC.CD=BC^2+CD^2$ (II 7)
adding and because $AC=CB$ we get
$AD^2+DB^2+2AC.CD=2AC^2+2CD^2+2AC.CD$
$\therefore AD^2+DB^2=2AC^2+2CD^2$



4 (*a*) Euclid III 20, 2nd case.
(*b*) Let ABC be the ⊙ D a pt without it
DA and DB are the tangents.
Join AB
Find E the centre of the ⊙
Join AE and BE.
Prod BE to meet the circum at the pt. C.
$\because$ all the interior $\angle$s of the quad. $AEBD=4$ rt. $\angle$s,
also $\angle$s EAD and EBD are rt $\angle$s (III. 18)
$\therefore \angle$s $AEB+\angle ADB=2$ rt $\angle$s
But $\angle AEB+\angle EBA+\angle BAE=2$ rt $\angle$s (I 32)
$\therefore \angle AEB+\angle ADB=\angle AEB+\angle EBA+\angle BAE$
or $\angle ADB=\angle EAB+\angle EBA$
But $\angle EAB=\angle EBA$ (I. 5) $\because EA=EB$,
$\therefore \angle ADB=2\angle EBA$

5 Inscribe a regular hexagon in the given ⊙ (IV 15). Through the angular points of the inscribed figure draw str lines touching the ⊙ (III 17). These str. lines will form an equilateral and equiangular hexagon. This may be demonstrated from what has been said of the pentagon in IV 12.

1876.—MORNING.

Examiners,— { MR. A W CROFT, M.A. / MR G. ROUSE, B.A.

1 Simplify $$\frac{5\frac{4}{7}-3\frac{3}{4}+4\frac{1}{14}}{3\frac{1}{3}+\frac{1+\frac{4}{5}}{2-\frac{5}{8}}}$$

Find the value of

$\frac{5}{36}$ of 16 Rs 14 as − 1·14 of Rs 5 0 as 3 p + $1\frac{5}{6}$ of Rs 9 6as. 6p.
Reduce (16·5 − 6·25) of a Rupee to the decimal of 22 Rs 1a

2 An equal number of men, women, and boys earned 39 Rs. 6 as in seven days Each boy received 2 as a day, each woman 3 as 6 p and each man 4as 6p How many were there of each ?
Find the square root of 531·065 to five places of decimals.

3. How many yards of matting 2 ft. 4 in wide will be required

for a square room, whose side is 9 ft. 4 in ? And what will be the price of it at 2as. 3p per yard ?

Find the value of 33 cwt 3 qrs 7lbs at £6 7s 8d per cwt.

4. If 4 000 men have provisions for 190 days, and if after 30 days 800 men go away, find how long the remaining provisions will serve the number left ?

5 At what rate per cent. simple interest, will 1,462 Rs. 8as. amount to 1,725 Rs. 12as in 4 years ?

6. Simplify the following expressions —

(1) $3a-[a+b-2\{a+b+c-(a-b+c-d)\}+a]$.

(2) $(x-y)^3+(x+y)^3+3(x+y)^2(x-y)+3(x-y)^2(x+y)$.

(3) $$\frac{\dfrac{a}{a-b}-\dfrac{a}{a+b}}{\dfrac{b}{a-b}-\dfrac{b}{a+b}}+\frac{\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}}{\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}}\times\frac{a^2}{a^2+b^2}$$

7. Find the Greatest Common Measure of $(2x^3+3x^2+2x-2)$ and $(4x^4-2x^3+2x-1)$..

Multiply (x^2-x+1) by $\left(\dfrac{1}{x^2}+\dfrac{1}{x}+1\right)$

Find the square root of $\left(4-4c+2b+c^2-bc+\dfrac{b^2}{4}\right)$

8. Solve the equations —

(1) $$\frac{2x-13}{9}-\frac{x-1}{11}=\frac{x}{8}+\frac{x}{7}-9.$$

(ii) $$\left.\begin{aligned}\frac{x+y}{2}+\frac{3x-5y}{4}&=2\\ \frac{x}{14}+\frac{y}{18}&=1\end{aligned}\right\}$$

9 If $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{e}{f}$, show that.

(i) $$\frac{ma+nb}{mc+nd}=\frac{b^2c}{d^2a}$$

(ii) $a^2+c^2+e^2 : b^2+d^2+f^2 . \quad ce \quad df$

10 A can do a piece of work in 9 days, B in twice that time, C can only do $\frac{3}{4}$ as much as A in a day ; how long would A, B, and C, working together, require to do the same piece of work ?

1876.—AFTERNOON

Examiners,—{Mr M Mowat, M A.
Mr E D Archibald, M A.}

1. Describe a parallelogram that shall be equal to a given angle, and have one of its angles equal to a given rectilineal angle

2 In an obtuse-angled triangle, if a perpendicular be dra from either of the acute angles to the opposite side produced, square on the side subtending the obtuse angle is greater than squares on the sides containing the obtuse angle, by twice rectangle contained by the side upon which, when produced, perpendicular falls, and the straight line intercepted without triangle between the perpendicular and the obtuse angle.

3 Draw a straight line from a given point, either without or the circumference, which shall touch a given circle

4 If two straight lines cut one another within a circle, rectangle contained by the segments of one of them, is equal to rectangle contained by the segments of the other Prove this *only the case in which one of the straight lines passes through the centre a cuts the other which does not pass through the centre but not at ri angles*

5 Describe a circle about a given square.

6 AB and CD are two straight lines intersecting at CA and, DB are perpendicular to AB, OB is double of O. Prove, without making use of the properties of similar triang that OD is double of OC

7 C is the centre of a given circle, A any other point with it AB is drawn at right angles to AC and meets the circumferen in B Prove that the circle described about the triangle AE touches the given circle, and that ABC is the greatest angle subtended by AC at any point in the circumference of the given circle

8, Give the construction given in your text book for dividing a straight line into any number of equal parts, and prove that the line is equally divided

SOLUTIONS

1876.—MORNING.

1. $$\frac{5\frac{6}{7}-3\frac{2}{3}+4\frac{1}{14}}{3\frac{1}{3}+\dfrac{1+\frac{4}{5}}{2-\frac{5}{8}}}=\frac{5\frac{12}{14}+4\frac{1}{14}-3\frac{2}{3}}{3\frac{1}{3}+\frac{9}{5}\times\frac{8}{11}}=\frac{9\frac{13}{14}-3\frac{2}{3}}{3\frac{1}{3}+\frac{72}{55}}$$

$$=\frac{6+\dfrac{39-28}{42}}{3\frac{1}{3}+1\frac{17}{55}}=\frac{6\frac{11}{42}}{4+\dfrac{55+51}{165}}=\frac{\frac{263}{42}}{4\frac{106}{165}}=\frac{263}{42}\times\frac{165}{766}$$

$$= \frac{263 \times 55 \times 3}{14 \times 3 \times 766} = \frac{14465}{10724} = 1\tfrac{3741}{10724}$$

(a) $\frac{5}{36}$ of Rs 16 14as $-1.\dot{1}\dot{4}$ of Rs. 5 0as 3p $+ 1\frac{5}{6}$ of Rs 9 6as. 6 pie

$= \text{Rs } \frac{5}{36} \text{ of } 16\frac{7}{8} - \text{Rs. } \frac{103}{90} \text{ of } 5\frac{1}{64} + \text{Rs } 1\frac{5}{6} \text{ of } 9\frac{13}{32}$

$= \text{Rs. } \frac{5}{36} \times \frac{135}{8} - \text{Rs } \frac{103}{90} \times \frac{321}{64} + \text{Rs } \frac{11}{6} \times \frac{301}{32}$

$= \text{Rs } \frac{75}{32} - \text{Rs. } \frac{11021}{1920} + \text{Rs } \frac{3311}{192}$

$= \text{Rs } 2\frac{11}{32} - \text{Rs. } 5\frac{1421}{1920} + \text{Rs. } 17\frac{47}{192}$

$= \text{Rs. } 14 + \frac{660 - 1421 + 470}{1920} = \text{Rs } 14 - \text{Re } \frac{291}{1920}$

$= \text{Rs } 13\frac{1629}{1920} = \text{Rs } 13 \text{ 13as } 6\frac{2}{15} \text{ pie.}$

(b) $(16\cdot0\dot{5} - 6\cdot25)$ of Re. 1.

$= (16\cdot055555 \ldots - 6\cdot25)$ of Re. 1

$= 9\cdot805555 \ldots$ of Re $1 = \text{Rs } 9\cdot80\dot{5} = \text{Rs } \frac{8825}{900}$

and Rs 22 1an $= \text{Rs } 22\frac{1}{16}$.

$\therefore$ fraction required $\dfrac{\frac{8825}{900}}{22\frac{1}{16}} = \dfrac{8825}{900} \times \dfrac{16}{353}$

$$= \frac{25 \times 353 \times 16}{25 \times 36 \times 353} = \frac{4}{9} = \cdot\dot{4}$$

2 1 man + 1 woman + 1 boy would earn 4as 6p + 3as 6p. + 2as = 10as.

$\therefore$ in 7 days they would earn 10as. $\times$ 7 or 70as.

$\therefore$ the number reqd. $= \dfrac{\text{Rs. } 30 \text{ 6as}}{73 \text{ as}} = \dfrac{630}{70} = 9$

$\sqrt{(531\cdot0650000000)} = 23\cdot04484 \ldots$

3 The area of the floor $= 9\frac{1}{3}$ ft $\times 9\frac{1}{3}$ ft $= \dfrac{28 \times 28}{3 \times 3}$ sq ft

$\therefore$ the length of matting $= \dfrac{28 \times 28}{3 \times 3} \times \frac{3}{7}$ ft $= \frac{112}{3}$ ft $= 37\frac{1}{3}$ ft

$= 12$ yds 1 ft 4 in.

The price $= 2\frac{1}{4}$as $\times \frac{112}{9} = \text{Rs } \left(\frac{9}{4} \times \frac{112}{9} \times \frac{1}{16}\right)$

$= \text{Rs } \frac{7}{4} = \text{Re } 1 \text{ 12as.}$

(a)

	£ s d.
2 qrs. = $\frac{1}{2}$ of 1 cwt	6 7 8 = value of 1 cwt
	11
	70 4 4 = value of 11 cwts
	3
	210 13 0 = value of 33 cwt.
	3 3 10 = value of 2 qrs.
1 qr = $\frac{1}{2}$ of 2 qrs	1 11 11 = value of 1 qr
7 lbs = $\frac{1}{4}$ of 1 qr	7 $11\frac{3}{4}$ = value of 7lbs
	£215 16s $8\frac{3}{4}d$ = value of 33 cwts 3qrs 7lbs.

4. After 30 days, there remain 3200 men, and 160 days.
3200 men   4000 men   160 days x

$$\therefore x = \frac{4000 \times 160}{3200} = 200 \text{ days.}$$

5. Interest = Rs 1725 12as − Rs 1462 8as = Rs 263 4as
∴ interest for 1 year = $\frac{1}{4}$ of Rs $263\frac{1}{4}$ = Rs $65\frac{13}{16}$.
Rs. $1462\frac{1}{2}$ Rs 100   Rs $65\frac{13}{16}$ the rate of interest

$$\therefore \text{the rate} = \text{Rs. } \frac{100 \times 1053 \times 2}{2925 \times 16} = \text{Rs. } \frac{9}{2} = 4\frac{1}{2} \text{ p c.}$$

6 (i) The expression

$$= 3a - [a + b - 2\{a + b + c - a + b - c + d\} + a]$$
$$= 3a - a - b + 4b + 2d - a = a - b + 4b + 2d = a + 3b + 2d.$$

(ii) Let $x - y = a$ and $x + y = b$ then $a + b = 2x$
The expression $= a^3 + b^3 + 3a^2b + 3b^2a$
$= (a + b)^3 = (2x)^3 = 8x^3$

(iii) The expression

$$= \frac{\dfrac{a^2 + ab - a^2 + ab}{a^2 - b^2}}{\dfrac{ab + b^2 - ab + b^2}{a^2 - b^2}} \div \frac{\dfrac{2(a^2 + b^2)}{a^2 - b^2}}{\dfrac{4ab}{a^2 - b^2}} \times \frac{a^2}{a^2 + b^2}$$

$$= \frac{2ab}{2b^2} + \frac{4ab}{2(a^2 + b^2)} \times \frac{a^2}{a^2 + b^4} = \frac{2a^4}{(a^2 + b^2)^2}.$$

7.
$$
\begin{array}{r|l}
2x^3 + 3x^2 + 2x - 2 & 4x^4 - 2x^3 + 2x - 1 \quad (2x - 4 \\
& 4x^4 + 6x^3 + 4x^2 - 4x \\
\hline
& -8x^3 - 4x^2 + 6x - 1 \\
& -8x^3 - 12x^2 - 8x + 8 \\
\hline
& 8x^2 + 14x - 9
\end{array}
$$

$$
\begin{array}{l}
8x^2+14x-9)\ 8x^3+12x^2+8x-8\ (\ x-1 \\
\qquad\qquad\qquad\ 8x^3+14x^2-9x \\
\hline
\qquad\qquad\qquad\quad -2x^2+17x-8 \\
\qquad\qquad\qquad\qquad\qquad\qquad\ \ 4 \\
\hline
\qquad\qquad\qquad\quad -8x^2+68x-32 \\
\qquad\qquad\qquad\quad -8x^2-14x+9 \\
\hline
\qquad\qquad\qquad\qquad 41\ \underline{)82x-41} \\
\qquad\qquad\qquad\qquad\qquad\quad 2x-1
\end{array}
$$

$$
\begin{array}{l}
2x-1)\ 8x^2+14x-9\ (\ 4x+9 \\
\qquad\quad\ 8x^2-4x \\
\hline
\qquad\qquad\quad 18x-9 \\
\qquad\qquad\quad 18x-9 \\
\hline
\end{array}
$$

$\therefore$ $2x-1$ is the G. C M.

(a) $(x^2-x+1)\left(\frac{1}{x^2}+\frac{1}{x}+1\right)=1-\frac{1}{x}+\frac{1}{x^2}+x-1+\frac{1}{x}$

$$+x^2-x+1=\frac{1}{x^4}+x^2+1$$

(b)

$$
\begin{array}{l}
4-4c+c^2+2b-bc+\frac{1}{4}b^2\ (\ 2-c+\frac{1}{2}b=\text{sq root} \\
4 \\
\hline
4-c\ \ \big|\ -4c+c^2 \\
\qquad\ \ \big|\ -4c+c^2 \\
\hline
4-2c+\frac{1}{2}b\ \big|\ 2b-bc+\frac{1}{4}b^2 \\
\qquad\qquad\quad \big|\ 2b-bc+\frac{1}{4}b^2 \\
\hline
\end{array}
$$

8. (1) $\frac{2x-13}{9}-\frac{x-1}{11}=\frac{x}{8}+\frac{x}{7}-9.$

Clearing of fractions, we get

$1232x-8008-504x+504=693x+792x-49896$

or $728x-7504=1485x-49896$

$\therefore$ $757x=42392$, $\qquad$ $\therefore$ $x=56.$

(2) $\frac{x+y}{2}+\frac{3x-5y}{4}=2$ ....(1), $\frac{x}{14}+\frac{y}{18}=1$......(2)

Multiply (1) by 4 and (2) by 126

$\therefore$ $2x+2y+3x-5y=8$ or $5x-3y=8$.... (3)

and $9x+7y=126$ . ..... (4)

Multiply (3) by 9, and (4) by 5

then $\left.\begin{array}{l} 45x-27y=72 \\ \text{and } 45x+35y=630 \end{array}\right\}$ By subtraction, $62y=558$, $\therefore y=9$.

From (3) $5x=3y+8=35$, $\therefore x=7$.

9. Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\lambda$, then $a=b\lambda$, $c=d\lambda$, $e=f\lambda$

(i) $\frac{ma+nb}{mc+nd}=\frac{mb\lambda+nb}{md\lambda+nd}=\frac{b(m\lambda+n)}{d(m\lambda+n)}$

$=\frac{b}{d}=\frac{b\,bd\lambda}{d\,bd\lambda}=\frac{b^2d\lambda}{d^2b\lambda}=\frac{b^2c}{d^2a}$.

(ii) $\frac{a^2+c^2+e^2}{b^2+d^2+f^2}=\frac{b^2\lambda^2+d^2\lambda^2+f^2\lambda^2}{b^2+d^2+f^2}=\frac{\lambda^2(b^2+d^2+f^2)}{b^2+d^2+f^2}$

$=\lambda^2=\frac{\lambda^2df}{df}=\frac{ce}{df}$

or $a^2+c^2+e^2 : b^2+d^2+f^2 :: ce : df$.

10 Let x = number of days required.

Then A's daily work $=\frac{1}{9}$, B's $=\frac{1}{18}$

and C's $=\frac{3}{4}$ of $\frac{1}{9}=\frac{1}{12}$

By the question,

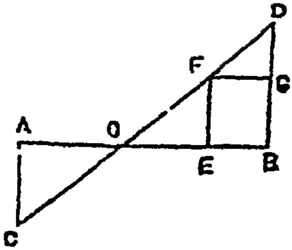
$(\frac{1}{9}+\frac{1}{18}+\frac{1}{12})x=1$, or $\frac{x}{4}=1$.

$\therefore x=4$ days.

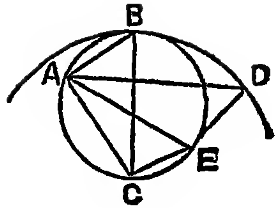
1876.—AFTERNOON.

1. Euclid I 42.
2. Euclid II 12
3. Euclid III 17.
4. Euclid III 35, third, case.
5. Euclid IV. 9

6 Bisect OB in E.
Draw EF $\perp$ AB meeting ODF.
Draw FG $\parallel$ AB.
$\because$ OB=2OA=2OE, $\therefore$ OA=OE;
but EB=FG $\therefore$ OE=FG.
Again $\because$ AO=OE,
$\angle$AOC=$\angle$FOE (I. 15)
also $\angle$OAC=$\angle$OEF (I 29)
$\therefore$ CO=OF (I 26)
Again $\because$ OE=FG
$\angle$FOE=$\angle$DFG, OFE=$\angle$FDG (I. 29)



$\therefore$ OF=FD (I 26)
$\therefore$ OD=2OF=2OC.



7. ∵ BAC is a rt ∠
∴ BAC is a ½ ⊙, (III 31)
∴ BC is a diameter

A tangent drawn to the original ⊙ is also a tangent to the ⊙BAC

∴ ⊙ ABC touches the original ⊙

Take any other pt D in the circum of the original ⊙

and join AD, CD cutting the smaller ⊙ in E.

Join AE

Again ∵ ∠ABC=∠AEC (III 21)
but ∠AEC>∠ADC, (I 16)
∴ ∠ABC>∠ADC.

8 See Hall and Stevens, First Book.

1877.—MORNING.

Examiners,— { Rev G. H Rouse
Mr A. M. Nash, M A.

Simplify $\dfrac{\frac{1}{4}+\frac{2}{9}}{4-\frac{5}{9}\text{ of }5\frac{1}{2}}-\dfrac{\frac{2}{5}+\frac{1}{4}}{\frac{7}{10}\text{ of }4\frac{1}{2}-2\frac{1}{2}}$,

and find the value of $\frac{4}{27}$ of 16*s* 11*d* + $\frac{3}{35}$ of £1 1*s* 4*d*. + £3 2$\dot{3}$

2. Find by Practice the value of $739\frac{1}{2}$ maunds of sugar at Rs. 1,231 4as per hundred maunds

3 Find the discount on £453 15*s* due 6 years hence at $3\frac{1}{2}$ per cent

4. A man sells 3 per cent. stock at 75, and invests the proceeds in 5 per cents; at what rate must he buy them in order that his income may be the same as before

5. If 7 men and 5 boys can reap 168 acres in 18 days, how many days will 15 men and 5 boys take to reap 700 acres, one man being able to do three times as much work as a boy?

6. In a rectangular area, 100 yards long and 50 yards broad, there are two paths crossing one another, each parallel to one side of the rectangle, and each 4 yards broad Find the cost of paving the area with stone at 12 annas per square yard, and of covering the paths with gravel at 6 annas per square yard

7 Simplify $\dfrac{x+2}{1+x+x^2}-\dfrac{x-2}{1-x+x^2}-\dfrac{2x^2-4}{1-x^2+x^4}$;

multiply together $a+b+c$, $b+c-a$, $c+a-b$, $a+b-c$;

and divide $x^4+x^3-24x^2-35x+57$ by x^2+2x-3.

8 Solve the equations —

(1) $\frac{2x-3}{6}+\frac{3x-8}{11}=\frac{4x+15}{33}+\frac{1}{2}$

(2) $2(x+2)=1+\sqrt{4x^2+9x+14}$.

(3) $3x+4y-11=0, \quad 5y-6z+8=0, \quad 7z-3x-13=0.$

9. Find the Greatest Common Measure of $x^4+x^3-11x^2-9x+18$, and $x^4-10x^3+35x^2-50x+24$

10. Find the first four terms of the square root of a^2+x^2, and from the result deduce the square root of 101 correct to six places of decimals

11. If $a : b :: c : d$, prove that $a^2+c^2 : b^2+d^2 :: ac : bd$.

12 A and B together can do a piece of work in 15 days A can do it alone in 24 days how long would B take to do it alone?

13 Two passengers have together 5 cwt of luggage, and are charged for the excess above the weight allowed 5s 2d and 9s 10d. respectively, but if the luggage had all belonged to one of them he would have been charged 19s 2d How much luggage is each passenger allowed to carry free of charge and how much luggage had each passenger?

1877.—AFTERNOON.

Examiners,—{REV J. P ASHTON, M A; MR E D. ARCHIBALD, M A.}

1 Define a square, a rhomboid, a straight line touching a circle, and an angle in a segment of a circle

2 If two triangles have two sides of the one equal to two sides of the other, each to each, but the base of the one, greater than the base of the other, the angle contained by the sides of that which has the greater base shall be greater than the corresponding angle of the other

3 If a straight line be divided into any two parts, the squares on the whole line and one of the parts are equal to twice the rectangle contained by the whole and that part, together with the square on the other part Prove this, and show to what algebraic formula it corresponds.

4 Prove that the straight line drawn at right angles to the diameter of a circle at its extremity touches the circle, according to your definition of a tangent.

5 Draw a common tangent to two given circles

6 All the interior angles of a rectilineal figure together with four right angles are equal to twice as many right angles as the figure has sides Prove this, and say what regular polygon has each of its angles equal to nine-tenths of two right angles

7. Prove that the side of a regular hexagon is equal to the radius of the circumscribed circle.

SOLUTIONS.

1877.—MORNING.

$$1.\quad \frac{\frac{1}{4}+\frac{2}{9}}{4-\frac{5}{9}\text{ of }5\frac{1}{2}} \div \frac{\frac{2}{5}+\frac{1}{4}}{\frac{7}{10}\text{ of }4\frac{1}{2}-2\frac{1}{2}} = \frac{\frac{9+8}{36}}{4-\frac{5}{9}\text{ of }\frac{11}{2}} \div \frac{\frac{8+5}{20}}{\frac{7}{10}\text{ of }\frac{9}{2}-2\frac{1}{2}}$$

$$= \frac{\frac{17}{36}}{4-\frac{55}{18}} \div \frac{\frac{13}{20}}{\frac{63}{20}-\frac{5}{2}} = \frac{\frac{17}{36}}{\frac{72-55}{18}} \div \frac{\frac{13}{20}}{\frac{63-50}{20}} = \frac{17}{36}\times\frac{18}{17}$$

$$\div\left(\frac{13}{20}\times\frac{20}{13}\right) = \frac{1}{2}\div 1 = \frac{1}{2}.$$

(*a*) $\frac{4}{29}$ of 16*s* 11*d*. + $\frac{3}{32}$ of £1 1*s* 4*d* + £3·23

= $\frac{4}{29}$ of 203*d*. + $\frac{3}{32}$ of $21\frac{1}{3}$*s*. + £3$\frac{7}{30}$

= 4 × 7*d* + $\frac{3}{32} \times \frac{64}{3}$*s*. + £3 4*s* 8*d*

= 28*d*. + 2*s* + £3 4*s*. 8*d*

= 2*s* 4*d* + 2*s*. + £3 4*s* 8*d*. = £3 9*s*

2. The value of 100 mds = Rs 1231 4*s*

∴ The value of 1 md. = Rs 12 5as

	Rs	as.	pie	
4as = $\frac{1}{4}$ of Re. 1	739	8	0	= value at Rs 1 per md.
			4	
	2958	0	0	= value at Rs 4 per md.
			3	
	8874	0	0	= value at Rs 12 per md.
1 an. = $\frac{1}{4}$ of 4as	184	14	0	= value at As 4 per md
	46	3	6	= value at As 1 per md.
	Rs 9105	1a.	6p.	= value at Rs 12 5as.

3. The int. on £100 at $3\frac{1}{2}$ p. c. for 6 yrs = £$(6\times\frac{7}{2})$ = £21.

∴ £121 : £$453\frac{3}{4}$ :: £21 : required discount.

∴ the discount reqd = £$\frac{21\times1815}{4\times121}$ = £$\frac{315}{4}$ = £78 15*s*.

4. 3 p o 5 p c 75 x

$$\therefore x=\frac{75\times 5}{3}=125$$

5 7 men $=7\times 3$ boys $=21$ boys and 15 men $=15\times 3$ boys $=45$ boys (21+5) boys $\times 18d$ (45+5) boys $\times$ xd. . 168ac ; 700ac.

$$\therefore x=\frac{26\times 18\times 700}{50\times 168}\text{ days}=39\text{ days.}$$

6. The area of the paths $=4\times 100$ sq. yds $+(50-4)\times 4$sq yds
$=(100+46)\times 4$ sq $=146\times 4$ sq yds $=584$ sq yds.
The area of the rectangle $=100\times 50$ sq yds $=5000$ sq yds.
The area of the part to be paved $=(5000-584)$ sq yds.
$=4416$ sq. yds
$\therefore$ the cost of paving $=4416\times 12$as $=3312$ Rs.
and that of gravelling $=584\times 6$as $=$ Rs 219.

7. Ans. $=\frac{(x+2)(1-x+x^2)-(x-2)(1+x+x^2)}{1+x^2+x^4}-\frac{2x^2-4}{1-x^2+x^4}$

$$=\frac{2x^2+4}{1+x^2+x^4}-\frac{2x^2-4}{1-x^2+x^4}$$

$$=\frac{(2x^2+4)(1-x^2+x^4)-(2x^2-4)(1+x^2+x^4)}{1+x^4+x^8}$$

$$=\frac{8+4x^4}{1+x^4+x^8}=\frac{4(2+x^4)}{1+x^4+x^8}$$

(a) $(a+b+c)(a+b-c)\{c+(b-a)\}\{c-(b-a)\}$
$=(a^2+b^2+2ab-c^2)(c^2-b^2-a^2+2ab)$
$=\{2ab+(a^2+b^2-c^2)\}\{2ab-(a^2+b^2-c^2)\}$
$=4a^2b^2-(a^4+b^4+c^4+2a^2b^2-2a^2c^2-2b^2c^2)$
$=2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4.$

(b) $x^2+2x-3)\ x^4+x^3-24x^2-35x+57\ (x^2-x-19.$

$$\begin{array}{l}
x^4+2x^3-3x^2 \\
\hline
-x^3-21x^2-35x \\
-x^3-2x^2+3x \\
\hline
-19x^2-38x+57 \\
-19x^2-38x+57 \\
\hline
\end{array}$$

8. (1) $\dfrac{2x-3}{6}+\dfrac{3x-8}{11}=\dfrac{4x+15}{33}+\frac{1}{2}$

Clearing of fractions, we get

$22x-33+18x-48=8x+30+33$

or $40x-81=8x+63$, $\therefore\ 32x=144$ $\therefore\ x=4\frac{1}{2}$

(2) $2(x+2)=1+\sqrt{(4x^2+9x+14)}$

transposing $2x+3=\sqrt{(4x^2+9x+14)}$

squaring $4x^2+12x+9=4x^2+9x+14$,

$\therefore\ 3x=5$ $\therefore\ x=1\frac{2}{3}$

(3) $3x+4y=11$. (1), $5y-6z=-8$ . . (2)

$7z-8x=13$ . . . . . (3)

Multiply (1) by 8, and (3) by 3,

then $\left.\begin{array}{l}24x+32y=88\\ 21z-24x=39\end{array}\right\}$ By addition $32y+21z=127$.. (4)

Multiply (4) by 5 and (2) by 32,

$\left.\begin{array}{l}160y+105z=635\\ 160y-192z=-256\end{array}\right\}$ By subtraction $297z=891$, $\therefore\ z=3$

From (2) $5y=6z-8=10$, $\therefore\ y=2$

From (1) $3x=11-4y=3$, $\therefore\ x=1$

9.
$$x^4+x^3-11x^2-9x+18\,\big)\,x^4-10x^3+35x^2-50x+24\,\big(1$$
$$x^4+x^3-11x^2-9x+18$$
$$-1\,\big|\,-11x^3+46x^2-41x+6$$

$$11x^3-46x^2+41x-6\,\big)\,11x^4+11x^3-121x^2-99x+198\,\big(x$$
$$11x^4-46x^3+41x^2-6x$$
$$57x^3-162x^2-93x+198$$
$$11$$
$$627x^3-1782x^2-1023x+2178\,\big(57$$
$$627x^3-2622x^2+2337x-342$$

$$840\,\big)\,840x^2-3360x+2520$$
$$x^2-4x+3$$

$$x^2-4x+3\big)\,11x^3-46x^2+41x-6\,\big(11x-2$$
$$\underline{11x^3-44x^2+33x}$$
$$-2x^2+8x-6$$
$$\underline{-2x^2+8x-6}$$

$\therefore$ G C M. $=x^2-4x+3$

10 $\quad a^2+x^2\big)\, a+\dfrac{x^2}{2a}-\dfrac{x^4}{8a^3}+\dfrac{x^6}{16a^5}-$&c sq root.

$$a^2$$

$$2a+\frac{x^2}{2a}\Big|\; x^2$$
$$\underline{x^2+\frac{x^4}{4a^2}}$$

$$2a+\frac{x^2}{a}-\frac{x^4}{8a^3}\Big|\; -\frac{x^4}{4a^2}$$
$$\underline{-\frac{x^4}{4a^2}-\frac{x^6}{8a^4}+\frac{x^8}{64a^6}}$$

$$2a+\frac{x^2}{a}-\frac{x^4}{4a^3}\Big|\;\frac{x^6}{8a^4}-\frac{x^8}{64a^6}$$
$$\frac{x^6}{8a^4}$$

(a) Again $101=(10)^2+(1)^2$. Now suppose $a=10$ and $x=1$

Then square root $=10+\dfrac{1}{2\times 10}-\dfrac{1}{8\times 10^3}+\dfrac{1}{16\times 10^5}$

$=10+\cdot05-\cdot000125=10\cdot049875.$

11. Since $\dfrac{a}{b}=\dfrac{c}{d}$, $\therefore \dfrac{a}{c}=\dfrac{b}{d}$, $\therefore \dfrac{a^2}{c^2}=\dfrac{b^2}{d^2}$

$\therefore \dfrac{a^2+c^2}{c^2}=\dfrac{b^2+d^2}{d^2}$ $\quad\therefore \dfrac{a^2+c^2}{b^2+d^2}=\dfrac{c^2}{d^2}=\dfrac{c}{d}\dfrac{c}{d}=\dfrac{a}{b}\dfrac{c}{d}=\dfrac{ac}{bd}$

$\therefore a^2+c^2 \quad b^2+d^2 \quad ac \cdot bd.$

12 Let $x=$B's time

A and B in one day can do $\frac{1}{15}$ of the work

A ... ... $\frac{1}{24}$...

$\therefore$ By the question

$\dfrac{1}{x}+\frac{1}{24}=\frac{1}{15}$ or $\dfrac{1}{x}=\frac{1}{15}-\frac{1}{24}=\frac{1}{40}$. $\quad\therefore x=40$ days.

13 Let x=luggage of first passenger in cwt
then $5-x$=luggage of second passenger in cwt.
and y=luggage allowed free in cwt.
Since charge of $(5-y)$ cwt of luggage is 19s 2d.
... . $(5-2y)$ ... . 15s.
$\therefore$ charge of y is 4s. 2d
... .5 cwt. is 19s 2d + 4s 2d = 23s. 4d.
....1 cwt.... ... 4s. 8d
$\therefore y = 4s.\ 2d \div 4s\ 8d = \frac{50}{56} = \frac{25}{28}$ cwt.

By the question,

since 5s 2d = 62d and 9s 10d = 118d

$$\frac{230(x-y)}{(5-y)} = 62. \text{ ....(1)}$$

$230x = 168y + 310 = 460, \qquad \therefore x = 2$

Hence luggage of 1st is 2 cwt and of 2nd 3 cwt

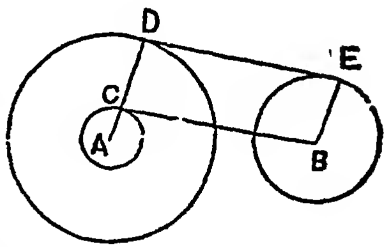
1877.—AFTERNOON.

1. Euclid I Defs. 32 and 35, Euclid III Defs. 2 and 6.
2. Euclid I. 24.
3. Euclid II 7

Let AB be represented by a, and CB, one of the parts by b.
Then $a-b$ represents AC.
By the prop $AB^2+BC^2=2AB\ BC+AC^2$
we have $a^2+b^2=2ab+(a-b^2)$ or $(a-b)^2=a^2+b^2-2ab$.

4. Euclid. 16.

5 Let A be the centre of the large ⊙ and B the centre of the smaller ⊙



With A as centre and with radius equal to the difference of the radii of the ⊙s, desc a ⊙
From B draw BC touching this ⊙ at C
Join AC and prod. it to meet the ⊙ at D
From B draw BE⊥CB meeting the ⊙ at E
Join DE ; DE is the required tangent
Then ∠DCB is a rt ∠ ; (III. 18)
but also CBE is a rt ∠ and DC=BE ;
$\therefore$ DCBE is a rectangle
$\therefore$ DE⊥AD and BE ;
Wherefore DE touches both ⊙s (III 16).

6 Euclid I 32, Cor I
Let n denote the number of sides as well as the angles of the regular polygon ; then each ∠ of this reg. polygon$=\left(\frac{2n-4}{n}\right)$ rt.∠s.